STANFORD

Lecture 1

Introduction & Dimensionality

April 3, 2023

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Instructor EE392AA – Spring 2023

Announcements & Agenda

- Announcements
  - People Introductions
  - Web site https://cioffi-group.stanford.edu/ee392aa/
  - Chapters 1-5 are used, on-line at class web site (Course Reader)
  - Review/scan Section 1.3.4-7 ; read 2.1-5 ; 4.1-3
  - Chapter 3 (not necessary, equalization and ISI)
    - Supplementary files at canvas for your interest/review (contact Yun if interested in special section)

John M. Cioffi

Room 363, David Packard

Welcome | Course Info | Course Reader | Lecture Notes | Handouts | Homework | Matlab Code

Spring Quarter 2023

EE 392AA - Multiuser Data Transmission

Instructor : Prof. John Cioffi

Teaching Assistant : Yun Liao

Course Secretary : Helen Niu

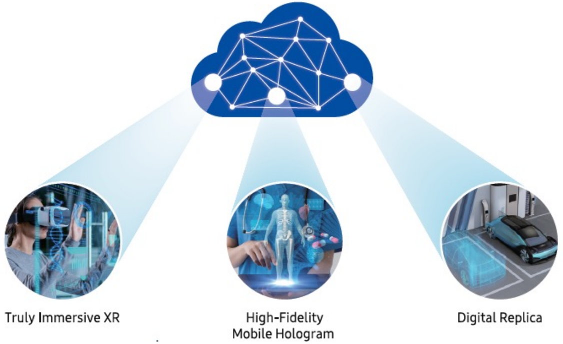
Lectures : Monday and Wednesday, 15:30-16:45, in class

- Today
  - Course introduction
  - The scalar AWGN channel (a foundation)
  - The matrix AWGN channel
  - Water-filling energy distribution
  - Projecting forward

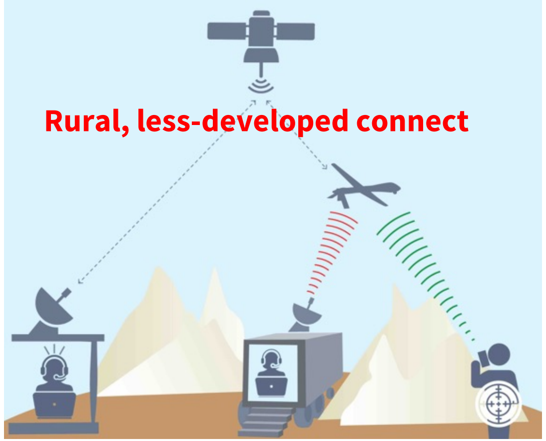
- Problem Set 1 = PS1 due Wednesday April 12 at 17:00
  1. 2.15 capacity refresher (read "subsymbol" = "symbol" here)
  2. 4.3 builds intuition on gap-based 1-dimensional channel analysis
  3. 4.18 DMT water-fill loading
  4. 4.7 Simple Water-fill Loading
  5. 4.25 Matrix AWGN & vector coding with water-fill

Why Communications?

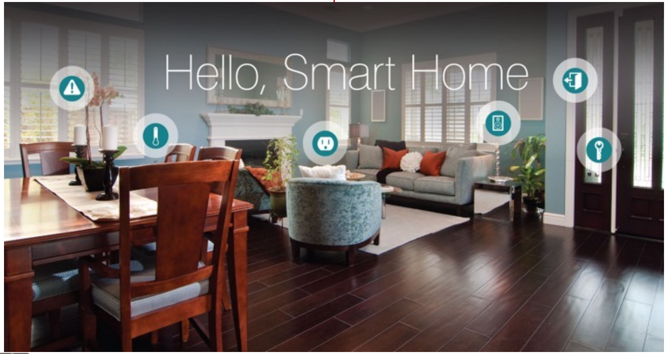
Next Generation Connectivity

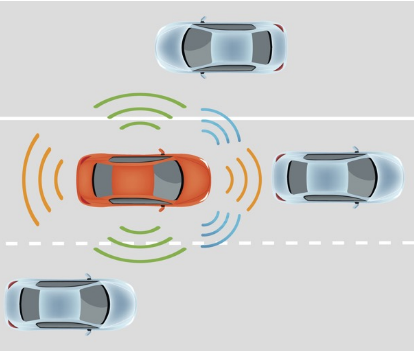


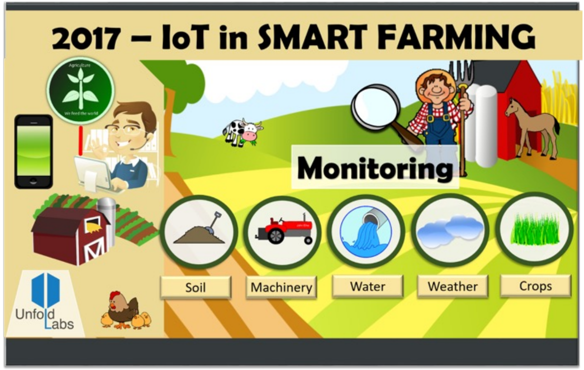
Beam "me" there, Scotty



Digital Twins used to forecast/emulate each







Defense ("5/6G.mil")
Clothing, Computing, …

Samsung: 6G "hyper connected"

Broadband Internet Access ($1.5T/year)

- Messages
  - Internet
  - Email
  - Text
  - video, audio
  - Sensor/camera images

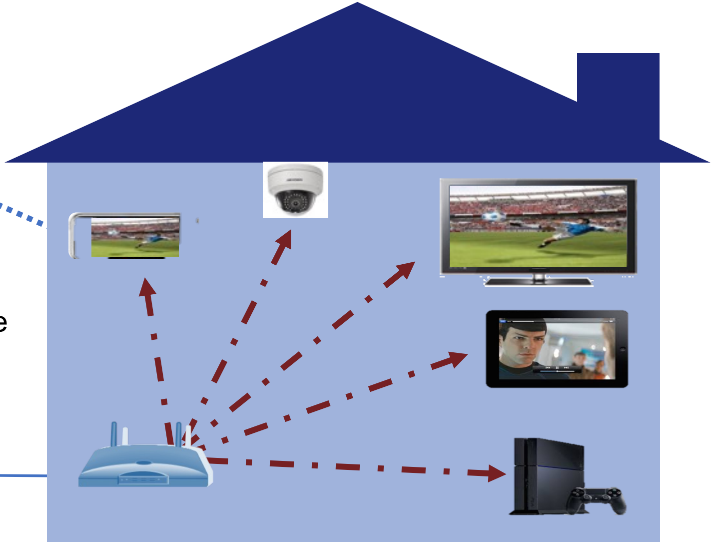
many antennas

“front/X”-haul
Split 7.2

(almost) All signal processing, modulation, and coding done in software at edge with shared computing facility

Network Edge (SDN)

Channel
- Fiber, copper
- Wireless
- Many or single
- Mesh



OSI Model

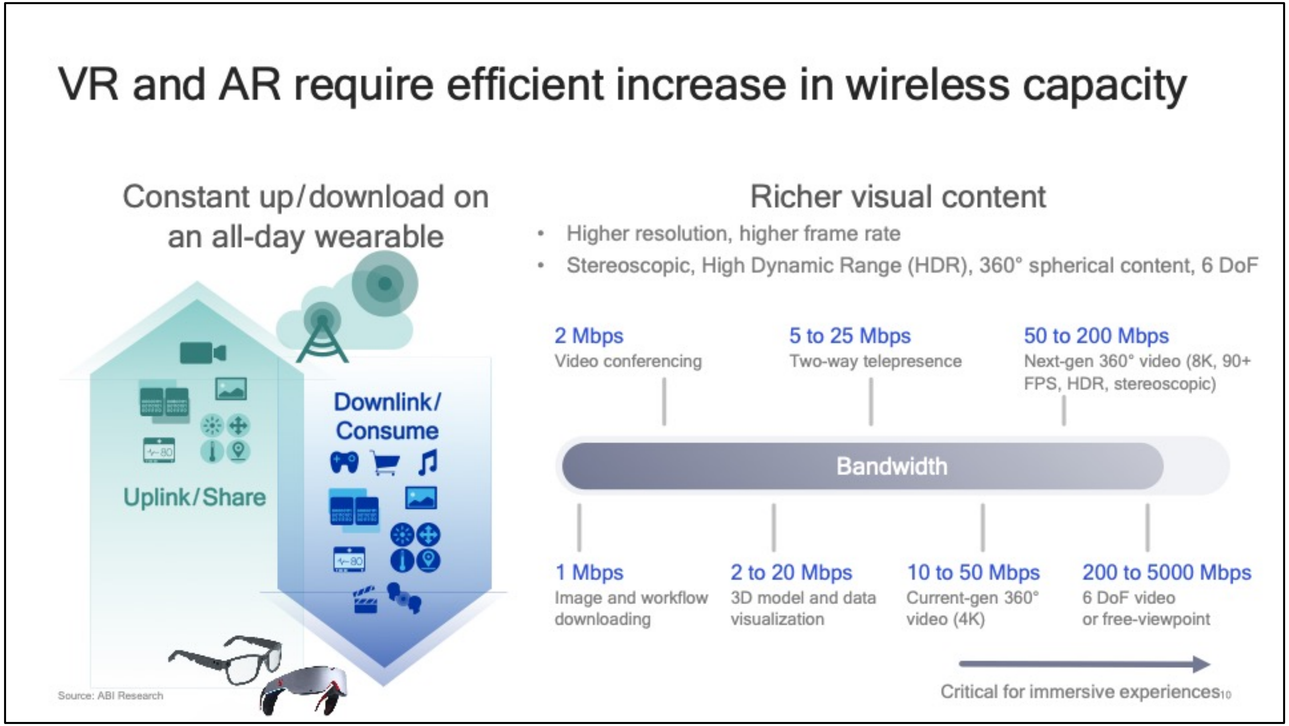
	Layer	Function	Example Protocols
7	Application Layer	network process to application	HTTP, SFTP, SSH
6	Presentation Layer	data representation & encryption	XML, JSON
5	Session Layer	interhost communication	Mostly theoretical
4	Transport Layer	end-to-end connections & reliability	TCP, UDP
3	Network Layer	path determination & logical addressing	IP Addresses
2	Data Link Layer	physical addressing	MAC Addresses
1	Physical Layer	medial signal & transmition	Ethernet, Bluetooth, Wireless

