STANFORD

Lecture 2

Channel Partitioning: Vector Coding & DMT

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Announcements & Agenda

- Announcements
  - Problem Set 1 due Wednesday, April 12 @ 17:00
  - Most relevant reading – Sections 2.5, 4.4-4.7
    - Supplementary lectures S1A and S1B may be also of interest
  - Questions?

- Agenda
  - Vector Coding in Time-Frequency
  - $1+.9D^{-1}$ Vector-Code Example
  - DMT/OFDM partitioning
  - DMT Waterfilling Software
  - Vector DMT/OFDM partitioning

Vector Coding in Time/Frequency

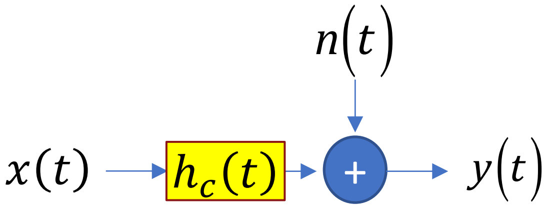
Section 4.6.1

See PS1.5 (Prob 4.25) (matrix AWGN and vector coding)

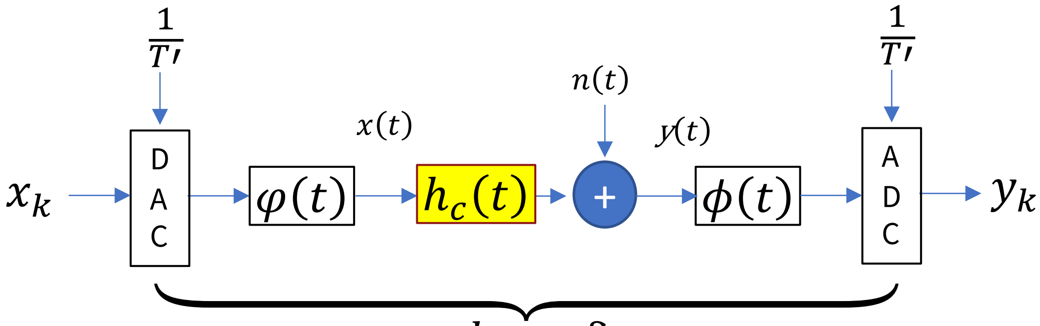
Scalar Time/Frequency (filtered) AWGN Channel

- Ideal AWGN channels just have noise, but

- Realistically, there is a filter
  - Attenuation
  - Band-limits
  - Spectrally shaped noise (See Sec 1.3.7)



- The $h_c(t)$ causes interference between successive transmissions – complicates and changes performance
- Sampled Equivalent ; $T' < T$ is the **sample period**



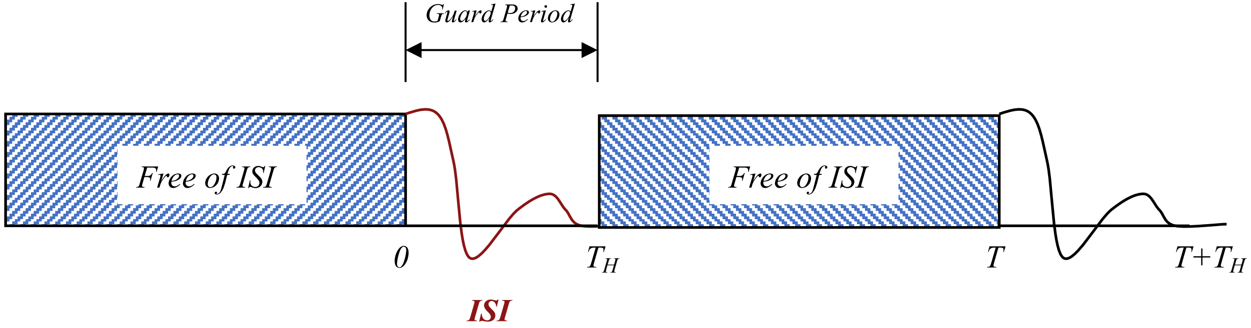
$$h_k \neq \delta_k$$

sampled pulse response

$$y_k = x_k * h_k + n_k$$

Sections 1.3.7.2, 4.5.1.1

Guard Periods for frequency-time

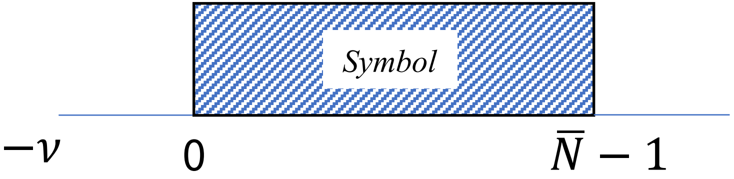


- The **guard period** T_H
  - Let ISI abate before next symbol

- Wastes T_H/T of resources (time dimensions)
  - If $T >> T_H$, then the guard period may be worth it
  - It may be zeroed or anything the receiver ignores

excess bandwidth $\alpha = \frac{T_H}{T - T_H}$

- A (scalar / SISO) symbol with $\bar{N}$ dimensions (samples)
  - $T_H = \nu \cdot T'$ so up to ν+1 non-zero samples/dimensions in h_k
  - $\bar{N}$ works for real ($\tilde{N} = 1$) or complex ($\tilde{N} = 2$)



