STANFORD

Lecture 6

Spatial Modulation & Wireless Examples

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Announcements & Agenda

- Announcements
  - Problem Set #3 is due Wed April 26 at 17:00
  - Readings 4.4, 4.6, 4.7
  - Midterm is 2 weeks from today, in class - May 3 (open book, notes, laptop, internet)
  - PS1 Solutions at web site (link to canvas) – PS2 when we get all turned in.
  - Chap4r0 is at canvas – no changes since class start. Current Chapter 4 (web site) has edits in Section 4.4 loading example and in Section 4.6 (some mislabeled figure text)
- Agenda
  - Spatial Modulation
  - Wireless Examples
    - Wi-Fi
    - Digital Video Broadcast
    - Cellular

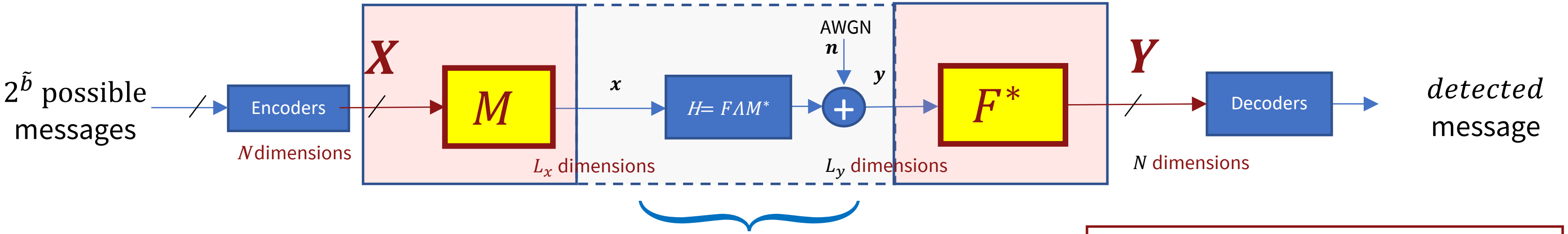
Spatial Modulation

“Space-Time Block Codes (STBC)”

Spatial Vector Coding is Optimal

- The symbols have crosstalk or inter-spatial-dimension interference
- However, symbols otherwise have no intersymbol interference, $\nu = 0$ (or ISI is separately handled)
  - With DMT/OFDM on all crosstalking channels

- Spatial Vector-Code channel partitioning remains for each tone n



$$\boxed{y_l = \lambda_l \cdot x_l + n_l}$$

- Parallel Channels
  - $l = 1, \ldots, L \leq \wp_H \leq \min\left(L_x, L_y\right)$

$$\boxed{SNR_l = \frac{\lambda_l^2 \cdot \bar{\mathcal{E}}_l}{\sigma^2}}$$

$$\mathbb{E}[\boldsymbol{n} \cdot \boldsymbol{n}^*] = R_{\boldsymbol{nn}} = R_{\boldsymbol{nn}}^{1/2} \cdot R_{\boldsymbol{nn}}^{*/2}$$

Noise-Equivalent Channel

$$\boldsymbol{y} \leftarrow R_{nn}^{-1/2} \boldsymbol{y} = \left(R_{nn}^{-1/2} \cdot H\right) \cdot \boldsymbol{x} + \tilde{\boldsymbol{n}}$$

Spatial Modulator (Matrix)

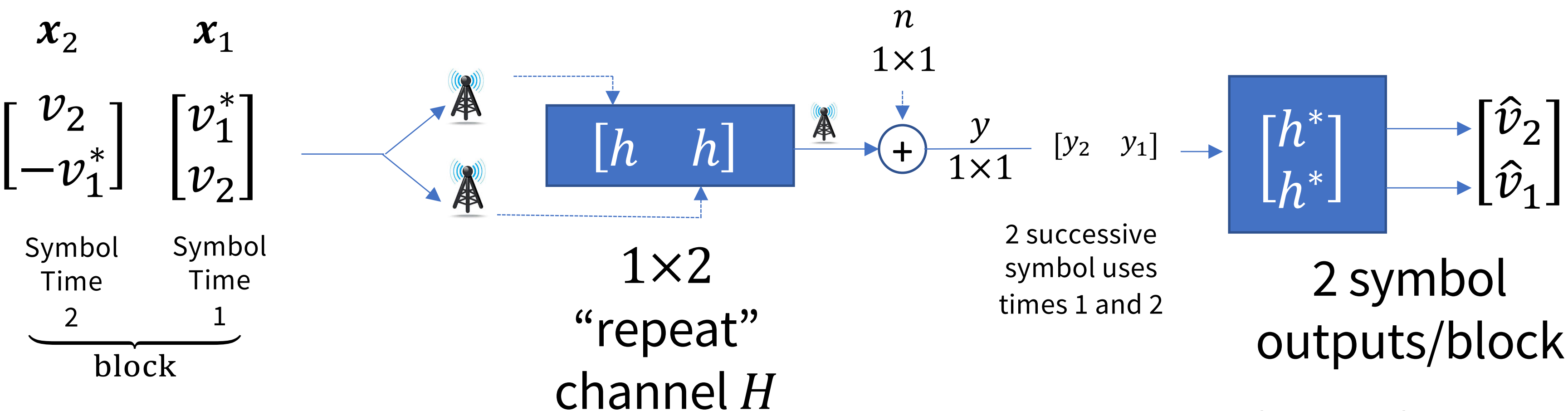
- Best matrix modulator is M for any energy distribution

- For this choice of M, it follows easily that the unbiased MMSE receiver matrix is F^*

- In practice, it may be difficult for the transmitter to know M, since only the receiver can measure it.
  - Need reverse control channel
  - May change by the time it is reversed communicated

- This leads to a variety of spatial approximations, among them Space-Time Block Codes (STBC)

Alamouti's Code (1998)



$$\underbrace{\begin{array}{cc} \boldsymbol{x}_2 & \boldsymbol{x}_1 \\ \begin{bmatrix} v_2 \\ -v_1^* \end{bmatrix} & \begin{bmatrix} v_1^* \\ v_2 \end{bmatrix} \\ \text{Symbol Time } 2 & \text{Symbol Time } 1 \end{array}}_{\text{block}}$$

n 1×1

$[h \quad h]$

y 1×1

$[y_2 \quad y_1]$

$\begin{bmatrix} h^* \\ h^* \end{bmatrix}$

$\begin{bmatrix} \hat{v}_2 \\ \hat{v}_1 \end{bmatrix}$

1×2 "repeat" channel H

2 successive symbol uses times 1 and 2

2 symbol outputs/block

4 complex input dimensions
(8 real dimensions)
some symbol values repeated

2 complex output dimensions
(4 real dimensions)
no repeats

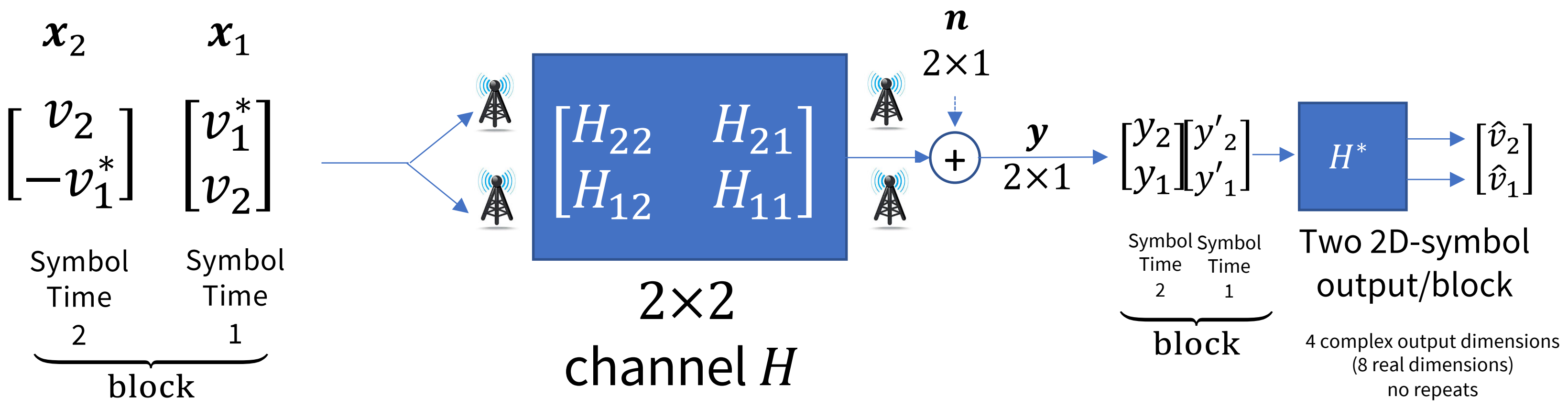
- The trivial 1 x 2 "repeat" channel is the ONLY channel for which Alamouti's code is Vector Coding
  - It obviously does better by 3 dB than a single channel use $2 \cdot [h]^2$ instead of $[h]^2$
- Basically, two line-of-sight paths to the same single-antenna receiver that must have the same gain

2x2 H ?? Some matrix basics

- Symmetric matrices $J_2 \triangleq \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad J_1 \triangleq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad R \triangleq \begin{bmatrix} a & c \\ c & b \end{bmatrix}$
- Identity $J_2 \cdot R \cdot J_2 + J_1 \cdot R \cdot J_1 = \begin{bmatrix} a+b & 0 \\ 0 & a+b \end{bmatrix}$
- Fixed modulator $\underbrace{\boldsymbol{x}}_{4\times1} = \underbrace{\begin{bmatrix} J_2 \\ J_1 \end{bmatrix}}_{4\times2} \cdot \underbrace{\begin{bmatrix} v_2 \\ v_1 \end{bmatrix}}_{\boldsymbol{v},2\times1} \quad \boldsymbol{y} = \underbrace{\begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}}_{\boldsymbol{H}} \cdot \boldsymbol{x} + \boldsymbol{n} = \begin{bmatrix} H \cdot J_2 \\ H \cdot J_1 \end{bmatrix} \cdot \boldsymbol{v} + \boldsymbol{n}$
- Forward channel autocorrelation is the block diagonal $\boldsymbol{R_f} = \boldsymbol{H}^* \cdot \boldsymbol{H} = \begin{bmatrix} R_f & 0 \\ 0 & R_f \end{bmatrix}$
- Each 2×2 sub-block diagonal is

$$\begin{aligned} R_f &= J_2^* \cdot H^* \cdot H \cdot J_2 + J_1 \cdot H^* \cdot H \cdot J_1 \\ &= \begin{bmatrix} H_{22}^2 + H_{21}^2 + H_{12}^2 + H_{11}^2 & 0 \\ 0 & H_{22}^2 + H_{21}^2 + H_{12}^2 + H_{11}^2 \end{bmatrix} \end{aligned}$$