this class

OSI = **Open** Systems Interconnect

SDN = **Software** Defined Network

VR/AR focus and Bandwidth



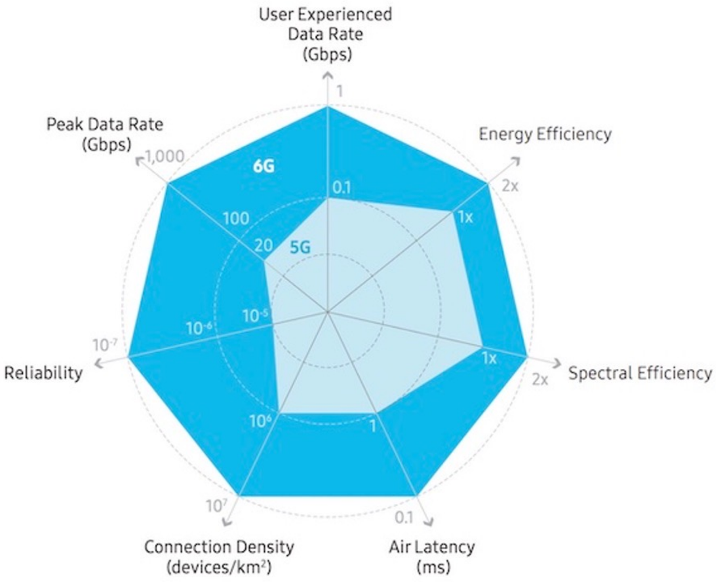
Latency:

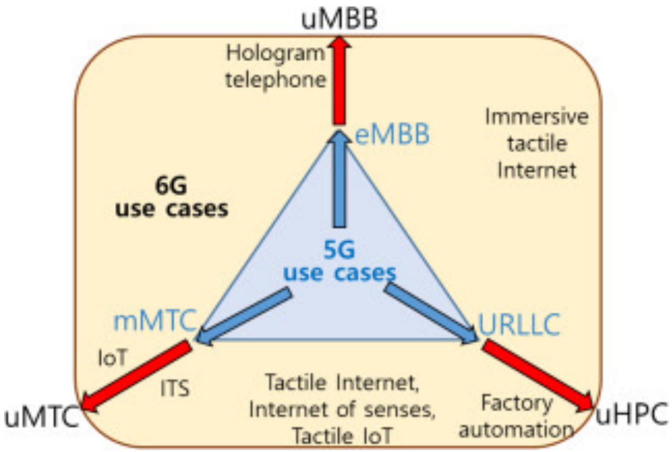
Edge ~ 1 ms

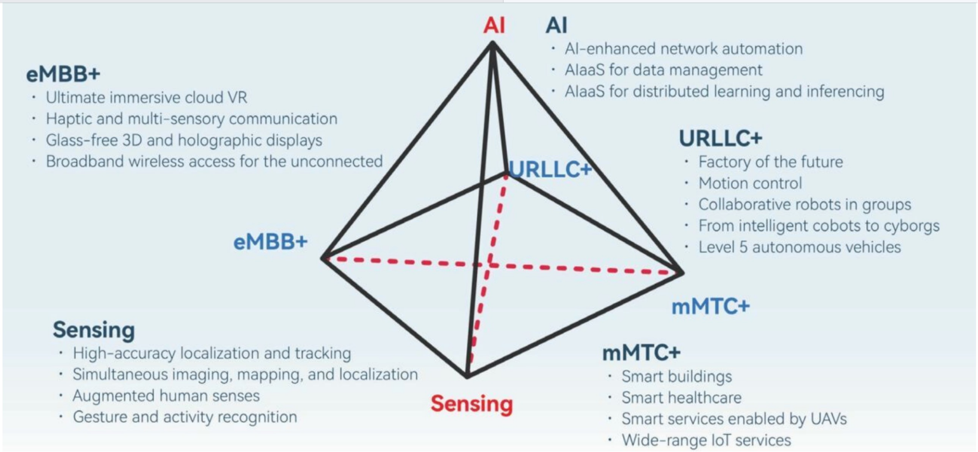
ISP Cloud 20-50 ms

Public Cloud 100 ms

Popular Com Standards Summaries

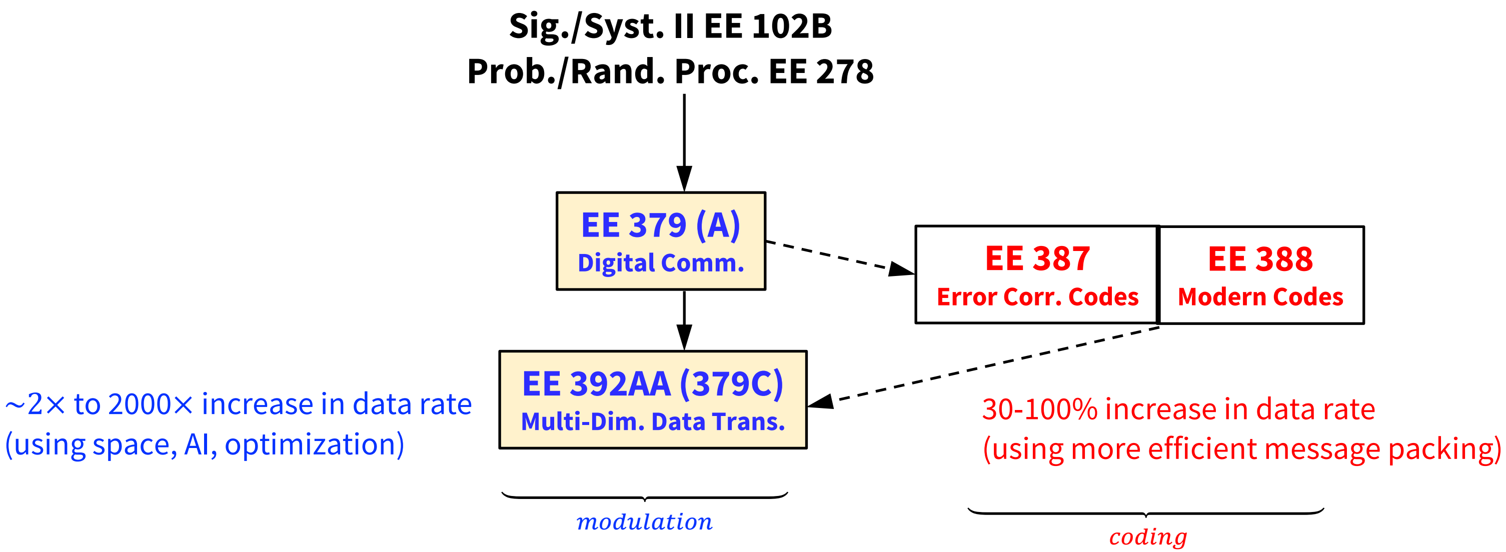






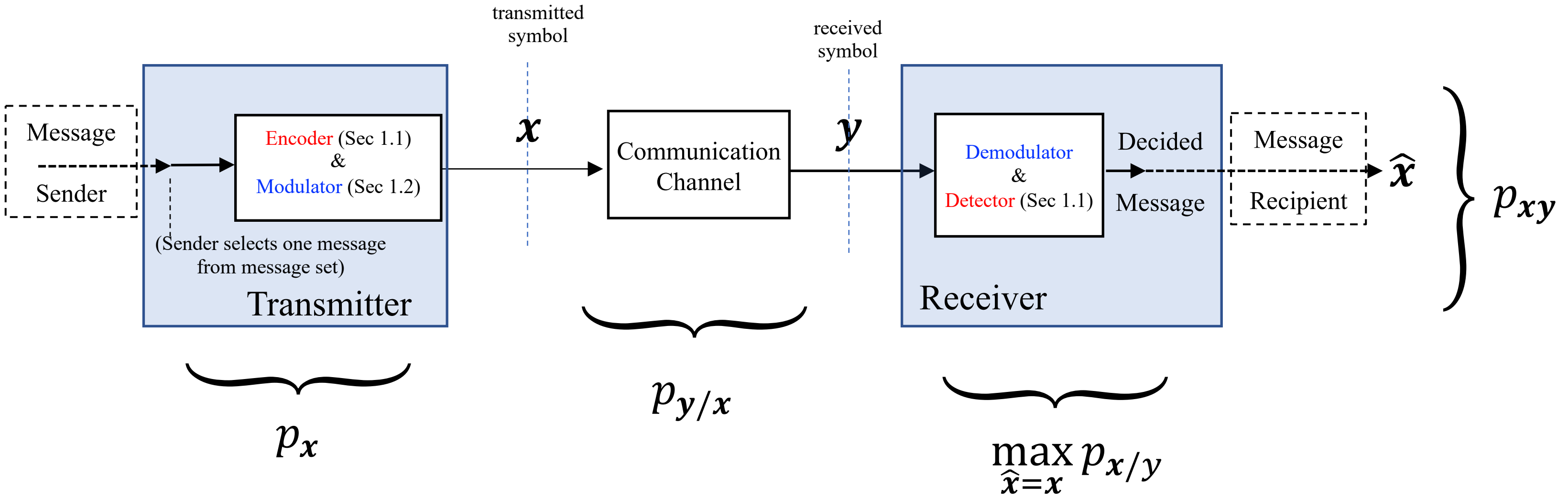
Course Introduction

Communications Depth Sequence



- Modulation and Coding compliment one another
  - Modulation = energy assignment to time/frequency/space, is separate from:
  - Coding = distinct message mapping
  - If both done well, they separate

Basic Communication (digital)



- The symbol x and messages are in some 1-to-1 relationship
- Finding the best $\hat{x}$ and designing x well → this class (good 1-to-1 assumed)
- Most general channel is represented by the conditional probability $p_{y/x}$.
- Most general source description is p_x - together, p_{xy} .
- Optimum detector (minimizes ave error probability) is Maximum a Posteriori (MAP), max $p_{x/y}$
  - When input distribution is uniform → ML (maximum likelihood), max $p_{y/x}$

3 Basic Problems to Solve

- **CHANNEL IDENTIFICATION** - what is $p_{\boldsymbol{y}/\boldsymbol{x}}$?

- **CODING & MODULATION** - What are good (best) $\boldsymbol{x}$ and $p_{\boldsymbol{x}}$ for a given channel?

- **DETECTION** – What is a good (best) receiver for deciding which $\boldsymbol{x}$?

Especially with more than 1 user (so expanding on 379A)

Dimensionality

- Input $\boldsymbol{x}$ and channel $\boldsymbol{y}$ are vectors

- Simple dimensions
  - **time** (samples, slots, packets)
  - **frequency** (carriers, tones/subcarriers, bands)
  - **space** ("antennas")

  } next slide

- Exotic Extensions from Physics
  - higher-order modes (TM(m,n))
  - orbital angular momentum
  - quantum communication

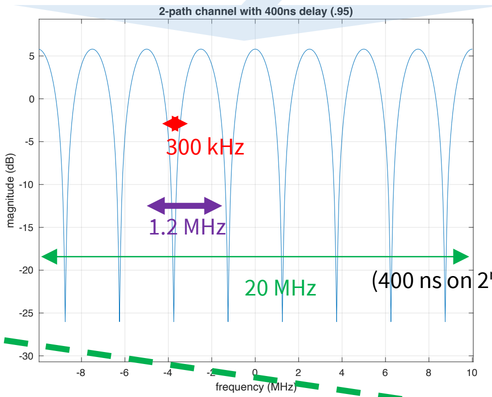
Communication *Dimensionality*

- Time-Frequency (any fixed location)
  - 2 x bandwidth = # of dimensions/sec (wireless or wired, including “optical” – all are EM waves)

2G (2 dim/ 3.7 μs)

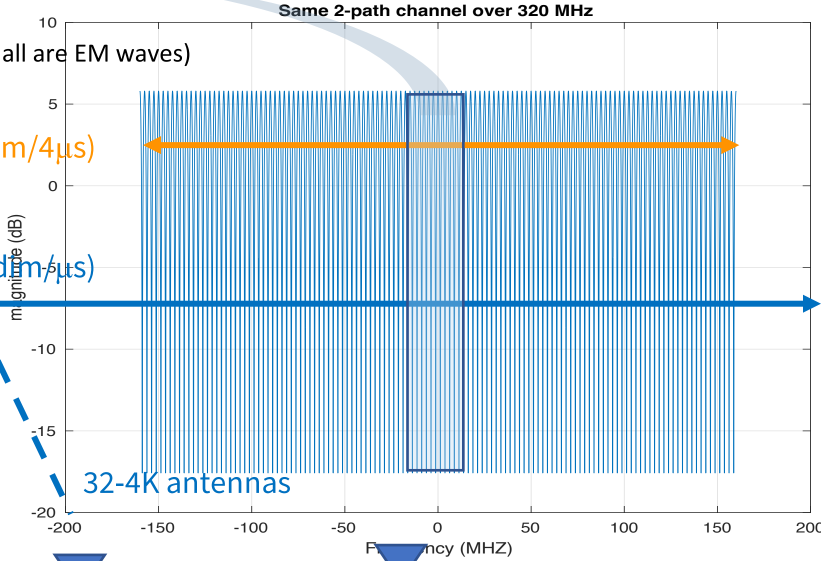
3G (128 dim/ 533μs)

4G/Wi-Fi (160 dim/4μs)



5G / Wi-Fi6 (1280 dim/4μs)

6G / Wi-Fi7 (~1000 dim/μs)

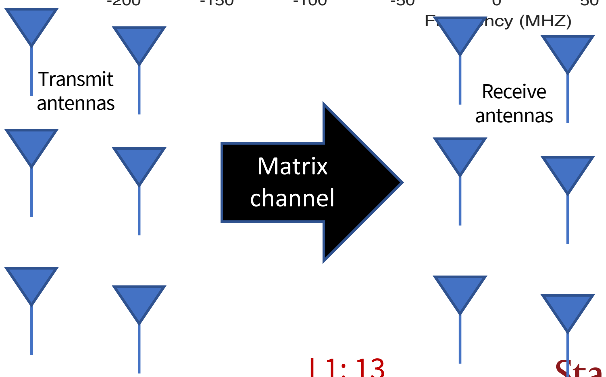


8-32 antennas

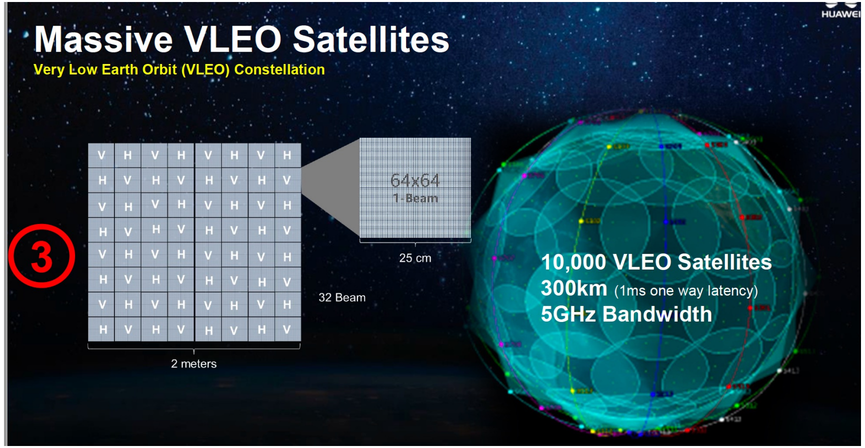
2-4 antennas

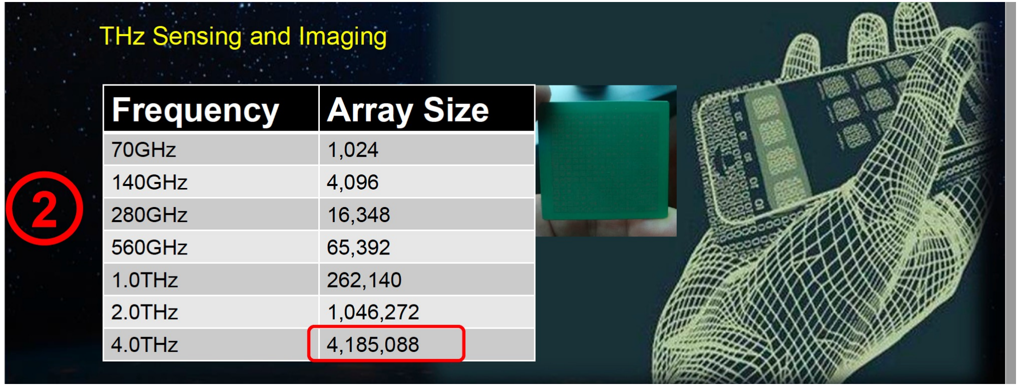
32-4K antennas

- Space-Time
  - 2D - 3D (at least …..)
  - Spacing of half wavelength or more
  - 10k – 1M dimensions per few microseconds
  - Number of channels can be up to # of antennas “spatial streams”
  - Also can be crosstalk between wires (e.g., ethernet, telco lines)
    - Optical xtalk is between “modes” at same frequency (a spatial effect)