reindex time in sample periods

SISO (time-dimension) Case

- Simple scalar convolutional matrix channel with guard band

$$\begin{bmatrix} y_{\bar{N}-1} \\ y_{\bar{N}-2} \\ \vdots \\ y_0 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \dots & h_\nu & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & h_0 & \ddots & h_{\nu-1} & h_\nu & \ddots & 0 & 0 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & 0 & 0 & h_0 & h_1 & \dots & h_\nu \end{bmatrix} \begin{bmatrix} x_{\bar{N}-1} \\ \vdots \\ x_0 \\ \boxed{\begin{matrix} x_{-1} \\ \vdots \\ x_{-\nu} \end{matrix}} \end{bmatrix} + \begin{bmatrix} n_{\bar{N}-1} \\ n_{\bar{N}-2} \\ \vdots \\ n_0 \end{bmatrix}$$

guard period could be anything, including 0 or cyclic

$$\boldsymbol{y} = H \boldsymbol{x} + \boldsymbol{n}$$

- Non-square shift "Toeplitz" matrix for convolution
  - More inputs than outputs when $\nu \neq 0$

SVD Again for the time-dimension case

- Singular Value Decomposition

$$H = F \cdot \left[\underbrace{\Lambda}_{\bar{N}\times\bar{N}} \quad \vdots \quad \underbrace{\mathbf{0}_{-1:-\nu}}_{\bar{N}\times\nu} \right] \cdot M^*$$

$N = \bar{N}$ if real subsymbols and $\tilde{N} = 1$
$N = 2 \cdot \bar{N}$ if complex subsymbols and $\tilde{N} = 2$

$$\underbrace{FF^* = F^*F = I}_{\bar{N}\times\bar{N}}$$

$$\Lambda = \begin{bmatrix} \lambda_{\bar{N}-1} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \lambda_1 & 0 \\ 0 & \cdots & 0 & \lambda_0 \end{bmatrix}$$

$$\underbrace{MM^* = M^*M = I}_{(\bar{N}+\nu)\times(\bar{N}+\nu)}$$

unique (real) singular values ≥ 0

- The vector-coding input construction

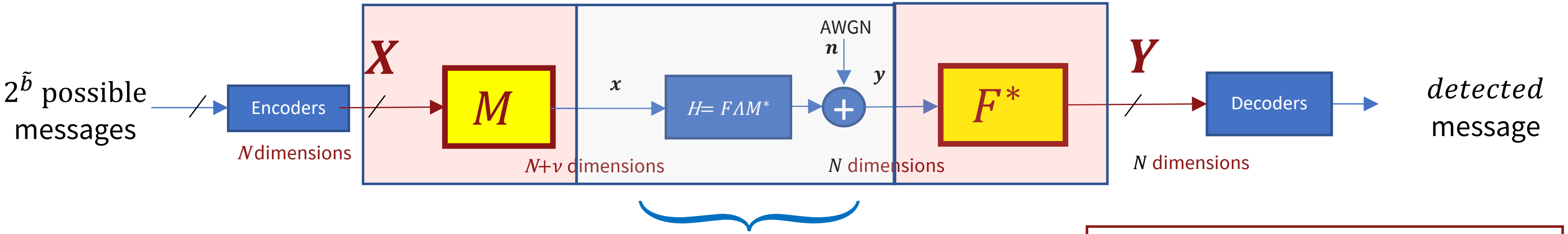
$$\boldsymbol{x} = M \begin{bmatrix} \boldsymbol{X} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = [\underbrace{\boldsymbol{m}_{\bar{N}-1}\ \boldsymbol{m}_{\bar{N}-2}\ \dots\ \boldsymbol{m}_1 \boldsymbol{m}_0}_{(\bar{N}+\nu)\times\bar{N}}\ \underbrace{\dots\ \boldsymbol{m}_{-\nu}}_{zeroed}] \begin{bmatrix} X_{\bar{N}-1} \\ X_{\bar{N}-2} \\ \vdots \\ X_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \sum_{n=0}^{\bar{N}-1} X_n \cdot \boldsymbol{m}_n$$

Vector-Coded Time Dimensions Only

- Channel Output into Matched Vectors

$$\boldsymbol{Y} = F^* \cdot \boldsymbol{y} = \begin{bmatrix} \boldsymbol{f}^*_{\bar{N}-1} \cdot \boldsymbol{y} \\ \vdots \\ \boldsymbol{f}^*_0 \cdot \boldsymbol{y} \end{bmatrix}$$

- Vector-Code Channel Partitioning



- Parallel Channels

$$Y_n = \lambda_n \cdot X_n + N_n$$

$$SNR_n = \frac{\lambda_n^2 \cdot \bar{\mathcal{E}}_n}{\sigma^2}$$

$$\mathbb{E}[\boldsymbol{n} \cdot \boldsymbol{n}^*] = R_{\boldsymbol{nn}} = R_{\boldsymbol{nn}}^{1/2} \cdot R_{\boldsymbol{nn}}^{*/2}$$

Noise-Equivalent Channel

$$\boldsymbol{y} \leftarrow R_{nn}^{-1/2} \boldsymbol{y} = \left(R_{nn}^{-1/2} \cdot H \right) \cdot \boldsymbol{x} + \tilde{\boldsymbol{n}}$$

Replaces H

Section 4.6.1.2

Data Rates and Mutual Information

- For any energies and consequent SNR's

$$\bar{b} = \frac{b}{\bar{N}+\nu} = \frac{1}{2}\cdot\log_2\left(1+\frac{SNR_{vc}}{\Gamma}\right)$$