SUBOPTIMAL Alamouti's Code Use, 2 x 2 channel



$\boldsymbol{x}_2$ $\boldsymbol{x}_1$

$\begin{bmatrix} v_2 \\ -v_1^* \end{bmatrix}$ $\begin{bmatrix} v_1^* \\ v_2 \end{bmatrix}$

Symbol Time 2

Symbol Time 1

$\underbrace{\qquad\qquad}_{\text{block}}$

$\begin{bmatrix} H_{22} & H_{21} \\ H_{12} & H_{11} \end{bmatrix}$

2×2 channel H

$\boldsymbol{n}$ 2×1

$\boldsymbol{y}$ 2×1

$\begin{bmatrix} y_2 \\ y_1 \end{bmatrix}\begin{bmatrix} y'_2 \\ y'_1 \end{bmatrix}$

Symbol Time 2

Symbol Time 1

$\underbrace{\qquad\qquad}_{\text{block}}$

H^*

$\begin{bmatrix} \hat{v}_2 \\ \hat{v}_1 \end{bmatrix}$

Two 2D-symbol output/block

4 complex output dimensions
(8 real dimensions)
no repeats

4 complex input dimensions
(8 real dimensions)
some symbol values repeated

$$\begin{aligned} R_f &= J_2^* \cdot H^* \cdot H \cdot J_2 + J_1 \cdot H^* \cdot H \cdot J_1 \\ &= \begin{bmatrix} H_{22}^2 + H_{21}^2 + H_{12}^2 + H_{11}^2 & 0 \\ 0 & H_{22}^2 + H_{21}^2 + H_{12}^2 + H_{11}^2 \end{bmatrix} \end{aligned}$$

$$\gamma_{repeat} = \frac{1}{2}$$

- The transmitter remains channel independent, and the receiver remains simple matched matrix
- The SNR is higher, BUT the data rate is halved. Vector coding is better.

STBC Codebooks

- Variety of different sizes, but rate loss $\gamma_{repeat} \geq$ ½ (with more than one receive antenna)
- They perform a lot worse than vector-coding in practice

Example: $\bar{\mathcal{E}}_x = 1\,;\ \sigma^2 = 1$

```
>> H
H =
    1.0000    0.9000
    0.8000    1.0000
>> [F,L,M]=svd(H)
F =
   -0.7245   -0.6892
   -0.6892    0.7245
L =
    1.8512         0
         0    0.1512
M =
   -0.6892   -0.7245
   -0.7245    0.6892
bvc=log2(det(L^2+eye(2))) = 2.1790
bstbc=log2(sqrt(norm(H,'fro')^2+1)) = 1.0769
10*log10((2^bvc-1)/(2^bstbc - 1))  =    5.0245 dB
```

Sqrt same as ½ in front,
must double data rate

At higher data rates, the VC advantage grows;

Wireless Raleigh fading

$$\langle P_e \rangle \propto (SNR)^{d_{free}}$$

But still large VC advantage

Wireless field use has STBC as “option”

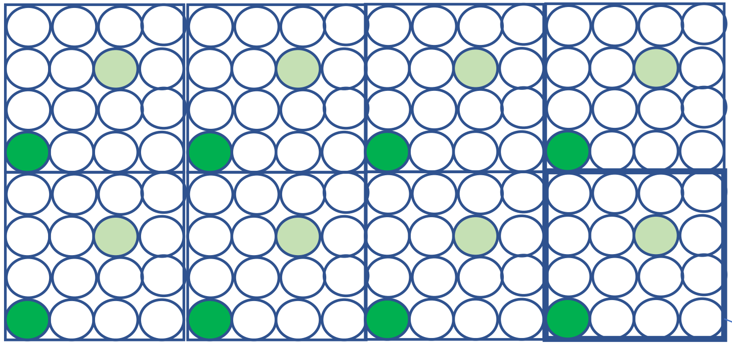
- STBC will essentially repeat symbols, so if the channel is very poor and the codes (nonideal) are fixed, then it can increase SNR and create a reliable link at significant data-rate loss

- This may be acceptable if no connection is otherwise reliably possible

- For ideal outer codes ($\Gamma = 0$ dB), there is no such repeat-energy advantage theoretically

- Most wireless systems (Cellular and Wi-Fi) allow use of STBC as an option, but also permit use of vector-coding approximations to avoid the STBC losses.

Cellular: Type 1 Precoders



$L_{x,2} \cdot O_2$ choices

$L_{x,1} \cdot O_1$ choices

O_i is oversampling factor (angle)

$L_x = L_{x,1} \cdot L_{x,2}$ = number used antennas

Each $O_1 \cdot O_2$ rectangle is "subarray" of choices

$Q_{L_{x,1} \cdot O_1} \otimes Q_{L_{x,2} \cdot O_2}$ = precoder factor is cartesian product of horizontal and vertical DFTs

$Q_{L_{x,i} \cdot O_i}$ is a DFT of size in subscript

- The precoder matrix A stacks the columns of $Q_{L_{x,1} \cdot O_1} \otimes Q_{L_{x,2} \cdot O_2}$ so each is L_x x 1
- These columns can be multiplied by $1, j, -1, -j$ for different streams to form an L_x x ss matrix

Cellular: Type 2 Precoders

- Use the columns of Type 1, call them $\boldsymbol{w}_{l,ss}$ and weight them (not necessarily unity gain)
- Weights $a_{l,ss}$ can have amplitudes $2^{-i/2}$ for $i \in \{0, \dots, 6\}$ and phases $\frac{2\pi}{j}$ $j \in \{0, \dots, L_x - 1\}$

$$W_{ss} = \sum_{\ell=1}^{L_x} \boldsymbol{w}_{\ell,ss} \cdot a_{\ell,ss}$$

- Receiver will also send back indices i and j along with indices for the Type 1 $\boldsymbol{w}_{l,ss}$
- Factors $a_{l,ss}$ are often split into wideband slower-varying factor and narrow band (frequency-dependent) faster-varying factor.

Wi-Fi Spatial Modulators

- Much simplier than cellular
  - Allows Alamouti on 2x2 antenna mix channels
    - And will repeat them for several 2x2 groupings

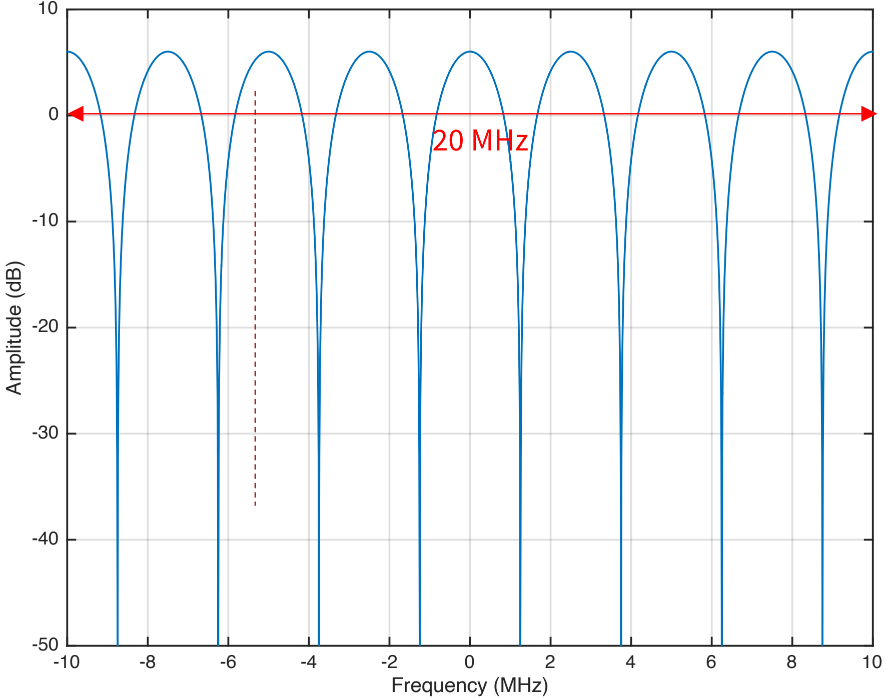
- Specifies a series of 2x2 rotations (complex, 2 angles) and the spatial antenna indices to which they apply to attempt to approximate M (or exactly realize it if there are enough sent).