Even More Dimensions (smaller wavelengths)





Frequency	Array Size
70GHz	1,024
140GHz	4,096
280GHz	16,348
560GHz	65,392
1.0THz	262,140
2.0THz	1,046,272
4.0THz	4,185,088

- How do we design these systems for best rates (per energy) use?
- How adaptive do they need to be?

The scalar AWGN channel

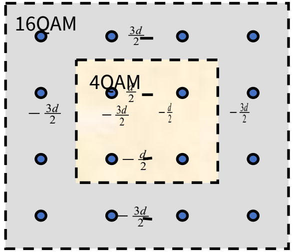
(a foundation: Section 1.3, Section 2.1-3
direct: 2.4.1, 2.4.3)

See PS1.1 (Prob 2.15 - capacity) and PS1.2 (Prob 4.3 gap)

Simple Additive White Gaussian Noise Channel

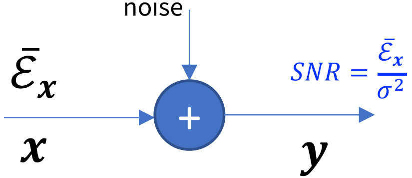
Detection Problem First, every T seconds (symbol period)

symbol/codeword
(M=16, # of messages)



$\mathcal{E}_x = \mathbb{E}[|x|^2]$ $\quad p_{\boldsymbol{x}}$

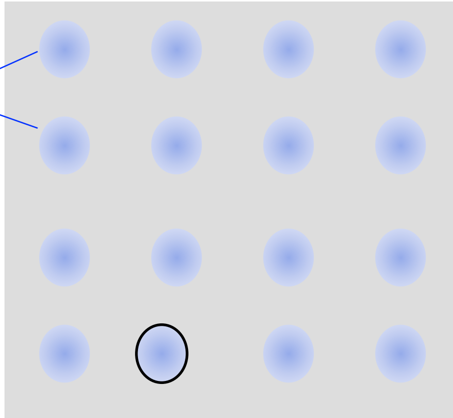
$\boldsymbol{n} \sim \sigma^2 \left(= \frac{\mathcal{N}_0}{2}\right)$



$p_{\boldsymbol{y}/\boldsymbol{x}}$

$= p_{\boldsymbol{n}}(\boldsymbol{y} - \boldsymbol{x})$

channel-output codeword



$\max_{\hat{\boldsymbol{x}} = \boldsymbol{x}} p_{\boldsymbol{y}/\boldsymbol{x}}$

$\hat{\boldsymbol{x}}$

- Min error probability P_e
- Pick closest point for AWGN
- Maximum Likelihood
  - Uniform input dist'n

"SISO"
(scalar)
$N = 1 \quad \text{or} \quad 2$

Subsymbol if coded

$\boldsymbol{x} \rightarrow \tilde{\boldsymbol{x}} \in \mathbb{R}\ ;\ \boldsymbol{x} \rightarrow \tilde{\boldsymbol{x}} \in \mathbb{C}$

$\boldsymbol{x}$ has N real dimensions in general, and has $\bar{N}$ subsymbols, of dim $\tilde{N}$

- QAM → 2 dimensional
- Uniform input (usually) $p_x = \frac{1}{M}$
- $b = \log_2 M$ **bits/symbol**
- $R = \frac{b}{T}$ bits/second (**data rate**)

- Add noise
- Zero mean
- Variance σ^2 (= 2-sided PSD)

$$P_e = 4 \cdot \left(1 - \frac{1}{\sqrt{M}}\right) \cdot Q\left(\sqrt{\frac{3 \cdot SNR}{M-1}}\right)$$

SNR, QAM, PAM reminders

$$SNR \triangleq \frac{\bar{\mathcal{E}}_x}{\sigma^2} = \frac{\text{single} - \text{sided psd}}{\text{single} - \text{sided psd}} = \frac{\text{two} - \text{sided psd}}{\text{two} - \text{sided psd}}$$

- SNR must have the same number of dimensions in numerator (signal) and denominator (noise)
- Thus, also $SNR \triangleq \frac{\bar{\mathcal{E}}_x}{\sigma^2} = \frac{2 \cdot \bar{\mathcal{E}}_x}{\mathcal{N}_0} = \frac{\mathcal{E}_x}{N \cdot \sigma^2}$ where $\bar{\mathcal{E}}_x$ is energy/real-dimension.
- Energy/dimension essentially generalizes the term power/Hz (= energy) so that is why these quantities are related to power-spectral densities (psd's)
  - 1-sided → power is integral over positive frequencies of psd
  - 2-sided → power is integral over all frequencies of psd
  - These two powers are the same
  - So -40 dBm/Hz (one-sided) psd over 1 MHz is 20 dBm, or 100 mWatts of power , *practice PS1.1 (Prob 2.15) and Homework Helper 1's first part*
- PAM is always real baseband. QAM is always complex baseband (2 real dimensions)
  - **When QAM** has only 1 bit (2 points) in constellation, it is called BPSK (not binary PAM).
  - PAM's positive-frequency bandwidth is [0, 1/2T) .... x $(1 + \alpha)$ when there is $(100 \cdot \alpha)$ percent excess bandwidth
  - QAM's positive-frequency bandwidth is [-1/2T + f_c , 1/2T + f_c) .... "
  - The PAM system looks like it uses only 1/2 the bandwidth, but the QAM system is really transmitting two dimensions per symbol (so really like 2 PAM systems in parallel with symbol rate 1/T each), so no wonder it takes twice the bandwidth of a single PAM to do so

Codes and Gaps

Shannon's maximum reliable data rate "capacity"

$$\mathcal{C} = log_2(1 + SNR) \text{ bits/complex-subsymbol}$$

AWGN Max bits/sub-sym for $P_e \to 0$ (reliably decodable)

$\tilde{\boldsymbol{x}}_1$	$\tilde{\boldsymbol{x}}_2$	$\tilde{\boldsymbol{x}}_3$	$\cdots$	$\tilde{\boldsymbol{x}}_{\bar{N}}$

$codeword\ (symbol)\ \boldsymbol{x}$

Good Code $\tilde{b} \to \mathcal{C}$ as $\bar{N} \to \infty$

$subsymbols$ $N = \bar{N} \cdot \tilde{N} =$ # subsymbols x (dim/subsymbol)

Section 2.1.1; also PS1.1 (Prob 2.15)

bits/dim = $\bar{b} = {}^{b}/_{N}$; **bits/subsym =** $\tilde{b} = {}^{b}/_{\bar{N}} = \tilde{N} \cdot \bar{b}$

Code construction

- QAM/PAM operates with given low P_e (10^{-6}) and at a "SNR gap" ($\Gamma = 8.8\ dB$ @10^{-6}) below capacity
  - See basics in Section 1.3.4 – *for practice, see Section 2.4; also PS1.2 (Prob 4.3)*

$$\tilde{b} = log_2\left(1 + \frac{SNR}{\Gamma}\right) \text{ bits/complex-subsymbol} \leq \mathcal{C}$$

$\frac{3}{2^{\tilde{b}}-1} \cdot SNR = 13.5\ dB$ (from $P_e = 10^{-6}$ formula)

- For all $\tilde{b} > 1$, simple square QAM constellations have constant gap (= 8.8 dB at $P_e = 10^{-6}$)

It's like noise increased or power decreased for P_e
(where Γ approaches 0 dB for best codes)
Gap is function of code and of P_e , not $\tilde{b}$

Margin

$$\tilde{b} = log_2\left(1 + \frac{SNR}{\Gamma \cdot \gamma_m}\right) \text{ bits/complex-subsymbol } \leq \mathcal{C}$$

See also PS1.2 (Prob 4.3)

- The designer wants a little "margin" protection against possible noise-power increase
- **MARGIN** γ_m is this protection (usually in dB), $\gamma_m = \frac{(SNR/\Gamma)}{2^{\tilde{b}}-1}$