- When the gap = 0 dB, this is the "mutual information" , reliable $b \leq \bar{\mathcal{I}}(\boldsymbol{x};\boldsymbol{y})$
  - The **mutual information** is the capacity, when (Gaussian) input has given autocorrelation $R_{\boldsymbol{xx}}$, or a more arbitrary energy distribution (say not WF)
  - Good scalar-AWGN code applies "outside" the parallel channel set

$$SNR_{VC} = 2^{2\cdot\bar{\mathcal{I}}} - 1$$

- Maximized SNR, and thus mutual information, occur when energy is water-filling → Capacity

$$\boxed{SNR_{VC,water-fill} = 2^{2\cdot\bar{C}} - 1}$$

- Highest reliable data rate that can be transmitted (Shannon 1948)
  - **For the given block size $\bar{N}$ and guard period ν**

$1+.9D^{-1}$ Vector-Code Example

Section 4.6

Matlab Example

- Form H and do SVD

```
>> C=[.9
zeros(7,1)];
>> R=[.9 1 zeros(1,7)];


>> H=toeplitz(C,R)

H =

  0.9000   1.0000        0        0        0        0        0        0        0
       0   0.9000   1.0000        0        0        0        0        0        0
       0        0   0.9000   1.0000        0        0        0        0        0
       0        0        0   0.9000   1.0000        0        0        0        0
       0        0        0        0   0.9000   1.0000        0        0        0
       0        0        0        0        0   0.9000   1.0000        0        0
       0        0        0        0        0        0   0.9000   1.0000        0
       0        0        0        0        0        0        0   0.9000   1.0000
```

```
>> [F,L,M]=svd(H)
F =
  -0.1612   0.3030  -0.4082   0.4642  -0.4642   0.4082   0.3030  -0.1612
  -0.3030   0.4642  -0.4082   0.1612   0.1612  -0.4082  -0.4642   0.3030
  -0.4082   0.4082   0.0000  -0.4082   0.4082   0.0000   0.4082  -0.4082
  -0.4642   0.1612   0.4082  -0.3030  -0.3030   0.4082  -0.1612   0.4642
  -0.4642  -0.1612   0.4082   0.3030  -0.3030  -0.4082  -0.1612  -0.4642
  -0.4082  -0.4082  -0.0000   0.4082   0.4082   0.0000   0.4082   0.4082
  -0.3030  -0.4642  -0.4082  -0.1612   0.1612   0.4082  -0.4642  -0.3030
  -0.1612  -0.3030  -0.4082  -0.4642  -0.4642  -0.4082   0.3030   0.1612
```

$\boldsymbol{f}_0$ $\boldsymbol{f}_7$

```
L =
   1.8712        0        0        0        0        0        0        0        0
        0   1.7857        0        0        0        0        0        0        0
        0        0   1.6462        0        0        0        0        0        0
        0        0        0   1.4569        0        0        0        0        0
        0        0        0        0   1.2237        0        0        0        0
        0        0        0        0        0   0.9539        0        0        0
        0        0        0        0        0        0   0.6566        0        0
        0        0        0        0        0        0        0   0.3443        0

M =
  -0.0775   0.1527  -0.2232   0.2868  -0.3414   0.3852   0.4153  -0.4214   0.4728
  -0.2319   0.4037  -0.4712   0.4182  -0.2608   0.0428  -0.1748   0.3238  -0.4255
  -0.3583   0.4657  -0.2480  -0.1415   0.4320  -0.4280  -0.1475  -0.1871   0.3830
  -0.4415   0.3099   0.2232  -0.4674   0.1108   0.3852   0.4008   0.0278  -0.3447
  -0.4714   0.0090   0.4712  -0.0208  -0.4705   0.0428  -0.4666   0.1348   0.3102
  -0.4445  -0.2960   0.2480   0.4602   0.0526  -0.4280   0.3140  -0.2812  -0.2792
  -0.3639  -0.4626  -0.2232   0.1806   0.4522   0.3852  -0.0146   0.3936   0.2513
  -0.2395  -0.4127  -0.4712  -0.3975  -0.2097   0.0428  -0.2917  -0.4586  -0.2261
  -0.0862  -0.1697  -0.2480  -0.3187  -0.3794  -0.4280   0.4615   0.4683   0.2035
```

$\boldsymbol{m}_0$ $\boldsymbol{m}_7$

Matlab continued

- Use singular values/channels [1.87 1.78 1.64 1.45 1.22 .95 .66 .34]
- Channel gains $$g_n = \frac{\lambda_n^2}{\sigma^2(=.181)} = [19.3\ \ 17.6\ \ 15.0\ \ 11.7\ \ 8.3\ \ 5.0\ \ 2.4\ \ 0.66]$$
- Water-filling (RA) with 0 dB gap $$K = \frac{1}{7} \cdot \left(9 + \sum_{n=0}^{6} \frac{\Gamma}{g_n}\right) = 1.43$$
- Energies [1.38 1.37 1.36 1.34 1.30 1.23 1.01 0] $$\mathcal{E}_n = K - \frac{\Gamma}{g_n}$$
- SNRs [26.2 24.2 20.4 15.8 10.6 6.2 2.4 0] $$\mathcal{E}_n \cdot g_n$$

Not assigned, but might find Prob 4.17 interesting for look

Overall performance and rate

- Product SNR

$$SNR_{VC} = \left[\prod_{n=0}^{6}(SNR_n + 1)\right]^{1/9} - 1 = 6.46 = 8.1\ dB$$