Wireless Use-Case Examples

Section 4.7

Wi-Fi Channel Variability/Range

- W-Fi Channels are 20 MHz wide (T’ = 50 ns)
- Example Channel: 1 extra path with delay = 200 ns = 60 m @ speed of light
- Sometimes higher gain, but
- Roughly ¼ - ⅓ of tones’ gains are below the previous single path threshold (red line)
- Code roughly needs at least ¼ parity
  - To recover ¼ lost information
- Thus ¾, ⅔, and ½ code rates of interest



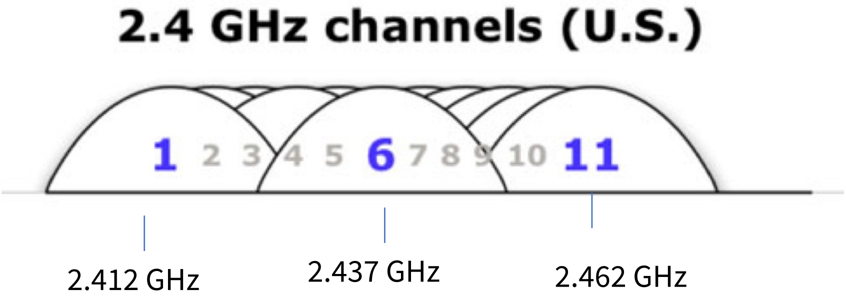
Coded OFDM

Wi-Fi’s 20-320 MHz Channels

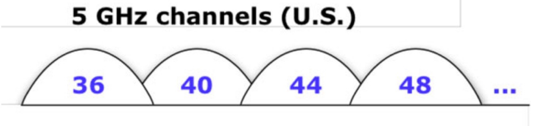
11g, n

“Wi-Fi 4”

n has $L \leq 4$



Only 1, 6, 11 avoid Overlap, so really 3 channels



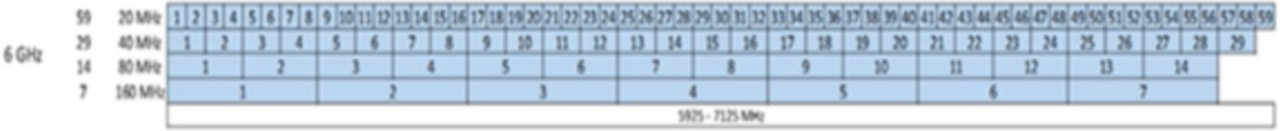
$(\max L)$

Wi-Fi 4 – 11a (1)
W0Fi 5 – 11ac (4)
Wi-Fi 6 - 11ax (8)
Wi-Fi 7 – 11be (16)

$f_c = 5108 + i \cdot 20, i = 4m, m = 0, \ldots 7, 16 \ldots 26$ or $5745 + i \cdot 20, i = 4m, m = 0, \ldots 4$

5 GHz band: used by 802.11n (Wi-Fi 4), 802.11ac (Wi-Fi 5), 802.11ax (Wi-Fi 6), and to a lesser extent by cellular in unlicensed (LWA, MultiFire)

Up to 71 channels (20 MHz)



WiFi 7 allows
$m = 5$

- Unlicensed – so multiple systems can collide – detect collect, retransmit after random wait

12 GHz carrier frequencies additional band?

Base Wi-Fi OFDM for 20 MHz

- Complex sampling rate

$$\frac{1}{T'} = 20 \text{ MHz}$$

- Number of carriers

$N=64$

- Carrier Spacing

$$\Delta f = \frac{20}{64} = 312.5 \text{ kHz}$$

- Cyclic Extension, Symbol Period $T = (N+\nu)T'$

$\nu=16$

$T=4$ μs & $1/T$ = 250 KHz

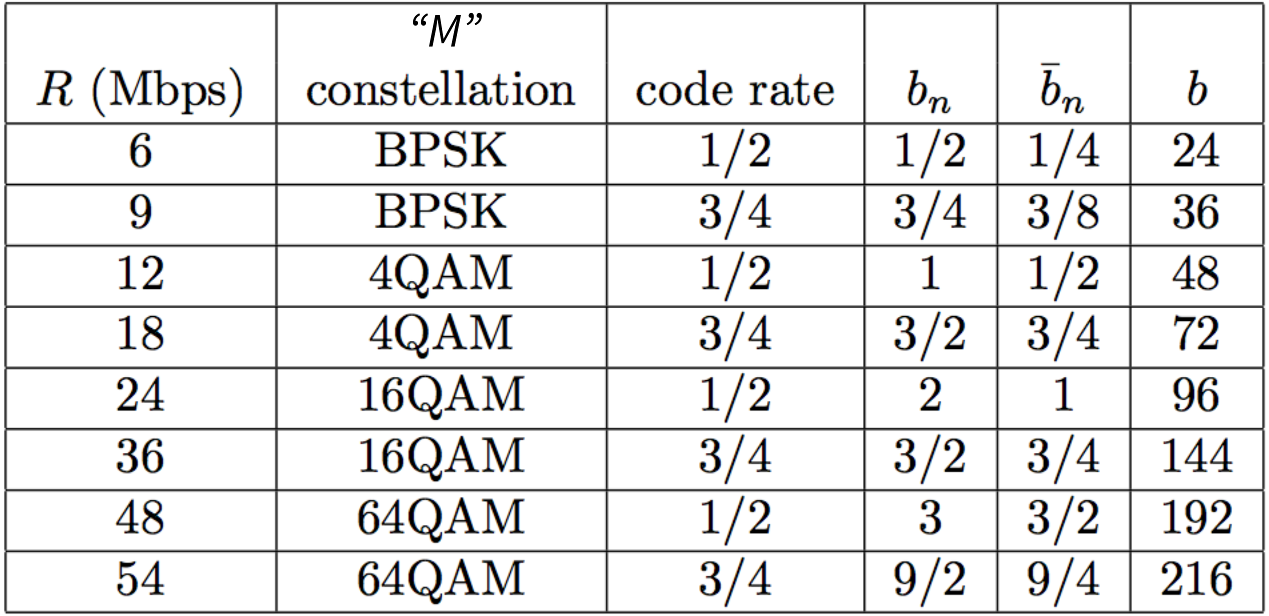
- Bits/tone

$$b_n \in \{2,4,6,8,10\}$$

- Used Carriers = 48
  - Tone 32 at edge is not used, nor are -27…-31, 27 … 31
  - Pilots are at -21, -7 , 7, 21 and 0 is not user data

802.11a, g Table

Statistical loaded
On a single
SNR$_{ofdm}$



R (Mbps)	"M" constellation	code rate	b_n	$\bar{b}_n$	b
6	BPSK	1/2	1/2	1/4	24
9	BPSK	3/4	3/4	3/8	36
12	4QAM	1/2	1	1/2	48
18	4QAM	3/4	3/2	3/4	72
24	16QAM	1/2	2	1	96
36	16QAM	3/4	3/2	3/4	144
48	64QAM	1/2	3	3/2	192
54	64QAM	3/4	9/2	9/4	216

$$R = \log_2(M) \cdot (\text{code rate}) \cdot (48 \text{ tones}) \cdot 250 \text{ kHz} = [0.5,\ 1,\ 2,\ \text{or } 3] \cdot (12 \text{ or } 18\ ) \text{ Mbps.}$$

- Equal energy on all used tones: Power is 16 dBm, 20 dBm, or 29 dBm
- Receiver (effectively) chooses 1 of these 8 loadings or "profiles" by reverse-channel indications to transmitter

Section 4.7.4.3

Example Computations & Codes

- 48 tones x 4 bits/tone (16QAM) x ¾ (code rate) x 250 kHz = 36 Mbps

- 48 tones x 6 bits/tone (64 QAM) x ½ (code rate) x 250 kHZ = 48 Mbps

- RSSI (received-signal-strength indication) is returned by rcvr to xmit via control/reverse channel
  - RSSI essentially is an SNR_{geo} that determines one of the 8 modulation parameter sets
  - This is a form of loading with all tones are equally excited $b_n = b_{ave}$ and $\mathcal{E}_n = \mathcal{E}_{ave}$.

- Codes are convolutional
  - 64-state rate-1/2 code (organized 6 of 12)
    - Punctured (2/3 - delete 4 bits from 12)
    - Punctured (3/4 – delete 3 bits from 12)

Code rate	Free distance	(gross) coding gain $10 \log (d_{free})$
½	10	10 dB
2/3	6	7.7 dB
¾	5	7 dB

Gap formula is for stationary channel, so coding gains with fading may have less performance impact

802.11 n, ac , ax

- n,ac,ax allow a shorter cyclic extension & up to 256 QAM.
- N, ac, ax allow 1/T’ = 40 MHz (N=128). The number of data-carrying tones is 108.
  - So 20 are used for pilots, or silenced at edges.

40 loading choices

constellation	code rate	$1/T' = 20$ $\nu = 16$ Mbps	$1/T' = 20$ MHz $\nu = 8$ Mbps	$1/T' = 40$ $\nu = 16$ Mbps	$1/T' = 40$ MHz $\nu = 8$ Mbps
BPSK	1/2	6.5	7.2	13.5	15
4QAM	1/2	13	14.4	27	30
4QAM	3/4	19.5	21.7	40.5	45
16QAM	1/2	26	28.9	54	60
16QAM	3/4	39	43.3	81	90
64QAM	2/3	52	57.8	108	120
64QAM	3/4	58.5	65	121.5	135
64QAM	5/6	65	72.2	135	150
256QAM	3/4	78	86.6	162	180
256QAM	5/6	86.7	96.3	180	200

- For 20 MHz,
  - Carriers -28,-27,27 and 28 are used, so data rates increase by 52/48 = 13/12 x (12 or 18) Mbps – so thus 13 or 19.5 Mbps