Positive margin – means performing well ; **Negative margin** – means not meeting design goals

- AWGN with SNR = 20.5 dB, then $\tilde{C} = log_2\,(1 + 10^{2.05})$ = 7 bits/subsymbol
- Suppose that 16-QAM ($\tilde{b} = 4$) is transmitted @ $P_e = 10^{-6}$ ($\Gamma = 8.8$ dB), then $\gamma_m = \frac{10^{2.05-.88}}{2^4-1} = 0$ dB
- Suppose instead QAM with $\tilde{b}$ =5 bits/complex-subsymbol with a code and gain 7 dB of gain ($\Gamma \rightarrow$ 8.8-7=1.8 dB)
  - $\gamma_m = \frac{10^{2.05-.18}}{2^5-1} = 3.8$ dB
- 6 bits/subsymbol with same code? → 0.7 dB margin – just barely below the desired P_e ; $\bar{P}_e = {}^{P_e}/_{N}$

EE 392AA (379C)

- The simple single-dimension AWGN is fundamental to most all designs

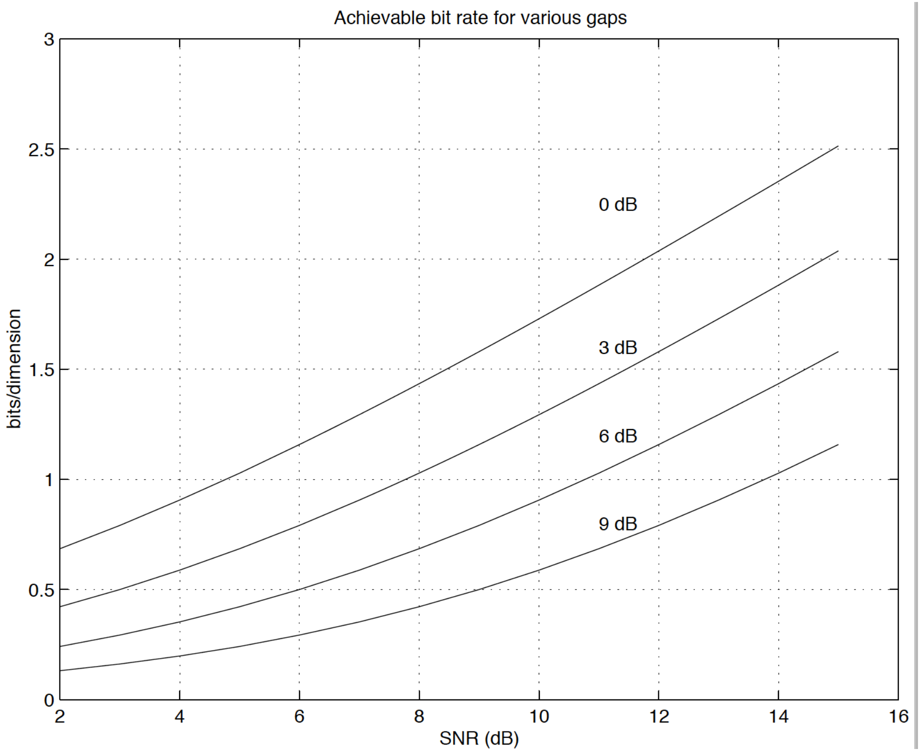
- All subsequent designs will depend on good codes (small or 0 dB gap) re-use on those single dimension AWGNs

- Designs can be optimized to get highest possible data rates for Gaussian noise
  - Single user (of course)
  - All multiuser
  - Channels with crosstalk between dimensions
    - Intersymbol interference
    - Crosstalk
    - Spatial reflections, multi-paths
  - Many users with many antennas, high/low data rates, crosstalking wires and different locations

- This is where the big gains occur

Gap Plot & Example

- The gap is constant, independent of the bits/dimension – greatly simplifies “loading” (adapting transmission codes to the channel)

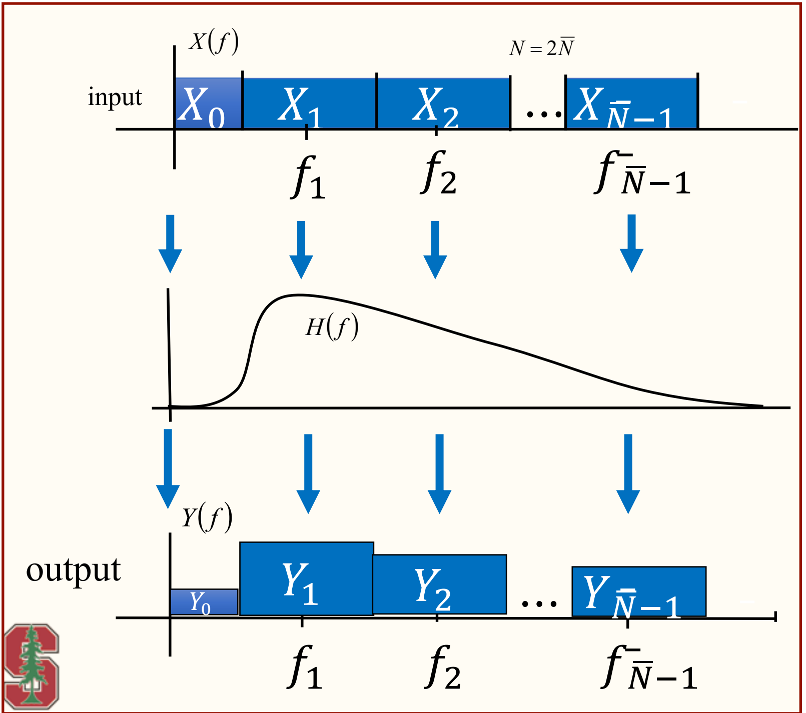


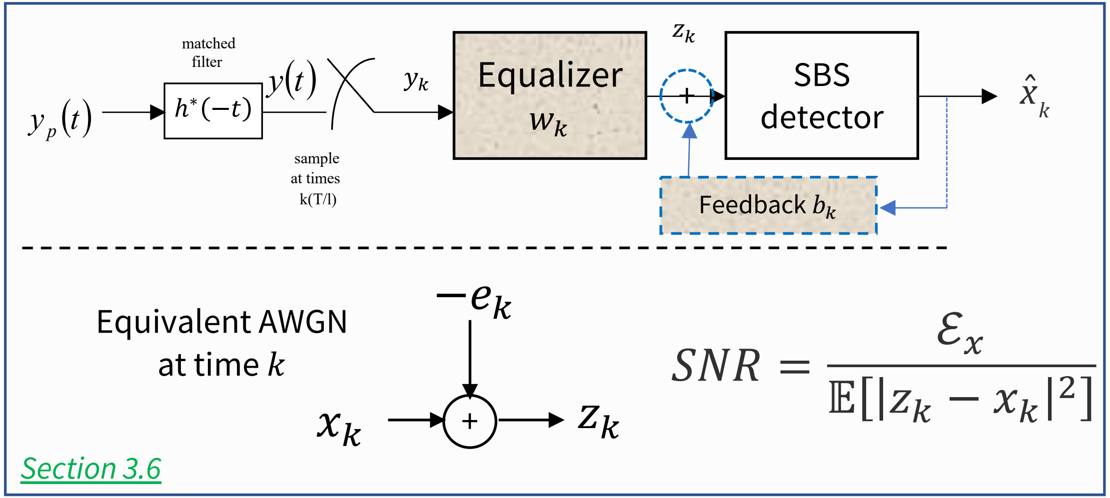
The Matrix AWGN Channel

Section 2.3.5
also supplementary lectures S1A and S1B
also 379Help files at Canvas site

Generating Parallel AWGNs

- Methods from EE379 ?
- An "equalizer" is one choice
  - Parallel channels in time
  - $z_k = x_k - e_k$