- Rate = capacity

$$\bar{C} = \frac{1}{9}\cdot\sum_{n=0}^{6}\frac{1}{2}\cdot\log_2(1 + SNR_n) = 1.45\ \text{bits/dimension}$$

```
>> R=[.9 1 zeros(1,99)];
>> C=[.9 zeros(1,99)];
>> H=(1/sqrt(.181))*toeplitz(C,R);
>> [F,L,M]=svd(H);
>> g=diag(L).*diag(L);
```

```
>> K=(1/89)*(101+sum(ones(1,89)./g(1:89)'))
K = 1.3294
>> K-1/g(89) =  -0.0219

>> K=(1/88)*(101+sum(ones(1,88)./g(1:88)'))
K =  1.3292
>> K-1/g(88) = 0.1627

So N* = 88
```

```
>> E=K*ones(1,88)-ones(1,88)./g(1:88)’;

>> snr=E.*g(1:88)';
>> b=(0.5/101)*(1/log(2))*sum(log(ones(1,88)+snr))

b =

   1.5360
```

$\bar{N} \to \infty$, then $\dfrac{\bar{N}+\nu}{\bar{N}} \to 1$, so no loss (max is 1.55)

DMT/OFDM Partitioning

Section 4.7.1-5

See PS1.3 (Prob 4.18)

Cyclic Extension

- Remember this convolution from vectoring coding?

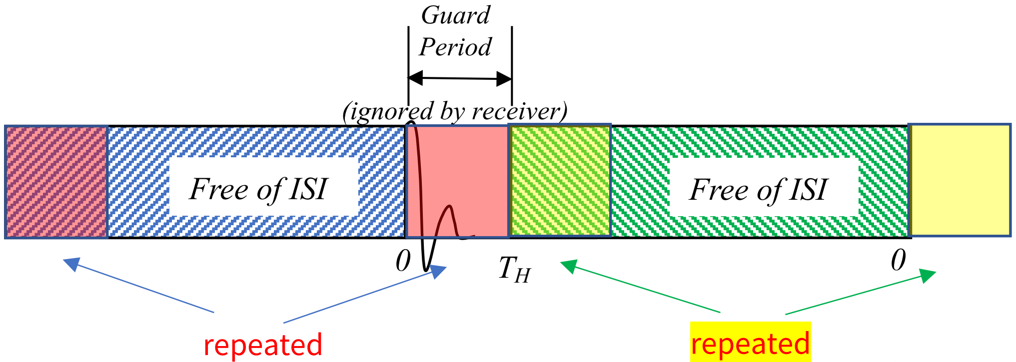
$$\begin{bmatrix} y_{\overline{N}-1}\\ y_{\overline{N}-2}\\ \vdots \\ y_0\end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \dots & h_\nu & 0 & \dots & 0\\ 0 & h_0 & \ddots & h_{\nu-1} & h_\nu & \ddots & 0\\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & h_0 & h_1 & \dots & h_\nu\end{bmatrix}\begin{bmatrix} x_{\overline{N}-1}\\ \vdots\\ x_0\\ x_{-1}\\ \vdots\\ x_{-\nu}\end{bmatrix} + \begin{bmatrix} n_{\overline{N}-1}\\ n_{\overline{N}-2}\\ \vdots\\ n_0\end{bmatrix}$$

guard period

could be anything, including 0 or cyclic

$$\boldsymbol{y} = \tilde{H} \quad \boldsymbol{x} + \boldsymbol{n}$$

- Let the guard period be such that $x_{-i} = x_{\overline{N}-i}$ for *i=1,…,* ν → **CYCLIC PREFIX** or **CYCLIC EXTENSION**
  - **The channel now appears periodic!**



- In fact, "Toeplitz-distribution" limiting results are based on this type of cyclic-extension concept

Ideally $\overline{N} >> \nu$

So small loss of dimensions

Cyclic Convolution

- The matrix expression now uses an *N x N* Circulant Matrix

$$\begin{bmatrix} y_{\overline{N}-1} \\ y_{\overline{N}-2} \\ \vdots \\ y_0 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \dots & h_\nu & 0 & \dots & 0 \\ 0 & h_0 & \ddots & h_{\nu-1} & h_\nu & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & h_0 & h_1 & \dots & h_\nu \\ h_\nu & 0 & \dots & 0 & h_0 & \dots & h_{\nu-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_1 & \dots & h_\nu & 0 & \dots & 0 & h_0 \end{bmatrix} \begin{bmatrix} x_{\overline{N}-1} \\ \vdots \\ x_0 \end{bmatrix} + \begin{bmatrix} n_{\overline{N}-1} \\ n_{\overline{N}-2} \\ \vdots \\ n_0 \end{bmatrix}$$

$\widetilde{H}$ is circulant $\overline{N}\times\overline{N}$

$$\boldsymbol{y} = \widetilde{H} \cdot \boldsymbol{x} + \boldsymbol{n}$$

- As far as output $\boldsymbol{y}$ is concerned, the input is periodic with the same period *N* as the output
- The cyclic prefix is added for each and every symbol
- ν dimensions are lost (both in terms of energy lost and no new information)

Cyclic Convolution and the DFT

- DFT = Discrete Fourier Transform
  - Normalized (maintains squared norm, energy from time ←→ frequency)
  - N or $\bar{N}$

$$X_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{-\jmath \frac{2\pi}{N} kn} \quad \forall \; n \in [0, N-1]$$

$$\boxed{\sum_{n=0}^{N-1} \left|X_n\right|^2 = \sum_{n=0}^{N-1} \left|x_k\right|^2}$$