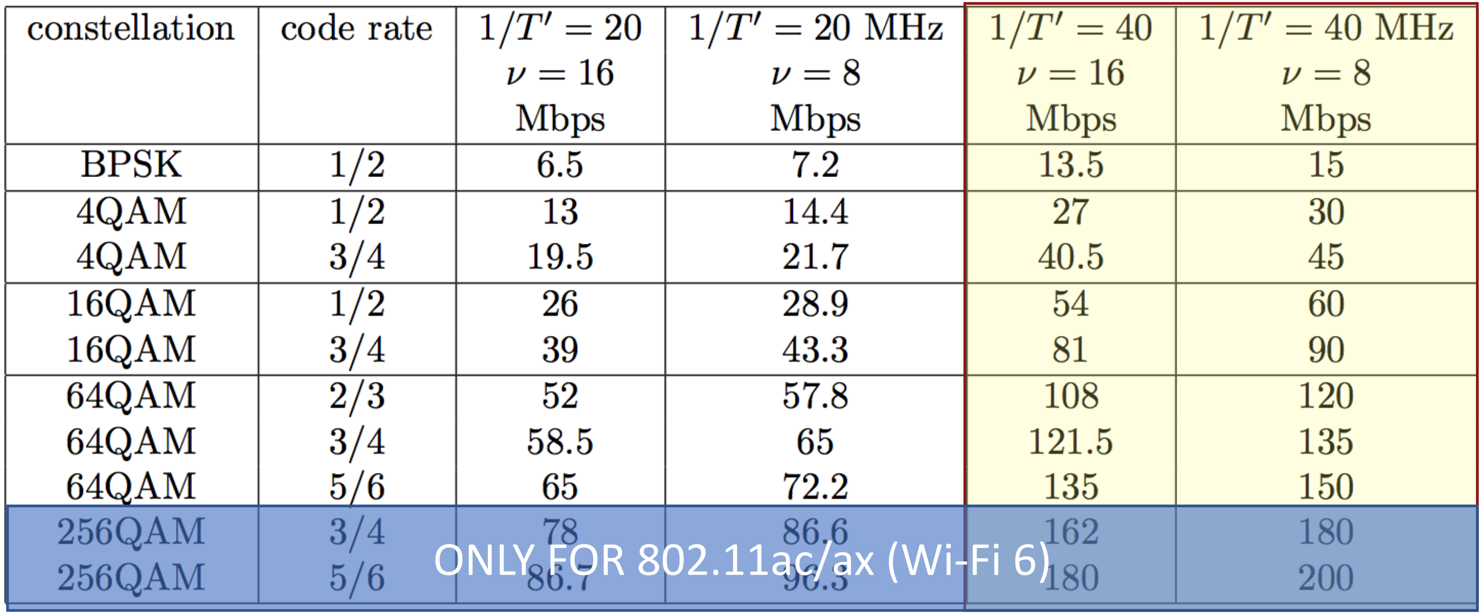
$$R = \log_2(M) \cdot (\text{code rate}) \cdot (48 \text{ tones}) \cdot 250 \text{ kHz} = [0.5,\ 1,\ 2,\ \text{or } 3] \cdot \text{(13 or 19.5)} \ \text{Mbps.}$$

- For 40 MHz:

$$R = \log_2(M) \cdot (\text{code rate}) \cdot (108 \text{ tones}) \cdot 250 \text{ kHz} = [0.5,\ 1,\ 2,\ \text{or } 3] \cdot (27 \text{ or } 40.5\ ) \,\text{Mbps}$$

Or 10/9 x these numbers for $\nu = 8$

802.11n Table with 4 x 4 MIMO



constellation	code rate	$1/T' = 20$ $\nu = 16$ Mbps	$1/T' = 20$ MHz $\nu = 8$ Mbps	$1/T' = 40$ $\nu = 16$ Mbps	$1/T' = 40$ MHz $\nu = 8$ Mbps
BPSK	1/2	6.5	7.2	13.5	15
4QAM	1/2	13	14.4	27	30
4QAM	3/4	19.5	21.7	40.5	45
16QAM	1/2	26	28.9	54	60
16QAM	3/4	39	43.3	81	90
64QAM	2/3	52	57.8	108	120
64QAM	3/4	58.5	65	121.5	135
64QAM	5/6	65	72.2	135	150
256QAM	3/4	78	86.6	162	180
256QAM	5/6	86.7	96.3	180	200

ONLY FOR 802.11ac/ax (Wi-Fi 6)

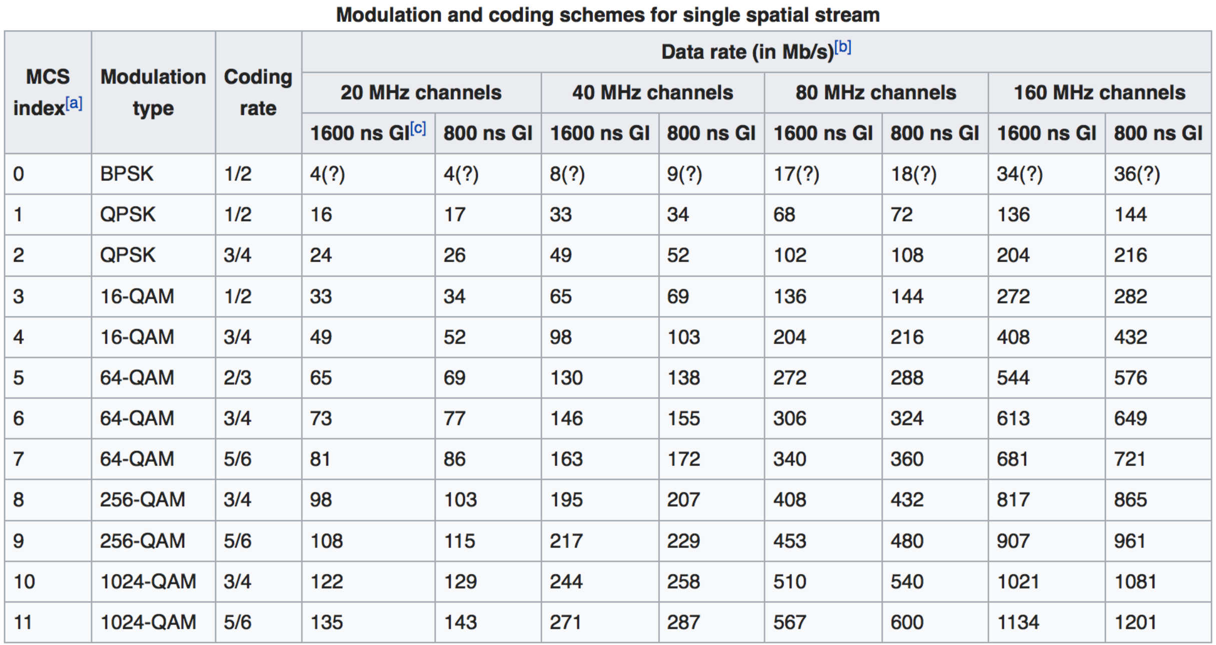
40 MHz
2 adjacent
Channels
as one

Makes collisions
Twice as likely,
Effectively
Reducing # of channels
by 2

- 802.11n allows 4 x 4 Vector OFDM, so data rates in any column can be multiplied by 4
  - Which means 600 Mbps on the 64 QAM (would be 800 Mbps if 256 QAM were used)

- While there is SVD on each tone, all 802.11n spatial dimensions use the same coding line chosen above

Wi-Fi 6 = 802.11ax – up to 4 channels bonded



Modulation and coding schemes for single spatial stream

MCS index[a]	Modulation type	Coding rate	Data rate (in Mb/s)[b]							
			20 MHz channels		40 MHz channels		80 MHz channels		160 MHz channels	
			1600 ns GI[c]	800 ns GI	1600 ns GI	800 ns GI	1600 ns GI	800 ns GI	1600 ns GI	800 ns GI
0	BPSK	1/2	4(?)	4(?)	8(?)	9(?)	17(?)	18(?)	34(?)	36(?)
1	QPSK	1/2	16	17	33	34	68	72	136	144
2	QPSK	3/4	24	26	49	52	102	108	204	216
3	16-QAM	1/2	33	34	65	69	136	144	272	282
4	16-QAM	3/4	49	52	98	103	204	216	408	432
5	64-QAM	2/3	65	69	130	138	272	288	544	576
6	64-QAM	3/4	73	77	146	155	306	324	613	649
7	64-QAM	5/6	81	86	163	172	340	360	681	721
8	256-QAM	3/4	98	103	195	207	408	432	817	865
9	256-QAM	5/6	108	115	217	229	453	480	907	961
10	1024-QAM	3/4	122	129	244	258	510	540	1021	1081
11	1024-QAM	5/6	135	143	271	287	567	600	1134	1201

160 MHz
8 adjacent
Channels
as one

Back to only
3 non-overlapping
channels,
So Wi-Fi 6E
(expands 5-7 GHz)

- 4 channels use N=256 with 234 carrying user data
- 8 channels use N=512 with 484 carrying user data

- Up to 8x8 MIMO on 11ax → 10 Gbits (almost)

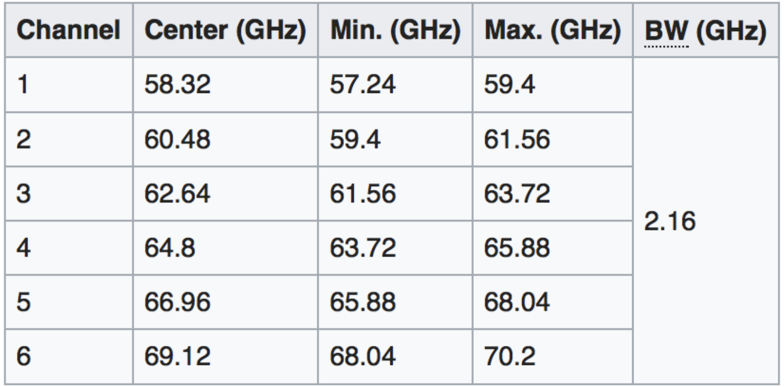
96 choices in loading

M also sent to xmit for MIMO
(one for each tone)

Web tutorial on
this by former
Student of this clas
R. Nabar

Wi-Gig is Wi-Fi, 802.11ad ~ 60 GHz

- Carrier frequencies (Six 2.16 GHz channels)



Channel	Center (GHz)	Min. (GHz)	Max. (GHz)	BW (GHz)
1	58.32	57.24	59.4	2.16
2	60.48	59.4	61.56	
3	62.64	61.56	63.72	
4	64.8	63.72	65.88	
5	66.96	65.88	68.04	
6	69.12	68.04	70.2	