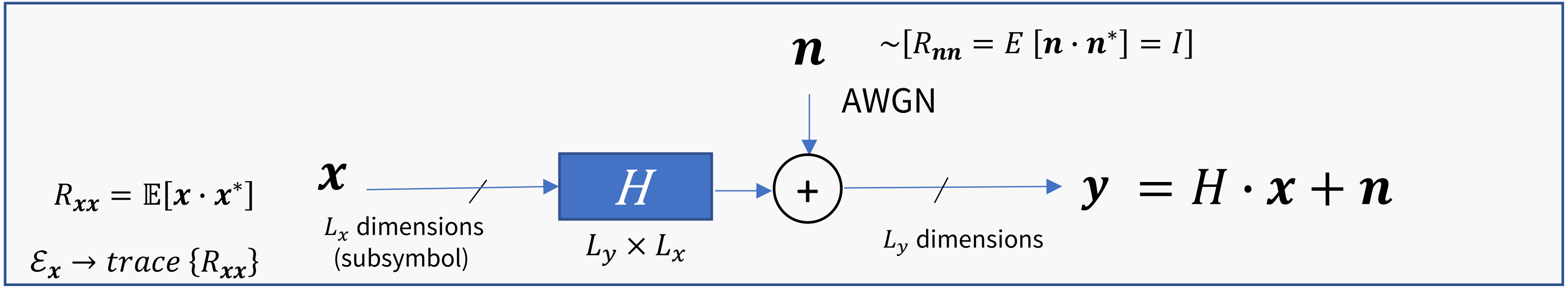


- Another?
- Multicarrier is another choice
  - Parallel channels in frequency

$$Y_n \cong H_n \cdot X_n \quad (+N_n)$$

Sections 1.3.8 and 4.2.1

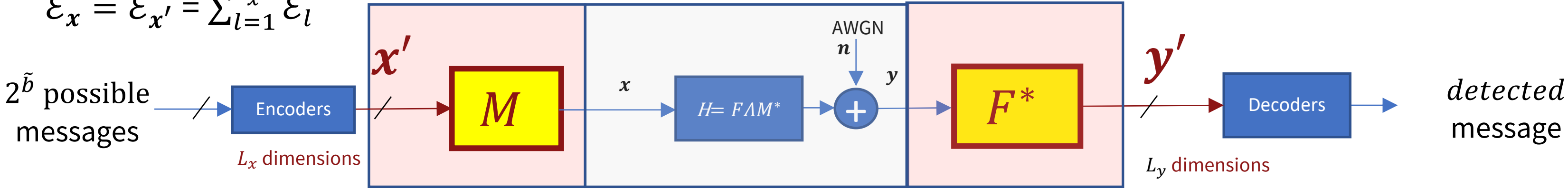
In general, a matrix AWGN channel

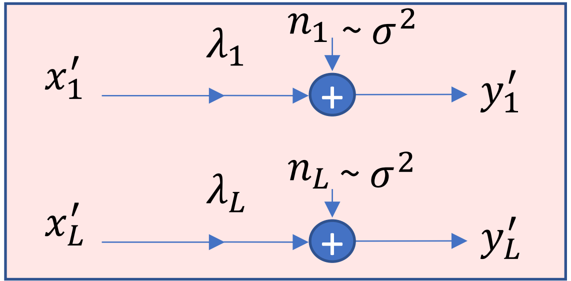


$H = F \cdot \Lambda \cdot M^*$

singular value decomposition (svd in matlab)
$F \cdot F^* = F^* \cdot F = I_{L_y}$; $M \cdot M^* = M^* \cdot M = I_{L_x}$
Λ. $(L_y \times L_x)$ is "diagonal" (real)

$\mathcal{E}_x = \mathcal{E}_{x'} = \sum_{l=1}^{L_x} \mathcal{E}_l$



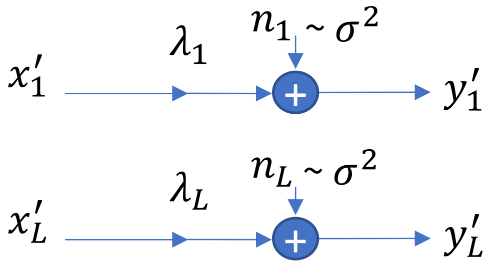


Vector Coding (MIMO)

$L \leq min(L_x, L_y)$ independent dimensions

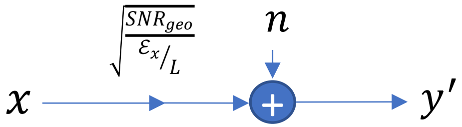
$R_{nn} \neq I \rightarrow (H \rightarrow R_{nn}^{-1/2} \cdot H)$

Geometric Equivalent Channel



$$\tilde{b}_l \;=\; \mathcal{C}_l = log_2(1 + SNR_l) \qquad SNR_l = \mathcal{E}_l \cdot \lambda_l^2 / \sigma^2 \;= \mathcal{E}_l \cdot g_l$$

$$\tilde{b} = \sum_{l=1}^{L} \tilde{b}_l = \sum_{l=1}^{L} log_2(1 + SNR_l) = L \cdot \log_2\left(1 + SNR_{geo}\right)$$



$$SNR_{geo} = \overbrace{\left[\prod_{l=1}^{L}(1 + SNR_l)\right]^{1/L}}^{\text{geometric average}} - 1$$

Use it L times like single constant AWGN

- Vector Coding – uses SVD to translate matrix AWGN to set of equivalent parallel AWGN's
  - Each can be individually encoded like AWGN (they are independent)
- Geometric-equivalent channel use L times
  - Any H and R_{nn}
  - Any set of input energies (that sum to allowed energy)

The Detection/Communication Issue

- MAP/ML receiver/detector implementation can be very complex

- An entire body of theory/practice has been devoted to reducing this complexity as well as projecting nearly attainable bounds
  - Communication Theory and Information Theory

- ***Decomposing into multiple channels can simplify design!***
  - Multiple dimensions are the key to this simplification
  - And today, used throughout digital communication (wires, wireless, soon fiber)

The Water-Filling Energy Distribution

Sections 2.3.5, 4.1-4.3

also supplementary lecture S1A

See PS1.3 (Prob 4.18), PS1.4 (Prob 4.7), and PS1.5 (Prob 4.25)

Rate Maximization and Dual

- Choose energy and bit allocation to maximize sum data rate over the dimensions

$$\max_{\mathcal{E}_l} \sum_{l=1}^{L} \log_2\left(1 + \frac{\mathcal{E}_l \cdot g_l}{\Gamma}\right) = \sum_{l=1}^{L} b_l$$

$$ST: \mathcal{E}_{\boldsymbol{x}} = \sum_{l=1}^{L_x} \mathcal{E}_l$$

Rate Adaptive (RA)

DUAL

$$\min_{b_l} \sum_{l=1}^{L} \mathcal{E}_l$$

$$ST: b = \sum_{l=1}^{L_x} b_l$$

Margin Adaptive (MA)

- Solution (basic calculus – see Section 4.2) ; see also matlab “waterfill.m” at web site to save hand calcs

$$\mathcal{E}_l + \frac{\Gamma}{g_l} = constant$$

WATER-FILLING

(Shannon 1948)

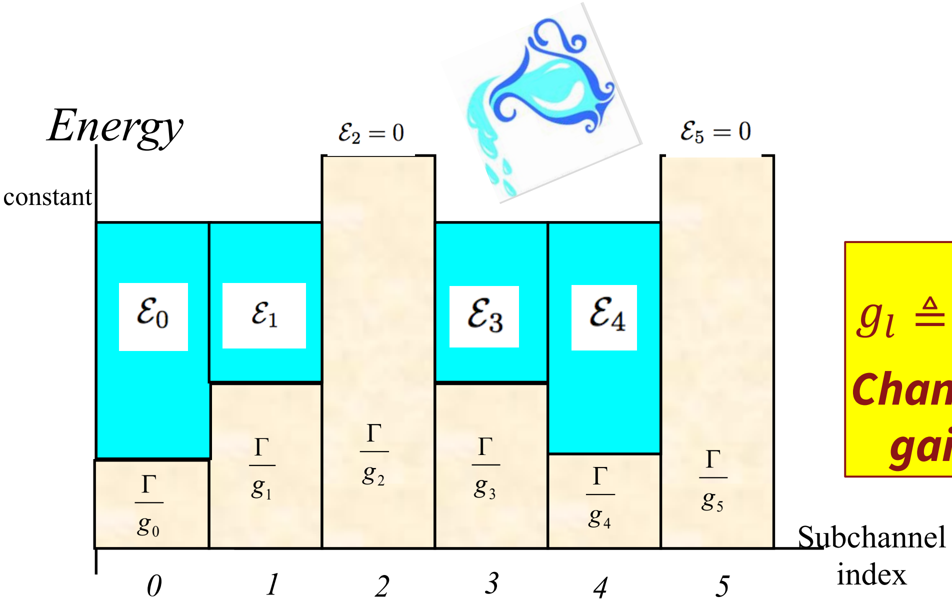
Neither energies allocated nor bits allocated can be negative

Water-filling Illustrated

- Energy available in a pitcher
  - Note re-indexed 0 (DC) to 5

RA: until all energy used
MA: until total bit rate attained

$$\tilde{b}_l = log_2\left(1 + \frac{SNR_l}{\Gamma}\right)$$



$$g_l \triangleq \frac{\lambda_l^2}{\sigma^2}$$

Channel gain

Rate Adaptive Solution

- Write and sum energy constraints

$$g_1 \geq g_2 \geq \ldots \geq g_L$$

$$\mathcal{E}_1 + {}^{\Gamma}/_{g_1} = K$$
$$\mathcal{E}_2 + {}^{\Gamma}/_{g_2} = K$$
$$\vdots$$
$$\mathcal{E}_L + {}^{\Gamma}/_{g_L} = K$$