- IDFT = Inverse Discrete Fourier Transform
  - Symmetrical form

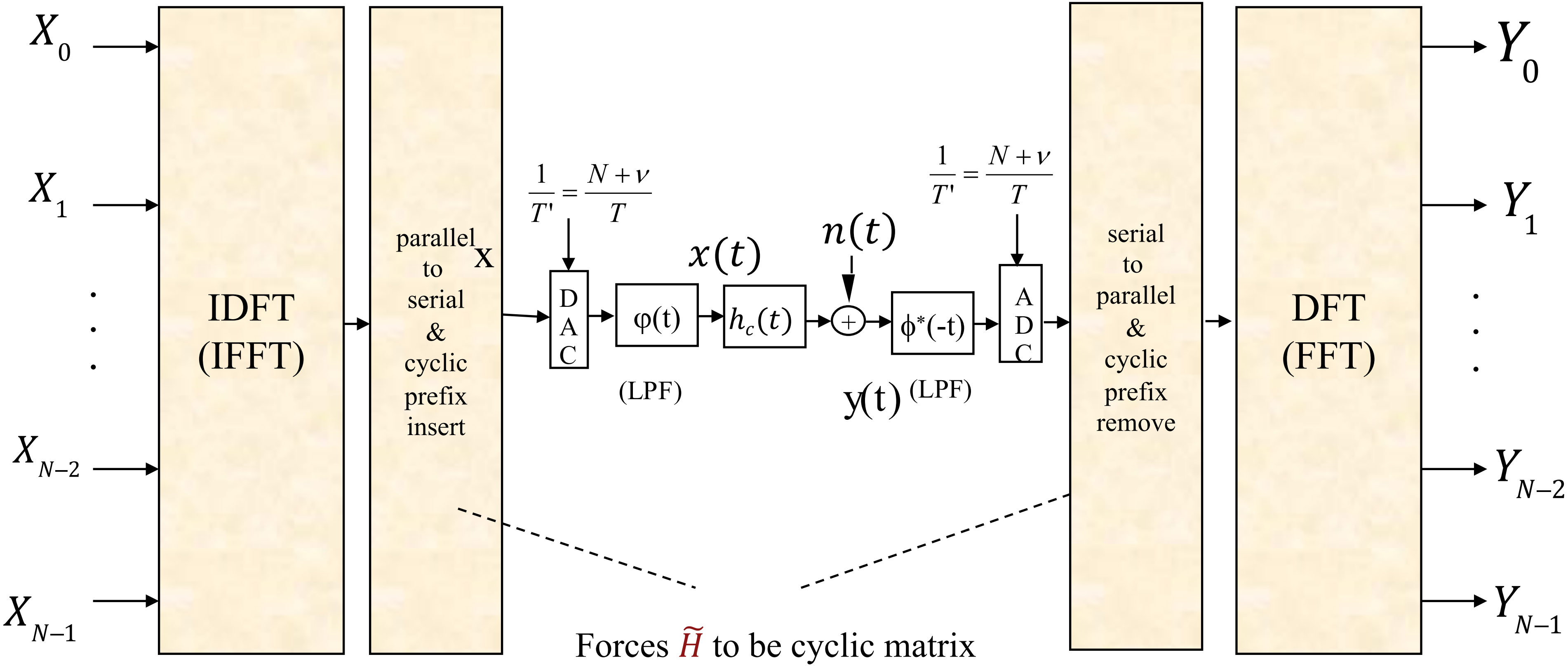
$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{\jmath \frac{2\pi}{N} kn} \quad \forall \; k \in [0, N-1]$$

- Subsymbol channel $Y_n = \widetilde{H}_n \cdot X_n \; (+ N_n)$
  - Vector coding with $M = F^* = Q$, but diagonal can be complex
  - *And the guard period must be cyclic*
  - Still parallel set of subchannels

$$\boxed{\widetilde{H} = Q \cdot \Lambda \cdot Q^*}$$

$$\Lambda = \begin{bmatrix} \widetilde{H}_{N-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \widetilde{H}_0 \end{bmatrix}$$

DMT / OFDM Transmission



X_0

X_1

$\vdots$

X_{N-2}

X_{N-1}

IDFT (IFFT)

parallel to serial & cyclic prefix insert

x

$\frac{1}{T'} = \frac{N+\nu}{T}$

D A C

φ(t)

(LPF)

$x(t)$

$h_c(t)$

$n(t)$

$+$

y(t)

ϕ*(-t)

(LPF)

$\frac{1}{T'} = \frac{N+\nu}{T}$

A D C

serial to parallel & cyclic prefix remove

DFT (FFT)

Y_0

Y_1

$\vdots$

Y_{N-2}

Y_{N-1}

Forces $\tilde{H}$ to be cyclic matrix

$X_{N-i} = X_i^*$ if $x(t)$ is real

If complex, use size $\bar{N}$

Heavily used, wireline & wireless

Product SNR

- 7 subchannels, DC plus 3 two-dimensional QAM subchannels, total of 9 dimensions used

$$\mathrm{SNR}_{DMT} = \left[\prod_{n=0}^{6}\left(1+\mathrm{SNR}_n\right)\right]^{1/9} - 1 = 7.6\ \mathrm{dB} \quad < \mathit{SNR}_{vc}$$

- But no channel-dependent partitioning, and much easier to implement (N log (N) vs N^2)

- How can we exploit this??