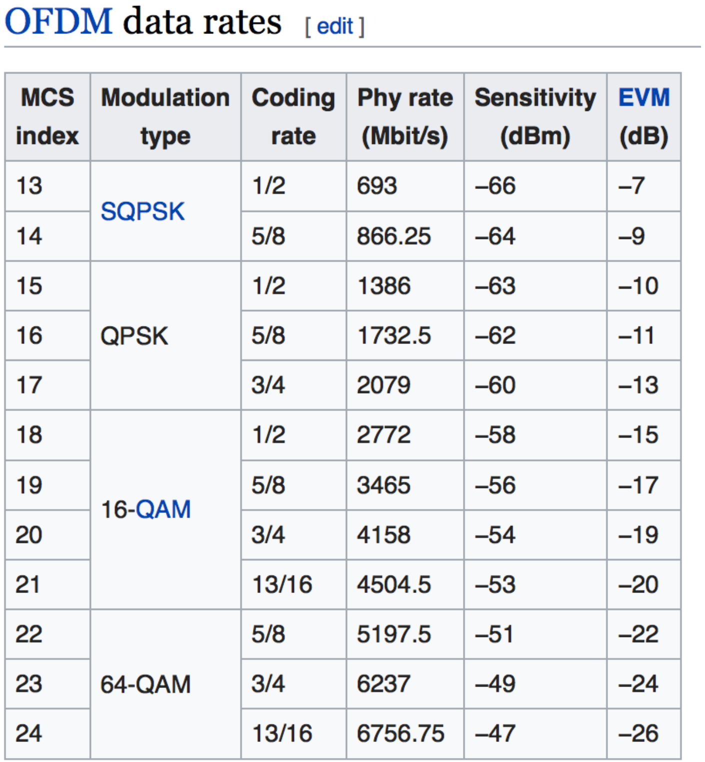
- Parameters:

$$\frac{1}{T'} = 2.64\text{ GHz} \qquad \Delta f = \frac{2640}{512} = 5.15625\text{ MHz}$$

N=512 with 336 used

$\nu=128$

$$\frac{1}{T} = \left(\frac{N}{N+\nu}\right)\cdot \Delta f = 4.125\text{ MHz}$$



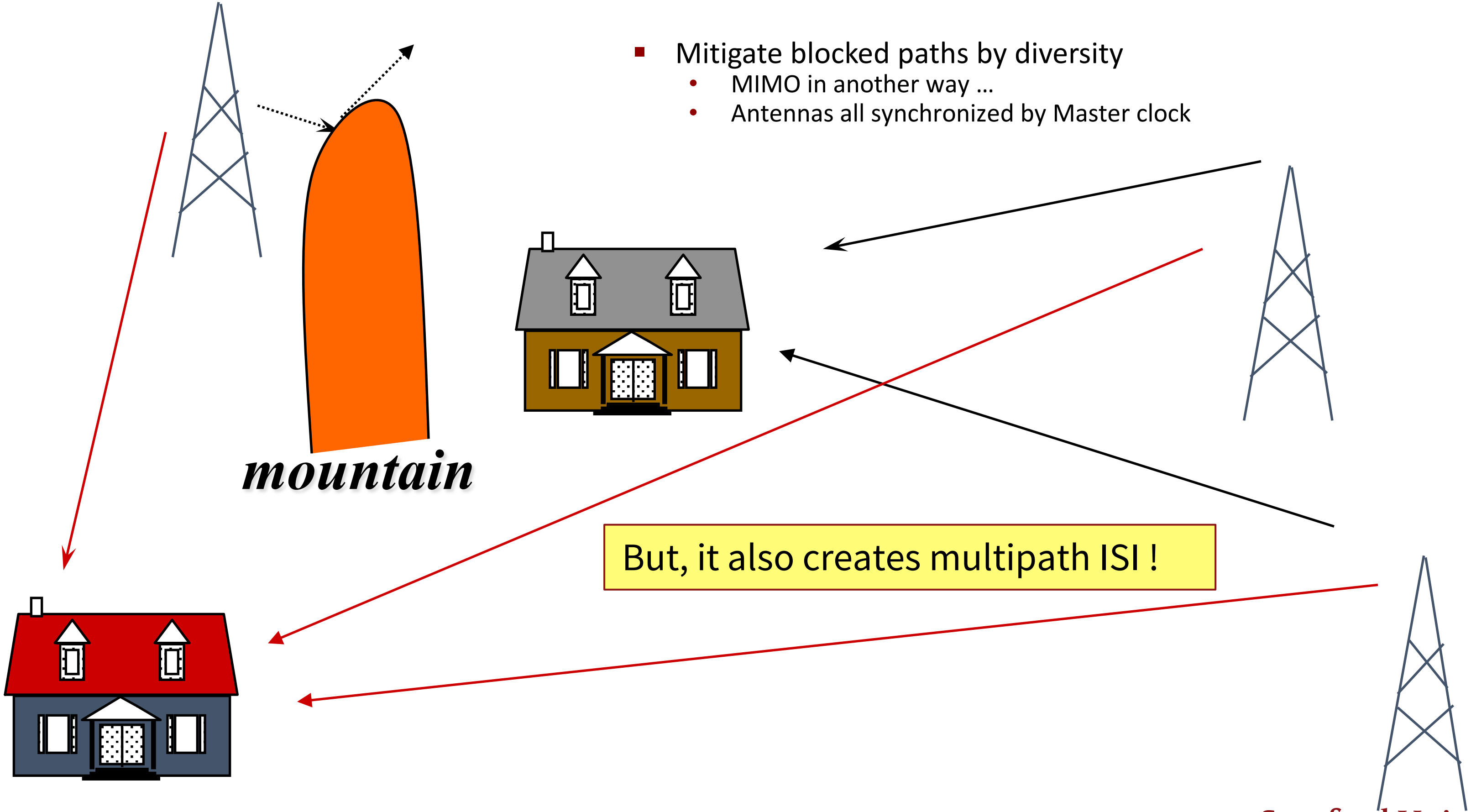
OFDM data rates [edit]

MCS index	Modulation type	Coding rate	Phy rate (Mbit/s)	Sensitivity (dBm)	EVM (dB)
13	SQPSK	1/2	693	–66	–7
14		5/8	866.25	–64	–9
15	QPSK	1/2	1386	–63	–10
16		5/8	1732.5	–62	–11
17		3/4	2079	–60	–13
18	16-QAM	1/2	2772	–58	–15
19		5/8	3465	–56	–17
20		3/4	4158	–54	–19
21		13/16	4504.5	–53	–20
22	64-QAM	5/8	5197.5	–51	–22
23		3/4	6237	–49	–24
24		13/16	6756.75	–47	–26

EVM is same as MSE

Digital Video Broadcast

Single-Frequency Network (SFN)



- Mitigate blocked paths by diversity
  - MIMO in another way …
  - Antennas all synchronized by Master clock

But, it also creates multipath ISI !

DVB Standard uses Coded OFDM

- Parameters

carrier(s)	Carrier frequencies are same as analog TV spacing 6 MHz, 7.61 MHz, 8 MHz	*52 MHz = channel 2*
sampling	$T' = 10.9375 \text{ ns}$	$\frac{1}{T'} \cong 9.14\text{MHz}$
symbol size	N=2048 or 8192	
tone width	$\Delta f = 44.64 \text{ kHz}$	Or 11.16 kHz
prefix (es)	$\nu = \left(\frac{1}{2^i}\right) \cdot N \quad i = 2,3,4,5$	
symbol rate	Symbol rates vary ~ 10 kHz	

LTE Examples 4G & 5G

4G and 5G wireless = LTE (licensed bands)

- Mobile/cellular connectivity also uses OFDM (and vector OFDM).
- 4G uses up to 4 xmit/rcvr antennas.
- 5G allows use of much larger number of antennas (Massive MIMO).
  - Best 32 of 128
- LTE uses 500 μs time "slots" (20 of them in a "frame" of 10 ms)
- Each time slot uses an integer tone-width index *m*=1,2,4,8, 12, and 16 to multply

$$\boxed{\Delta f = 15 \text{ kHz}}$$

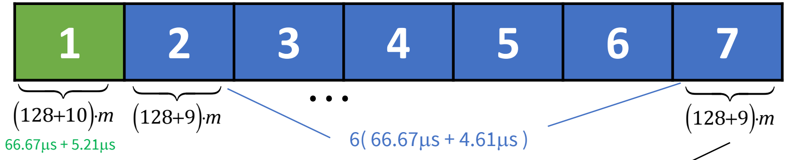
$$\frac{1}{T'} = \left(1.92 \text{ MHz}\right) \cdot m \quad N = 128 \cdot m \quad , \quad m = 1,2,4,8,12,16$$

$$m \cdot 960 \text{ samples/slot} \quad , \quad m = 1,2,4,8,12,16$$

Short and Long Cyclic Prefixes

Short Prefix

500 μs – LTE Slot of 7 OFDM symbols



$(128+10)\cdot m$ $(128+9)\cdot m$ $(128+9)\cdot m$

$66.67\mu s + 5.21\mu s$

$6(\,66.67\mu s + 4.61\mu s\,)$

$m\cdot\left[(128+10)+6\cdot(128+9)\right]$ samples

$m\cdot 960$ samples/slot

data rates are multiples of 14 kbps

$$\left(\frac{7}{500\mu s}\right)$$

Long Prefix

500 μs – LTE Slot of 6 OFDM symbols

$(128+32)\cdot m$ $(128+32)\cdot m$

$6(\,66.67\mu s + 16.67\mu s\,)$

$6m\cdot(128+32)$ samples

$m\cdot 960$ samples/slot

data rates are multiples of 12 kbps

$$\left(\frac{6}{500\mu s}\right)$$

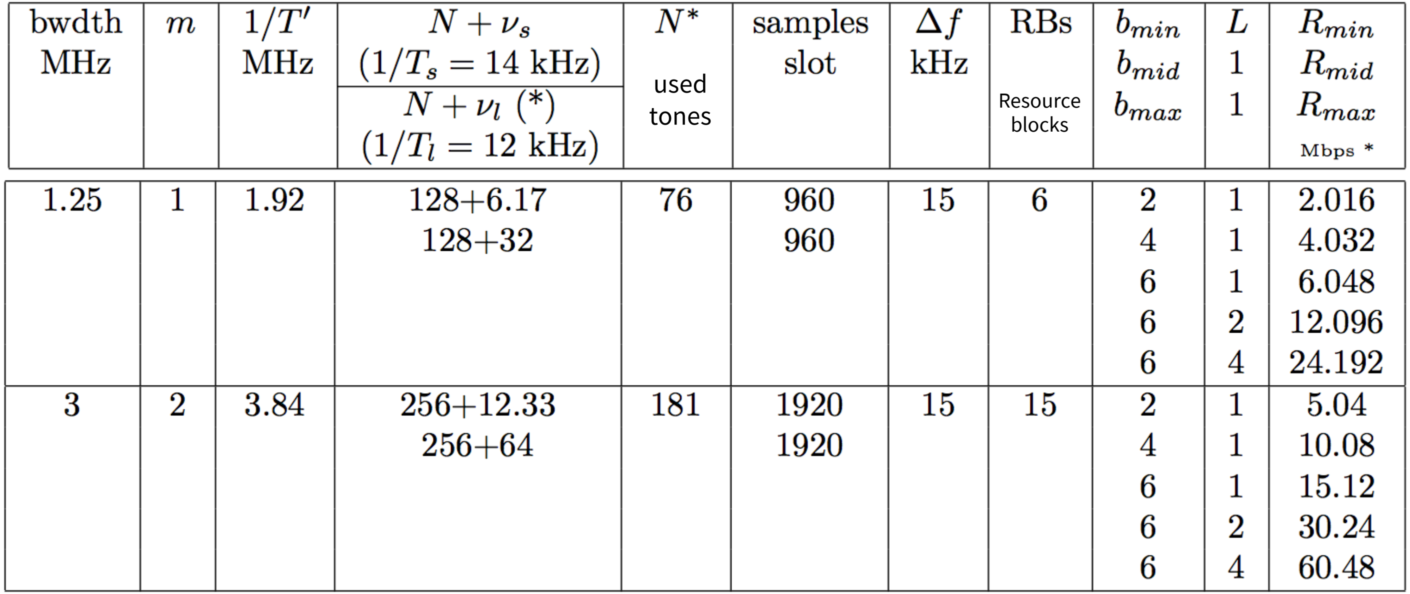
- Each symbol decomposes into “resource elements” and “resource blocks”