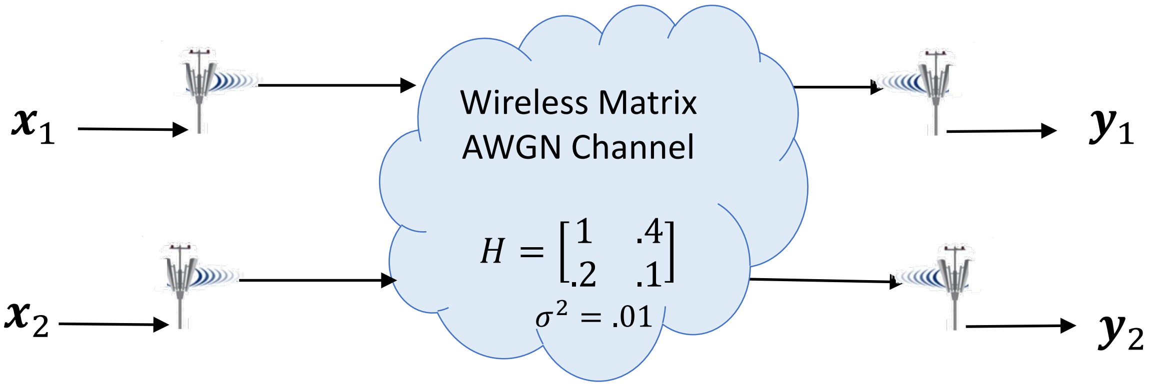
$$\sum_{l=1}^{L} \mathcal{E}_l + \Gamma \cdot \sum_{l=1}^{L} {}^{1}/_{g_l} = L \cdot K$$

- Solve for Water-Fill Constant

$$K = \frac{\mathcal{E}_x}{L^*} + \frac{\Gamma}{L^*} \cdot \sum_{l=1}^{L^*} {}^{1}/_{g_l}$$

L^* is largest L such that $\mathcal{E}_l > 0$ for all $l = 1, \ldots, L^*$

2 x 2 Antenna System with 0 dB gap



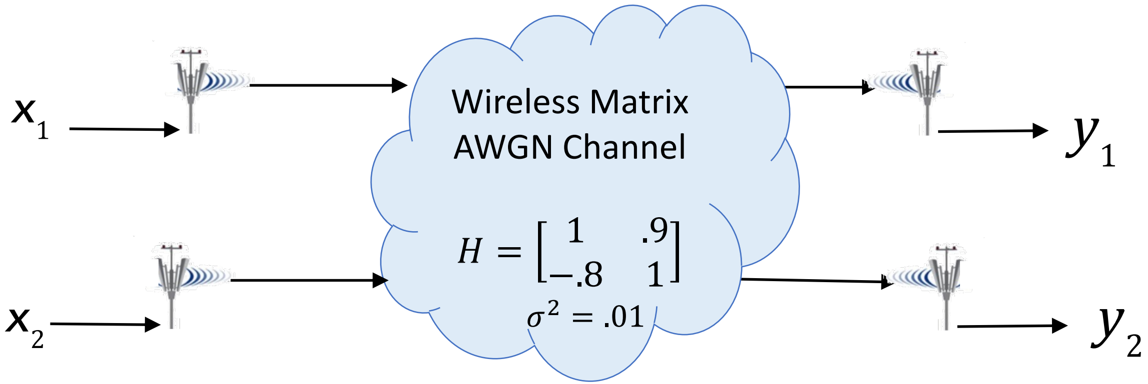
- There is crosstalk between dimensions and $\mathcal{E}_x$=2
  - Kind of sounds like a problem then, right?

```
>> H=[10 4
2 1];
>> [F , Lambda , Mstar]=svd(H);
>> Lambda =
  10.9985        0
        0   0.1818
>> g2=Lambda(1,1)^2  =  120.9669
>> g1=Lambda(2,2)^2  =    0.0331
>> K=1+0.5*(1/g1+1/g2) = 16.1250
>> E2=K-1/g2  =   16.1167
>> E1=K-1/g1 =  -14.1167   < 0 (whoops)
```

Just use dimension 2 → $\tilde{b} = \log_2(1 + 2 * g_2) = 6.93$ bits/subsymbol

In this case water-fill simply puts all energy on the best dimension (returns to scalar/SISO if that is best)

2 x 2 Antenna System



- There is stronger crosstalk between dimensions
  - Maybe worse, right? ???

```
>> H=[10 9
-8 10];
>> [F , Lambda , Mstar]=svd(H);
>> Lambda =
  13.6244        0
        0  12.6244
>> g2=Lambda(1,1)^2  =  185.6244
>> g1=Lambda(2,2)^2  =  159.3756
>> K=1+0.5*(1/g1+1/g2) =    1.0058
>> E2=K-1/g2  =    1.0004
>> E1=K-1/g1  =    0.9996
>> btilde = log2(1+E2*g2)+log2(1+E1*g1)  =   14.8693
```

Actually this is close to 2x the data rate for the previous case
Clearly, the use of both dimensions, and somewhat stronger crosstalk and signal.

In general, the increase is roughly a factor of L in data rate.

Energy-minimizing Margin-Adaptive Solution

- Energy and sum-bit constraints

$$\mathcal{E}_l = K - {}^{\Gamma}/_{g_l}$$

$$g_1 \geq g_2 \geq \ldots \geq g_L$$

$$\tilde{b} = \sum_{l=1}^{L} \tilde{b}_l = \sum_{l=1}^{L} \log_2\left(1 + \frac{\mathcal{E}_l \cdot g_l}{\Gamma}\right)$$

$$= \sum_{l=1}^{L} \log_2\left(\frac{K \cdot g_l}{\Gamma}\right)$$

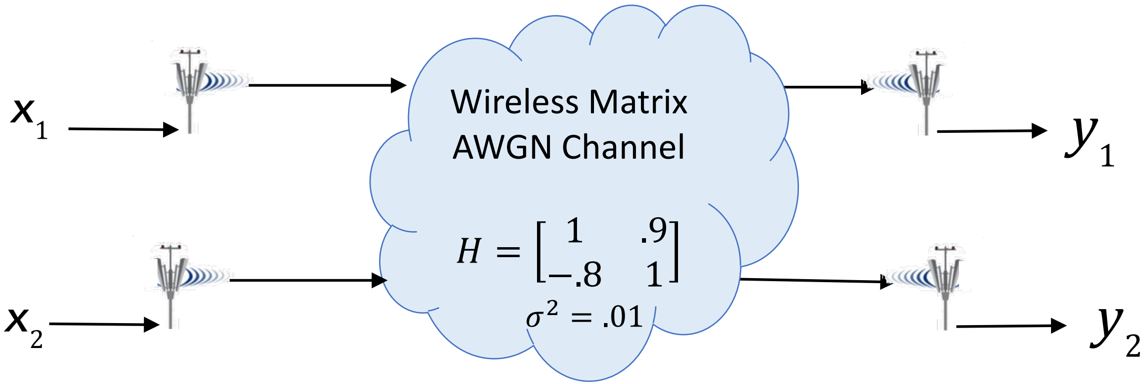
$$= \log_2\left(\prod_{l=1}^{L} \frac{K \cdot g_l}{\Gamma}\right)$$

- Solve for Water-Fill Constant

$$K = \Gamma \cdot \left(\frac{2^{\tilde{b}}}{\prod_{l=1}^{L^*} g_l}\right)^{1/L^*}$$

L^* is largest L such that $\mathcal{E}_l > 0$ for all $l = 1, \ldots, L^*$

2 x 2 Antenna System with MA



- Attempt $\tilde{b} = 14 \frac{\text{bits}}{\text{Hz}}$; The use of 2 antennas exploited channel's crosstalk,
  - Without the crosstalk, this channel supports only 7 bits/Hz (either channel has then SNR = 10)

```
>> H=[10 9
-8 10];
>> K=sqrt((2^14)/(g1*g2)) =   0.7442
>> E2=K-1/g2 =   0.7388
>> E1=K-1/g1 =   0.7379
>> margin = 10*log10(2/(E1+E2)) =   1.3 dB
```