INCREASE N !

- DMTra and DMTma will help

$1{+}.9D^{-1}$ revisited

- Circulant Channel is 8 x 8 (but wastes 1 dimension in cyclic extension, and loses its energy)

$$\tilde{H} = \begin{bmatrix} .9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .9 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .9 & 1 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & .9 \end{bmatrix}$$

- Channel FFT (size 8) leads to

Water-fill (Γ=0 dB) with $\mathcal{E}_x = 8$

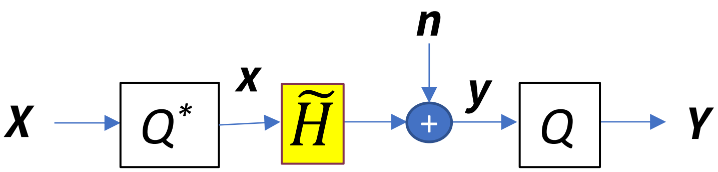
n	$\lambda_n = \|P_n\|$	$g_n = \frac{\|P_n\|^2}{.181}$	$\mathcal{E}_n$	SNR_n	b_n
0	1.90	20	1.24	24.8	2.34
1	1.76	17	1.23	20.9	2.23
6	1.76	17	1.23	20.9	2.23
2	1.35	9.8	1.19	11.7	1.85
5	1.35	9.8	1.19	11.7	1.85
3	.733	3	.96	2.9	.969
4	.733	3	.96	2.9	.969
7	.100	.05525	0	0	0

- $\bar{b} = 1.38$

See PS1.4 (Prob 4.7)

Partitioning with DFT

- DFT Partitioning



- Set of Parallel Channels – A "Discrete MultiTone" Partitioning
    - Some call it OFDM, but there is a difference in loading (DMT optimizes loaded energy, OFDM fixes equal energy)

$$\boxed{SNR_n = \frac{\mathcal{E}_n \cdot \left|\widetilde{H}_n\right|^2}{\sigma^2}}$$

Noise-Equivalent Channel

$$E\left[\boldsymbol{n}\boldsymbol{n}^*\right] = R_{nn}\sigma^2 = R_{nn}^{1/2} R_{nn}^{-1/2}\sigma^2$$

$$\boldsymbol{y} \leftarrow R_{nn}^{-1/2}\boldsymbol{y} = \left(R_{nn}^{-1/2} H\right)\boldsymbol{x} + \tilde{\boldsymbol{n}}$$

- Receiver is DFT (and noise remains white)

- Transmitter is IDFT (no power increase)

Neither is a function of the channel

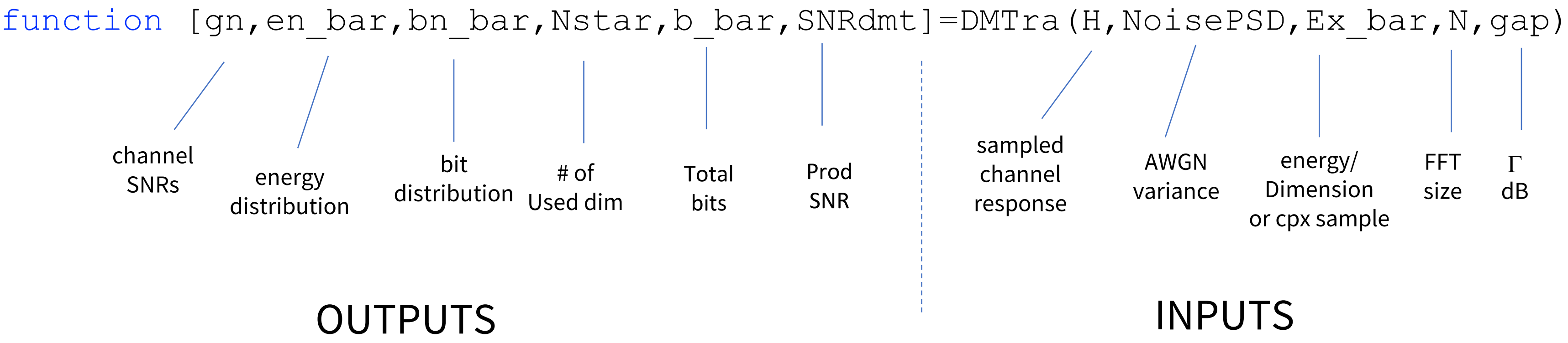
Can use efficient "FFT"

DMT Water-Filling Software

Subsection 4.7.4

See PS1.4 (Prob 4.7) and PS1.5 (Prob 4.9)

Rate Adaptive DMT



```
function [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra(H,NoisePSD,Ex_bar,N,gap)
```

```
>> [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,8,0)

gn =   19.9448   17.0320   10.0000    2.9680    0.0552    2.9680   10.0000   17.0320
en_bar =    1.2415    1.2329    1.1916    0.9547         0    0.9547    1.1916    1.2329
bn_bar =    2.3436    2.2297    1.8456    0.9693         0    0.9693    1.8456    2.2297
Nstar =     7
b_bar =    1.3814
SNRdmt =    7.6247 dB
```

In using the program, know if your channel is truly baseband or complex baseband equivalent

In real case, each dimension shown is a real dimension

Increase N

- [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,16,0);
  - >> SNRdmtSNRdmt =   8.1152
- [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,1024,0);
  - >> SNRdmtSNRdmt =   8.7437
- Feel free to experiment, PS1.4 goes better if you use this.