LTE Resource Blocks

User could be control channel

- Smallest unit that can be assigned to a user – vector loading can apply to these blocks (but not to individual tones) – 12 tones within a single symbol
- There can be pilots, synch symbols, and other overhead scattered throughout a slot so total number of tones need not be a multiple of 12

Low Bandwidth (small devices) LTE



bwdth MHz	m	$1/T'$ MHz	$N+\nu_s$ $(1/T_s = 14$ kHz$)$ $N+\nu_l$ (*) $(1/T_l = 12$ kHz$)$	N^* used tones	samples slot	Δf kHz	RBs Resource blocks	b_{min} b_{mid} b_{max}	L 1 1	R_{min} R_{mid} R_{max} Mbps *
1.25	1	1.92	128+6.17	76	960	15	6	2	1	2.016
			128+32		960			4	1	4.032
								6	1	6.048
								6	2	12.096
								6	4	24.192
3	2	3.84	256+12.33	181	1920	15	15	2	1	5.04
			256+64		1920			4	1	10.08
								6	1	15.12
								6	2	30.24
								6	4	60.48

- Individual users
  - 12 x 12 =144 kbps for RB=1, lcp
  - 12 x 14 = 168 kbps for RB=1, scp

Example (6 RBs x 12 tones/RB x 2 bits/tone x 14 KHz = 2.016 Mbps

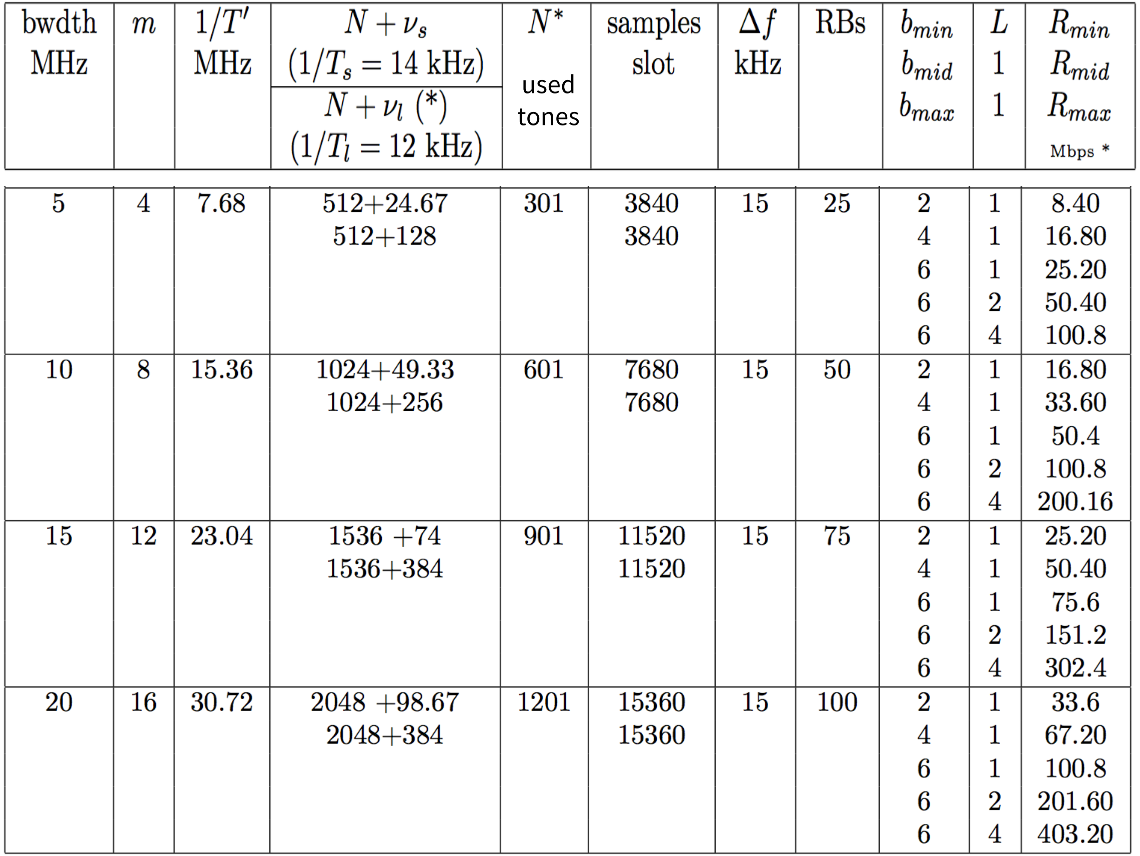
Code overhead included

- LTE attempts to address low-bandwidth uses where power may be very limited
  - MIMO can reduce power to get same data rate or can also permit more narrow bandwidth use for same rate

Wider Bands need more (licensed) spectra

- Wider bandwidths for higher speeds
  - MIMO helps that also

- LTE-U / LAA
  - 20 MHz option can use a Wi-Fi channel
  - “Look before talk”



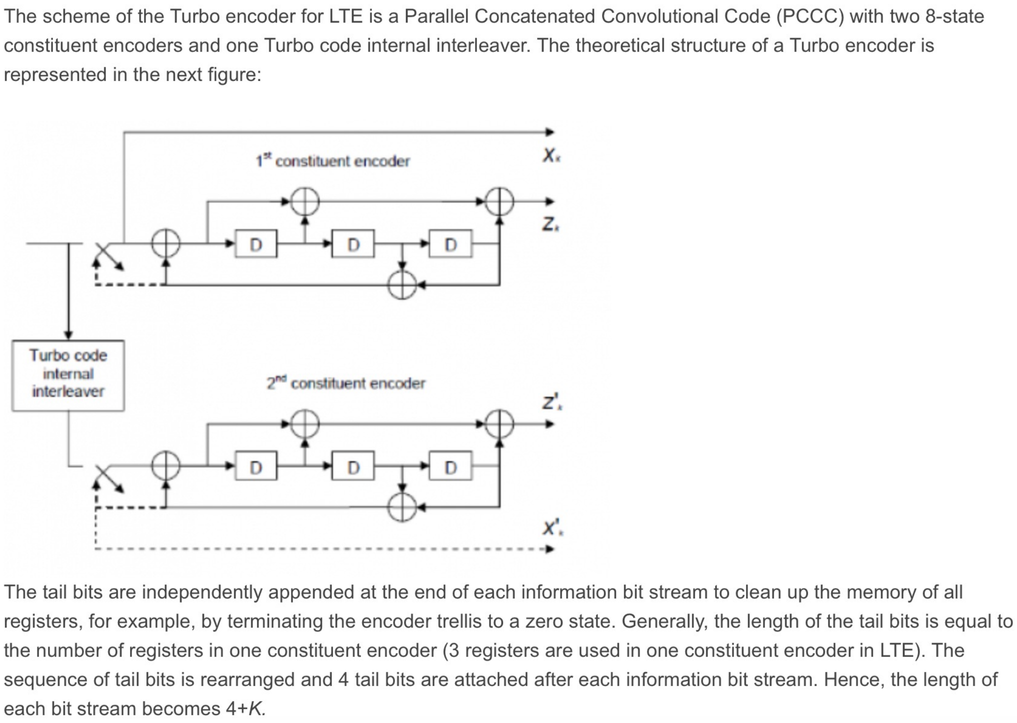
bwdth MHz	m	$1/T'$ MHz	$N+\nu_s$ $(1/T_s = 14\text{ kHz})$ $N+\nu_l$ (*) $(1/T_l = 12\text{ kHz})$	N^* used tones	samples slot	Δf kHz	RBs	b_{min} b_{mid} b_{max}	L 1 1	R_{min} R_{mid} R_{max} Mbps *
5	4	7.68	512+24.67	301	3840	15	25	2	1	8.40
			512+128		3840			4	1	16.80
								6	1	25.20
								6	2	50.40
								6	4	100.8
10	8	15.36	1024+49.33	601	7680	15	50	2	1	16.80
			1024+256		7680			4	1	33.60
								6	1	50.4
								6	2	100.8
								6	4	200.16
15	12	23.04	1536 +74	901	11520	15	75	2	1	25.20
			1536+384		11520			4	1	50.40
								6	1	75.6
								6	2	151.2
								6	4	302.4
20	16	30.72	2048 +98.67	1201	15360	15	100	2	1	33.6
			2048+384		15360			4	1	67.20
								6	1	100.8
								6	2	201.60
								6	4	403.20

LTE Coding and Loading

- Previous data rates were encoder-output data rates
- A rate 1/3 “turbo code” (8 states in each constituent code, see Chapter 11)
  - Can be punctured from 1/3 up to 95%
- MIMO systems do not return M from SVD on each tone in Vector OFDM (unlike Wi-Fi)
  - Instead one of 16 pre-defined M’s is selected during training/adaptation (called a “codebook”
- Loading will select code-puncturing, power level for RB, and the constellation size.