How about non-zero gap?

- SNR can look higher, but bit rate is overall lower

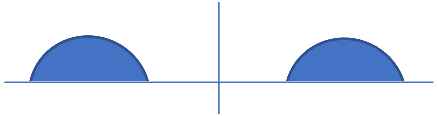
```
>> [gn,En,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,8,8.8)

gn = 19.9448   17.0320   10.0000    2.9680    0.0552    2.9680   10.0000   17.0320

En = 1.7773    1.7123    1.3991         0         0         0    1.3991    1.7123

bn_bar = 1.2521    1.1382    0.7540         0         0         0    0.7540    1.1382

Nstar = 5

b_bar = 0.5596

SNRdmt = 9.4904 dB  (remember this gets divided by the gap)

>> En.*gn = 35.4481   29.1634   13.9907         0         0         0      13.9907  29.1634
               24.7613   20.9991   11.9163    2.8336    0    2.8336   11.9163   20.9991 (Γ=0)

>> 10*log10(ans)  = 15.4959   14.6484   11.4584     -Inf     -Inf     -Inf   11.4584   14.6484
   (these subchannel SNR's also get divided by gap)
```

Data rate is roughly 1/3 of before

Note SNRdmt increase for nonzero gap

Suppose $1+.9D^{-1}$ were complex baseband?

- The channel effectively has twice as many dimensions
  - Same results would be for 8 complex dimensions
  - So bbar, ebar are really bits/tone and energy/tone
  - Same values mean the constellations on each tone are two dimensional though



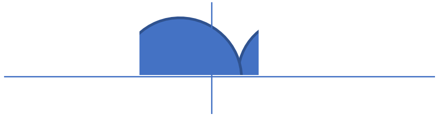
```
>> [gn,En,bn,Nstar,b,SNRdmt]=DMTra([.9 1],.181,2,8,0)

gn =  19.9448   17.0320   10.0000    2.9680    0.0552    2.9680   10.0000   17.0320

En = 2.3843    2.3758    2.3345    2.0976         0    2.0976    2.3345    2.3758

bn = 2.8008    2.6869    2.3028    1.4266         0    1.4266    2.3028    2.6869

Nstar = 7

b = 1.7370

SNRdmt = 10.0484 dB

>> sum(En) = 16.0000      ;     >> sum(bn) =   15.6332
```

- The bits/tone though is slightly larger.
  - 1.73 > 1.38
- What happened?
- The “DC” tone is now complex and has an additional good dimension.

Suppose $1+.9jD^{-1}$ - *must be* complex baseband

- The channel definitely has twice as many real dimensions

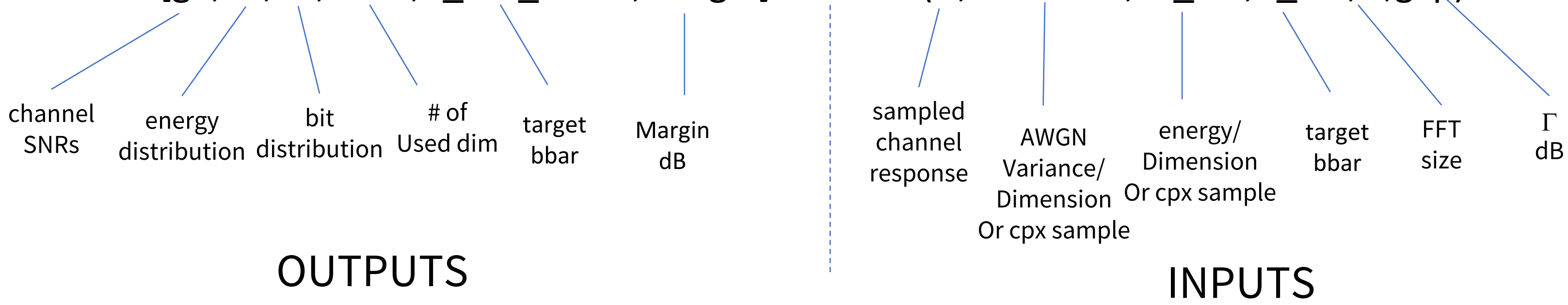
```
>> [gn,En,bn,Nstar,b,SNRdmt]=DMTra([.9*i 1],.181,2,8,0)

gn = 10.0000   17.0320   19.9448   17.0320   10.0000    2.9680    0.0552    2.9680

En = 2.3345    2.3758    2.3843    2.3758    2.3345    2.0976         0    2.0976

bn = 2.3028    2.6869    2.8008    2.6869    2.3028    1.4266         0    1.4266

Nstar = 7

b = 1.7370

SNRdmt = 10.0484

>> sum(En) = 16.0000      ;     >> sum(bn) = 15.6332
```



This channel rotates the earlier one in time, so circular shift in frequency

Margin Adaptive DMT

- function [gn,en,bn,Nstar,b_bar_check,margin]=DMTma(H,NoisePSD,Ex_bar,b_bar,N,gap)



channel SNRs

energy distribution

bit distribution

# of Used dim

target bbar

Margin dB

sampled channel response

AWGN Variance/ Dimension Or cpx sample

energy/ Dimension Or cpx sample

target bbar

FFT size

Γ dB

OUTPUTS

INPUTS

```
>> [gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,8,0)
gn =   19.9448   17.0320   10.0000    2.9680    0.0552    2.9680   10.0000   17.0320
en =    0.6043     0.5958     0.5545     0.3175        0        0.3175    0.5545     0.5958
bn =    1.8532    1.7393     1.3552     0.4790        0        0.4790    1.3552    1.7393
Nstar =      7
b_bar_check =      1
margin =    3.5410
```