The LTE Turbo Code

- IET Engineering Community



The scheme of the Turbo encoder for LTE is a Parallel Concatenated Convolutional Code (PCCC) with two 8-state constituent encoders and one Turbo code internal interleaver. The theoretical structure of a Turbo encoder is represented in the next figure:

The tail bits are independently appended at the end of each information bit stream to clean up the memory of all registers, for example, by terminating the encoder trellis to a zero state. Generally, the length of the tail bits is equal to the number of registers in one constituent encoder (3 registers are used in one constituent encoder in LTE). The sequence of tail bits is rearranged and 4 tail bits are attached after each information bit stream. Hence, the length of each bit stream becomes 4+*K*.

Coding, see
Chaps 10 & 11

Uplink LTE

- Aggregates RB's into a single carrier (with same cyclic prefix)
  - Presumably saves upstream energy (although not clear that is really true – peak/average with filters - See 4.10)
- The receiver is what is known as a "Cyclic DFE" (see Chapter 5)
  - This was miss-named "SC-OFDM" (single carrier– OFDM long after cyclic DFE name introduced)
- Same data rates, FFT sizes, etc – just computation executed for minimum number of RB's)

5G New Radio (NR)

- LTE Rev 15: 5G NR adds some capabilities that extend 4G
  - Lower band (FR1): 450 MHz -- 6 GHz (FDD/TDD)
  - Millimeter Wave Band: (FR2): 24.25 GHz – 52.6 GHz (TDD only)

  - Δf now increased to
    - 15 (same), 30 and 60 kHz in FR1
    - 60 and 120 kHz in FR2
    - Also allows 256 QAM

  - Channel Bandwidths now extend to as much as 400 MHz (depends on band)

  - Number of antennas is unlimited (Massive MIMO), but …
    - Maximum layers (so significant singular values or used dimensions) remains at 8 for a SINGLE user (4 for Uplink)
    - Maximum number of virtual antenna ports for which ”SVD-like” info can be supplied is 32 (4 for Uplink)

- See Tables to Follow and also Lecture 12 (Section 7.3)

3GPP TS38.101-1 Table 5.2-1: NR operating bands in FR1/2

Band Name	Uplink	Downlink	Duplex
n1	1920 MHz – 1980 MHz	2110 MHz – 2170 MHz	FDD
n2	1850 MHz – 1910 MHz	1930 MHz – 1990 MHz	FDD
n3	1710 MHz – 1785 MHz	1805 MHz – 1880 MHz	FDD
n5	824 MHz – 849 MHz	869 MHz – 894MHz	FDD
n7	2500 MHz – 2570 MHz	2620 MHz – 2690 MHz	FDD
n8	880 MHz – 915 MHz	925 MHz – 960 MHz	FDD
n20	832 MHz – 862 MHz	791 MHz – 821 MHz	FDD
n28	703 MHz – 748 MHz	758 MHz – 803 MHz	FDD
n38	2570 MHz – 2620 MHz	2570 MHz – 2620 MHz	TDD
n41	2496 MHz – 2690 MHz	2496 MHz – 2690 MHz	TDD
n50	1432 MHz – 1517 MHz	1432 MHz – 1517 MHz	TDD
n51	1427 MHz – 1432 MHz	1427 MHz – 1432 MHz	TDD
n66	1710 MHz – 1780 MHz	2110 MHz – 2200 MHz	FDD
n70	1695 MHz – 1710 MHz	1995 MHz – 2020 MHz	FDD
n71	663 MHz – 698 MHz	617 MHz – 652 MHz	FDD
n74	1427 MHz – 1470 MHz	1475 MHz – 1518 MHz	FDD
n75	N/A	1432 MHz – 1517 MHz	SDL
n76	N/A	1427 MHz – 1432 MHz	SDL
n78	3300 MHz – 3800 MHz	3300 MHz – 3800 MHz	TDD
n77	3300 MHz – 4200 MHz	3300 MHz – 4200 MHz	TDD
n79	4400 MHz – 5000 MHz	4400 MHz – 5000 MHz	TDD
n80	1710 MHz – 1785 MHz	N/A	SUL
n81	880 MHz – 915 MHz	N/A	SUL
n82	832 MHz – 862 MHz	N/A	SUL
n83	703 MHz – 748 MHz	N/A	SUL
n84	1920 MHz – 1980 MHz	N/A	SUL

Band Name	Uplink	Downlink	Duplex
n257	26500 MHz – 29500 MHz	26500 MHz – 29500 MHz	TDD
n258	24250 MHz – 27500 MHz	24250 MHz – 27500 MHz	TDD
n260	37000 MHz – 40000 MHz	37000 MHz – 40000 MHz	TDD

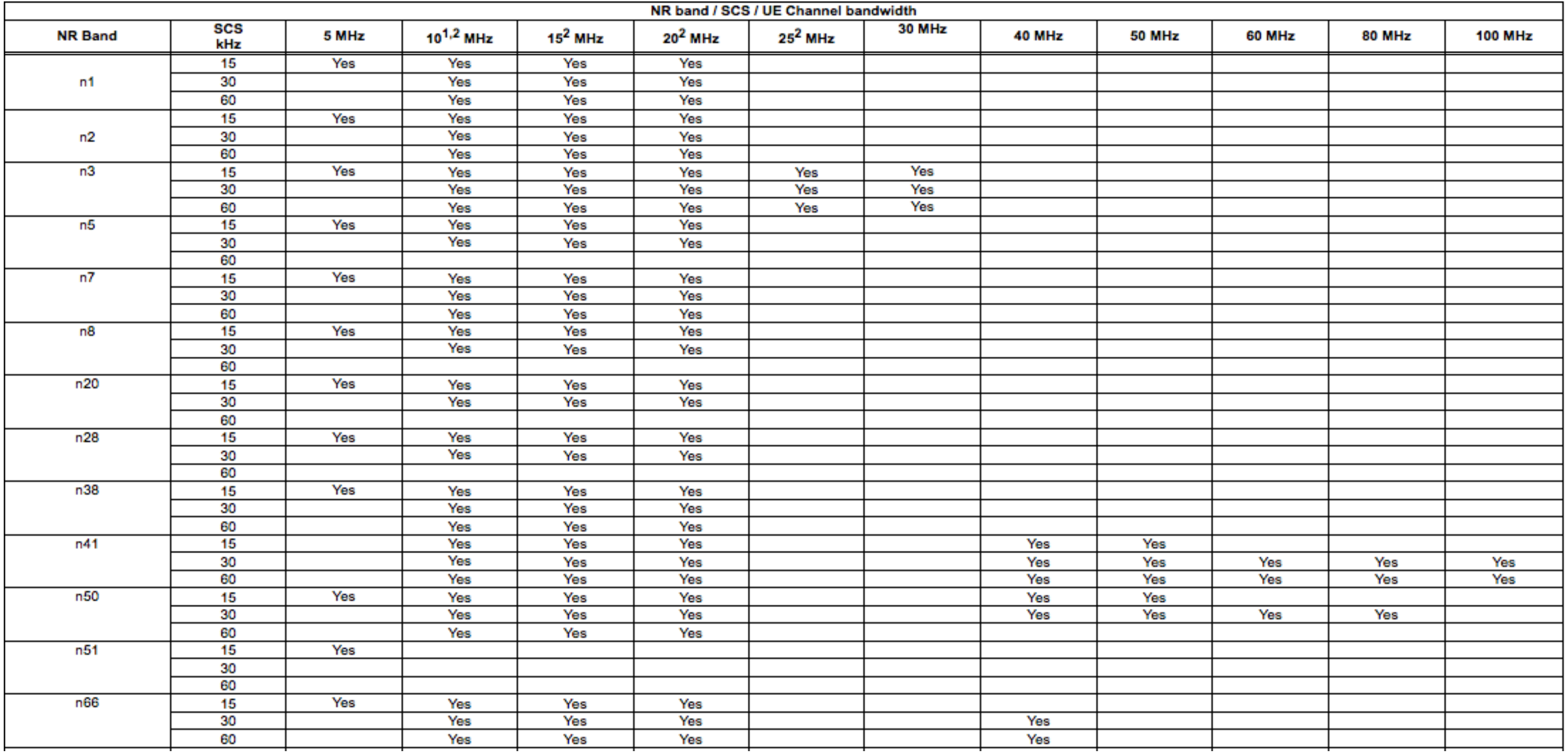
Downlink Powers

BS class	**$P_{rated,c,AC}$**
Wide Area BS	(Note)
Medium Range BS	< 38 dBm
Local Area BS	< 24 dBm
NOTE: There is no upper limit for the $P_{rated,c,AC}$ rated output power of the Wide Area Base Station.	

Source: 3GPP TS38.104 Table 6.2.1-1: *BS type 1-C* rated output power limits for BS classes

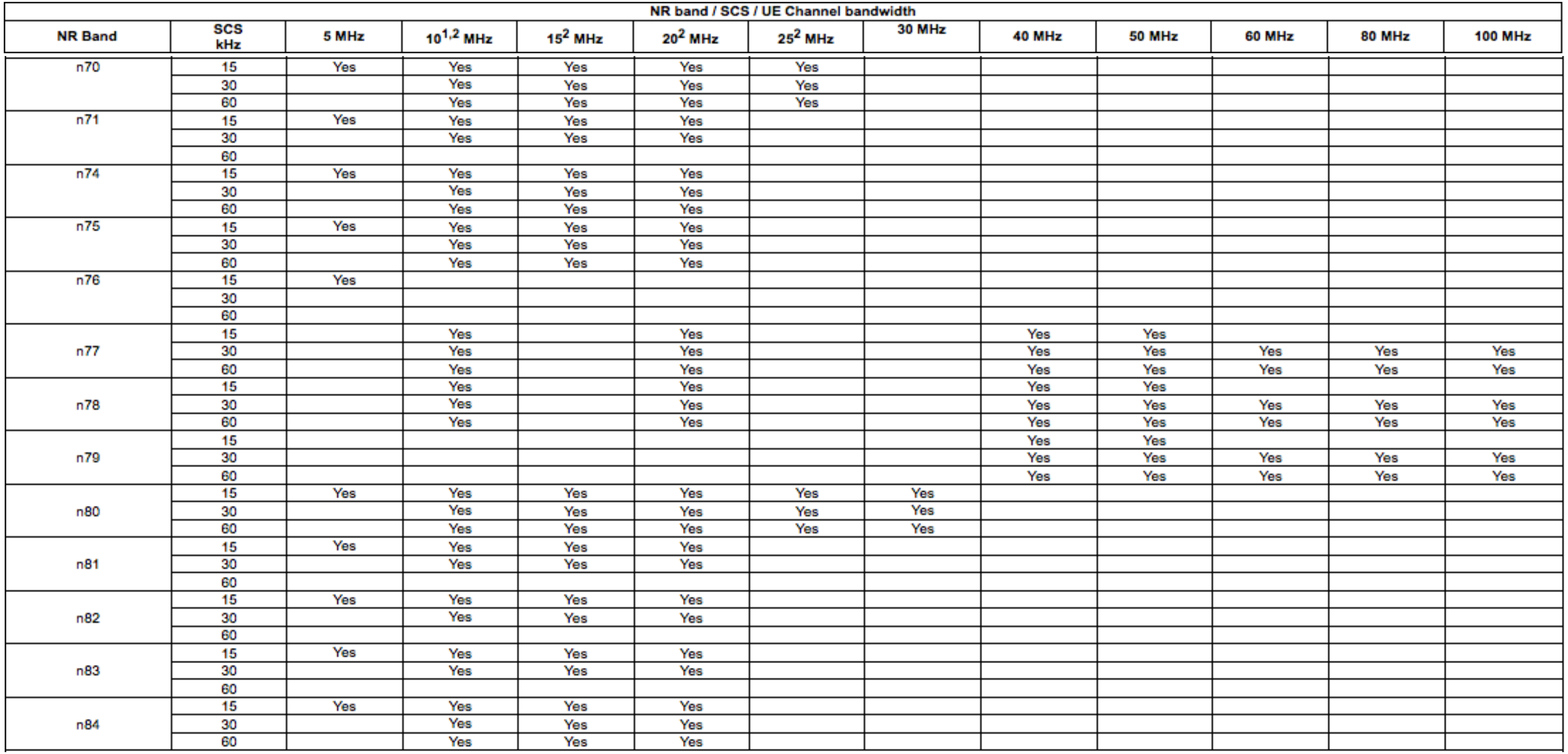
Uplink Power limit is 23 dBm, except n41, which is 26 dBm

3GPP TS38.101-1 Table 5.3.5-1: Channel Bandwidths for Each NR FR1 band



NR band / SCS / UE Channel bandwidth												
NR Band	SCS kHz	5 MHz	$10^{1,2}$ MHz	15^{2} MHz	20^{2} MHz	25^{2} MHz	30 MHz	40 MHz	50 MHz	60 MHz	80 MHz	100 MHz
n1	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60		Yes	Yes	Yes							
n2	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60		Yes	Yes	Yes							
n3	15	Yes	Yes	Yes	Yes	Yes	Yes					
	30		Yes	Yes	Yes	Yes	Yes					
	60		Yes	Yes	Yes	Yes	Yes					
n5	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n7	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60		Yes	Yes	Yes							
n8	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n20	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n28	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n38	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60		Yes	Yes	Yes							
n41	15		Yes	Yes	Yes			Yes	Yes			
	30		Yes	Yes	Yes			Yes	Yes	Yes	Yes	Yes
	60		Yes	Yes	Yes			Yes	Yes	Yes	Yes	Yes
n50	15	Yes	Yes	Yes	Yes			Yes	Yes			
	30		Yes	Yes	Yes			Yes	Yes	Yes	Yes	
	60		Yes	Yes	Yes							
n51	15	Yes										
	30											
	60											
n66	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes			Yes				
	60		Yes	Yes	Yes			Yes				

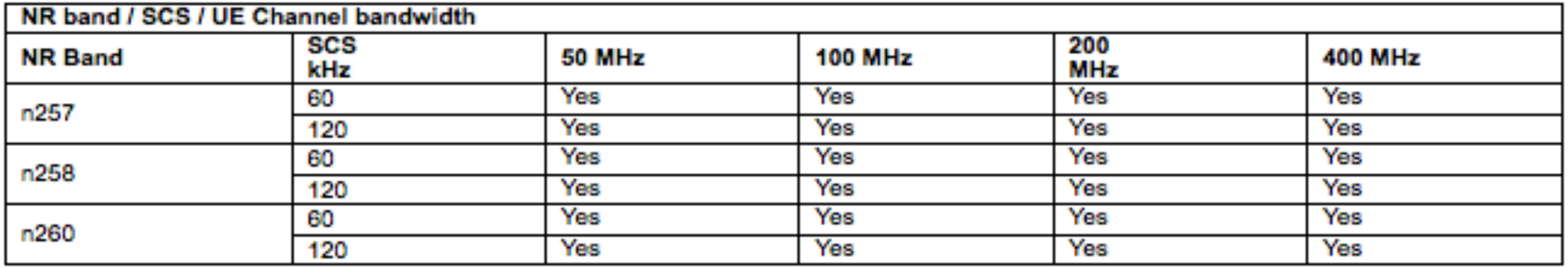
Continued from previous slide



NR band / SCS / UE Channel bandwidth												
NR Band	**SCS kHz**	**5 MHz**	**$10^{1,2}$ MHz**	**15^{2} MHz**	**20^{2} MHz**	**25^{2} MHz**	**30 MHz**	**40 MHz**	**50 MHz**	**60 MHz**	**80 MHz**	**100 MHz**
n70	15	Yes	Yes	Yes	Yes	Yes						
	30		Yes	Yes	Yes	Yes						
	60		Yes	Yes	Yes	Yes						
n71	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n74	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60		Yes	Yes	Yes							
n75	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60		Yes	Yes	Yes							
n76	15	Yes										
	30											
	60											
n77	15		Yes		Yes			Yes	Yes			
	30		Yes		Yes			Yes	Yes	Yes	Yes	Yes
	60		Yes		Yes			Yes	Yes	Yes	Yes	Yes
n78	15		Yes		Yes			Yes	Yes			
	30		Yes		Yes			Yes	Yes	Yes	Yes	Yes
	60		Yes		Yes			Yes	Yes	Yes	Yes	Yes
n79	15							Yes	Yes			
	30							Yes	Yes	Yes	Yes	Yes
	60							Yes	Yes	Yes	Yes	Yes
n80	15	Yes	Yes	Yes	Yes	Yes	Yes					
	30		Yes	Yes	Yes	Yes	Yes					
	60		Yes	Yes	Yes	Yes	Yes					
n81	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n82	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n83	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60											
n84	15	Yes	Yes	Yes	Yes							
	30		Yes	Yes	Yes							
	60		Yes	Yes	Yes							

Section 4.7.4.3

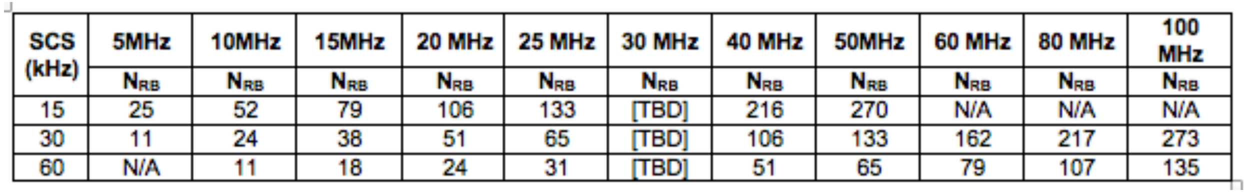
FR2 band: 3GPP TS38.101-2 Table 5.3.5-1: Channel bandwidths



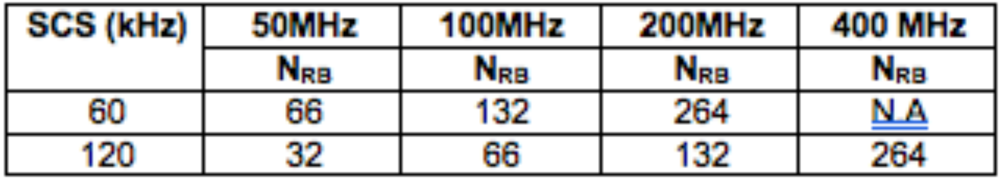
NR band / SCS / UE Channel bandwidth					
NR Band	**SCS kHz**	**50 MHz**	**100 MHz**	**200 MHz**	**400 MHz**
n257	60	Yes	Yes	Yes	Yes
	120	Yes	Yes	Yes	Yes
n258	60	Yes	Yes	Yes	Yes
	120	Yes	Yes	Yes	Yes
n260	60	Yes	Yes	Yes	Yes
	120	Yes	Yes	Yes	Yes

- At mmW frequencies of FR2, these wider-bandwidth channels are only a small fraction of total bandwidth in this range.

Resource Blocks



SCS (kHz)	5MHz	10MHz	15MHz	20 MHz	25 MHz	30 MHz	40 MHz	50MHz	60 MHz	80 MHz	100 MHz
	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}
15	25	52	79	106	133	[TBD]	216	270	N/A	N/A	N/A
30	11	24	38	51	65	[TBD]	106	133	162	217	273
60	N/A	11	18	24	31	[TBD]	51	65	79	107	135



SCS (kHz)	50MHz	100MHz	200MHz	400 MHz
	N_{RB}	N_{RB}	N_{RB}	N_{RB}
60	66	132	264	N.A
120	32	66	132	264

$$R = 168\ \mathrm{kHz} \times N_{RB} \times b \times L \times \frac{\Delta f}{15\ \mathrm{kHz}}$$

Example $R = 168 \times 133 \times 4 \times 4 \times 2 = 715.008\mathrm{Mbps}$

- Little more efficient in more RB’s allocated at lower frequencies too (than 4G)

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End Lecture 6