Works real or complex, but (again) careful

Continuing

- `>> [gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,8,8.8)`
- `gn =   19.9448   17.0320   10.0000    2.9680    0.0552    2.9680   10.0000   17.0320`
- `en =    4.5844    4.5193    4.2061    2.4088         0    2.4088    4.2061    4.5193`
- `bn =    1.8532    1.7393    1.3552    0.4790         0    0.4790    1.3552    1.7393`
- `Nstar =     7`
- `b_bar_check =    1.0000`
- `margin =   -5.2590`

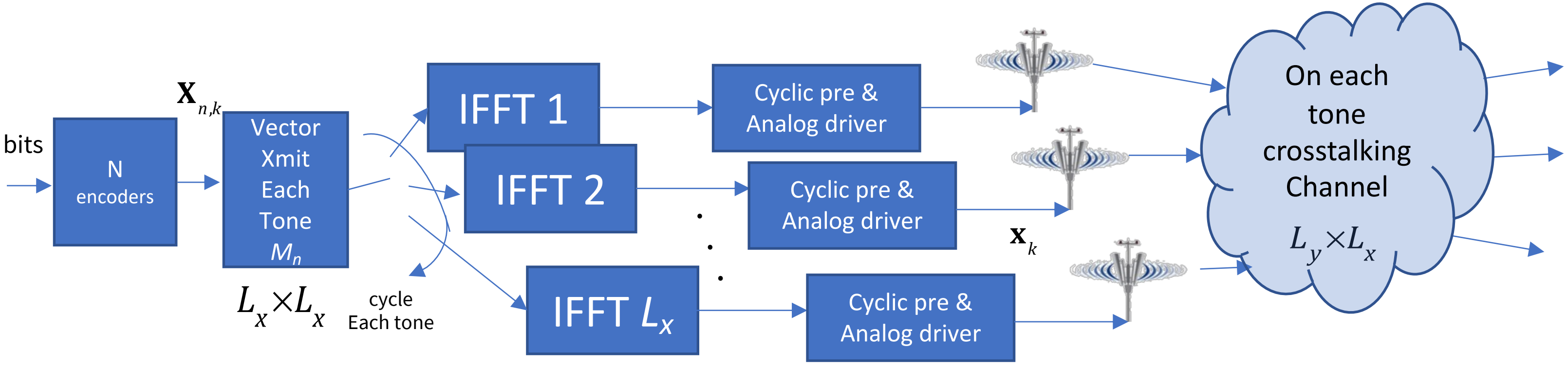
Negative margin – can’t do it!

```
[gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,16,0);

>> margin =    4.1445
[gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,1024,0);

>> margin =    4.7267
```

Vector DMT/OFDM

Section 4.7

Vector DMT/OFDM Transmitter



bits

N encoders

$\mathbf{X}_{n,k}$

Vector Xmit Each Tone M_n

$L_x \times L_x$

cycle Each tone

IFFT 1

IFFT 2

IFFT L_x

Cyclic pre & Analog driver

Cyclic pre & Analog driver

Cyclic pre & Analog driver

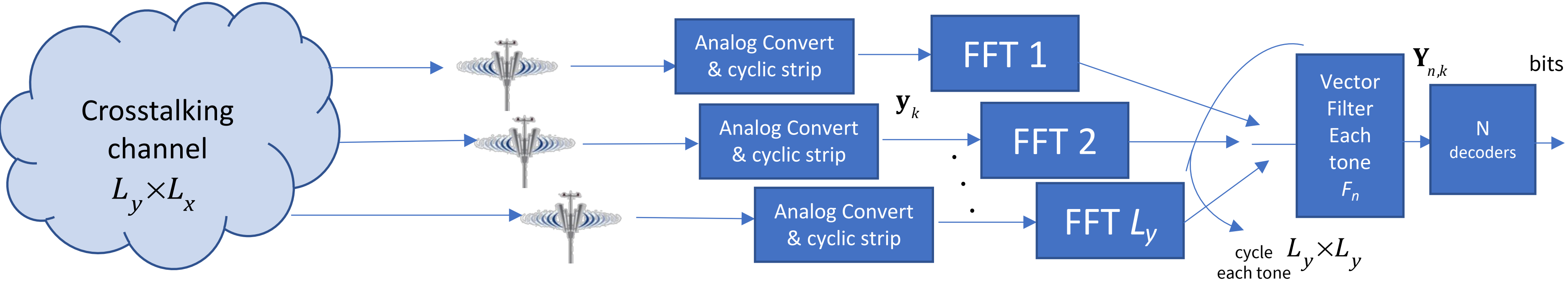
$\mathbf{x}_k$

On each tone crosstalking Channel

$L_y \times L_x$

$$NL_x^2 + L_x N \log_2(N) << (L_x N)^2$$

Vector DMT/OFDM Receiver



$$\widetilde{H}_n = F_n \cdot \Lambda_n \cdot M_n^*$$

$$\boxed{NL_y^2 + L_y N \log_2(N) << (L_y N)^2}$$

- Just much larger number of dimensions, each a scalar AWGN, $L = min(L_x, L_y)$
- $L \cdot N$ dimensions
- Can water-fill over them all (if total energy constraint, which is common)

STANFORD

End Lecture 2