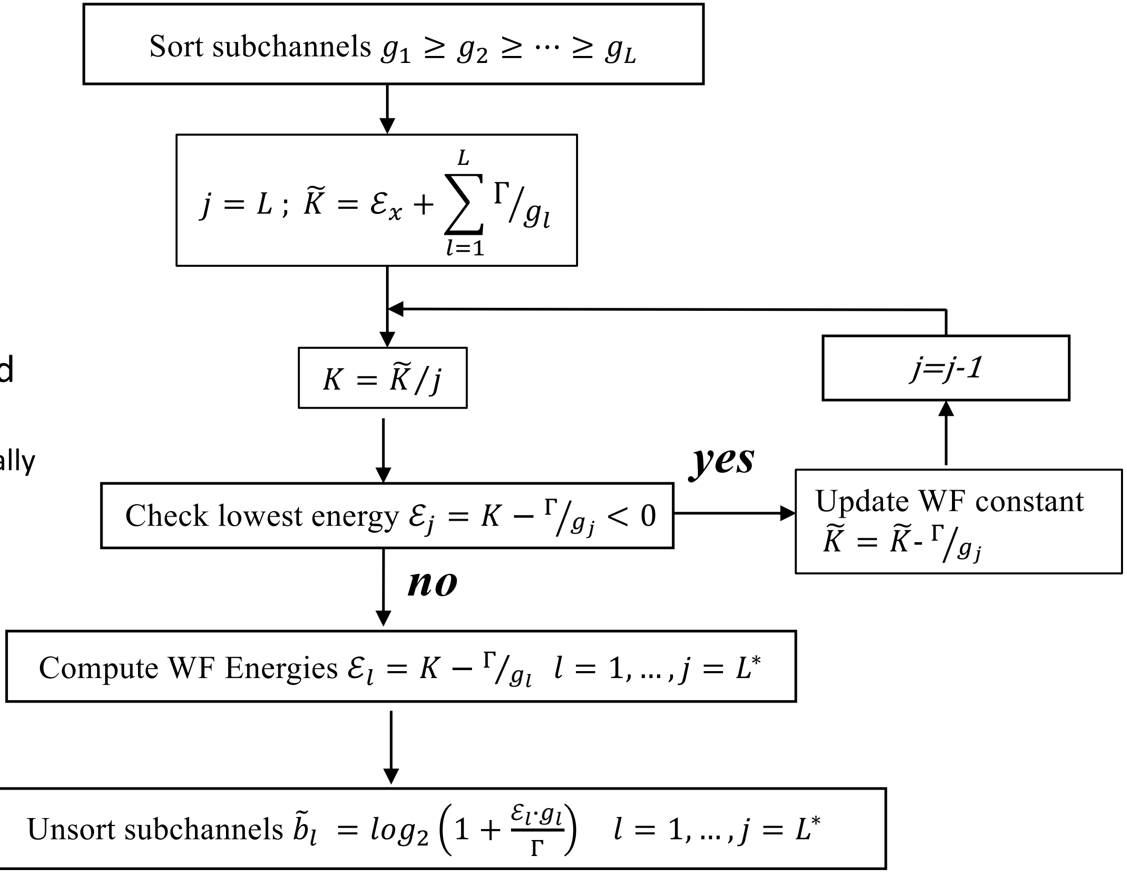
This effect magnifies as long as most of the singular values are "decent"

RA Water-Fill Flow Chart

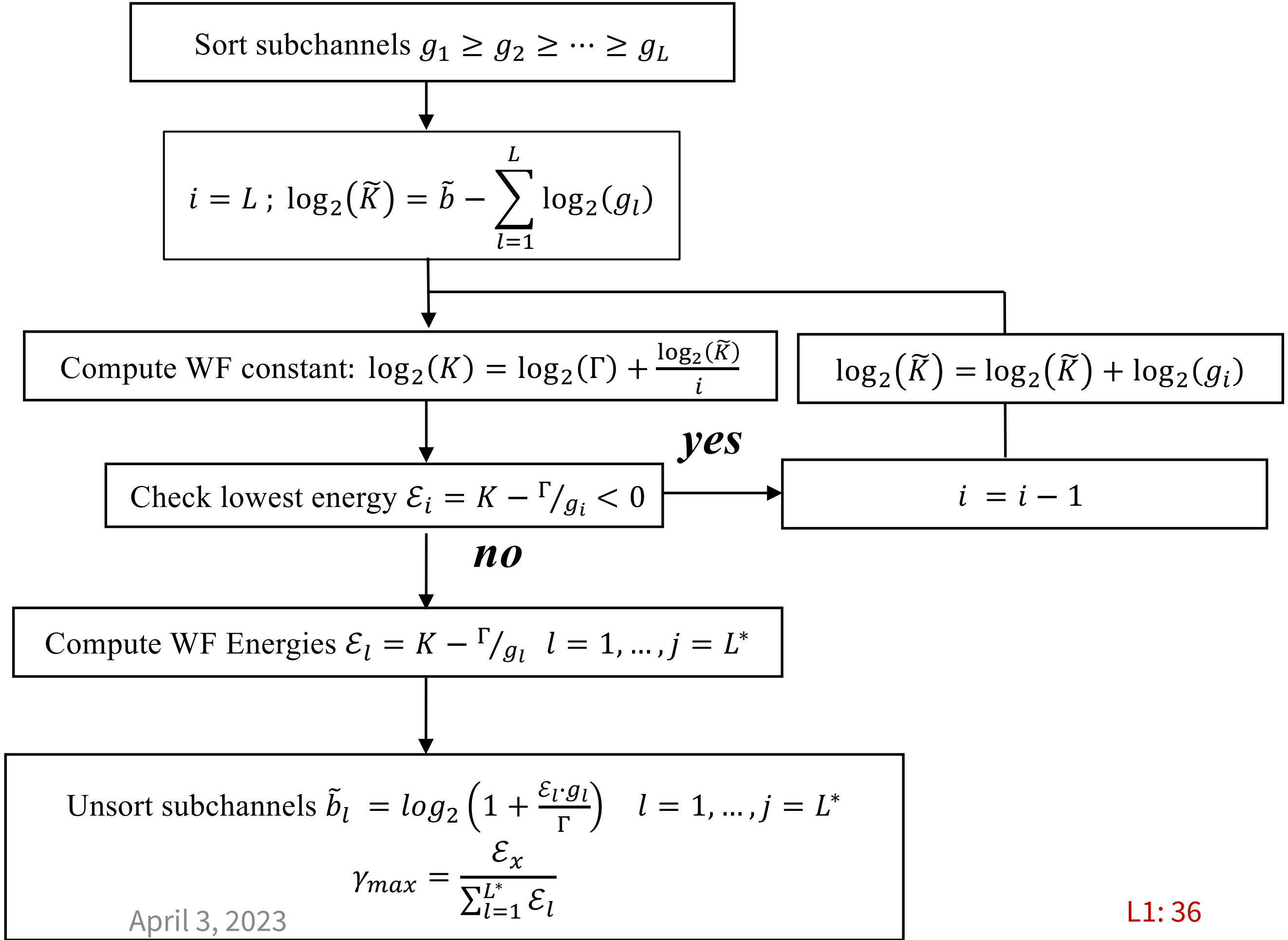
- Can start with all channels energized
  - Compute K, test lowest energy
  - Reduce number of dimensions incrementally

- Can also start with 1 channel energized
  - Compute K, test lowest energy
  - Increase number of dimensions incrementally

- The sort is most complex part
  - Can use pivots and bi-section
  - Avoids sort



Margin Adaptive Flowchart

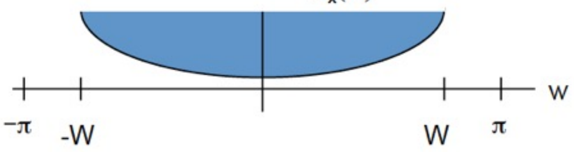


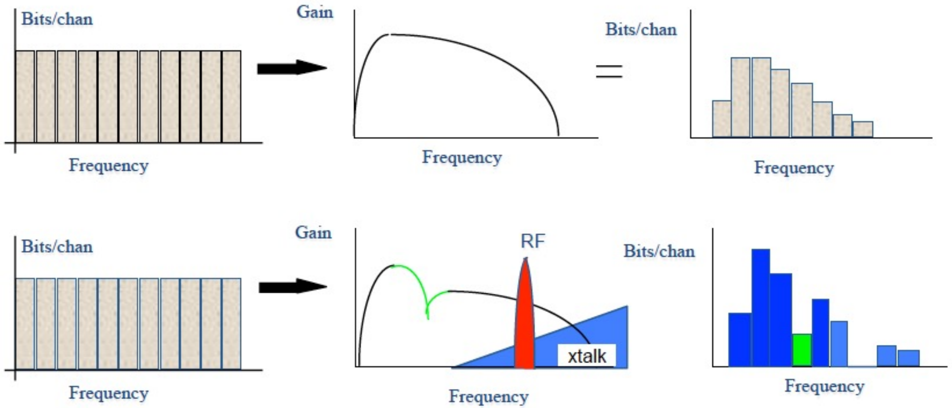
Section 4.3.2

Projecting Forward

Water-filling from 100k feet

“water-filling”



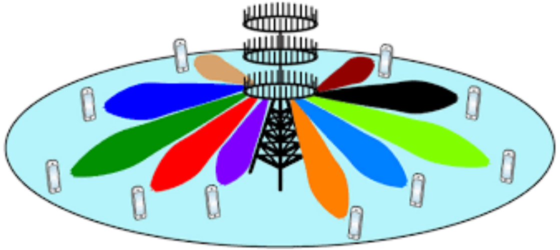


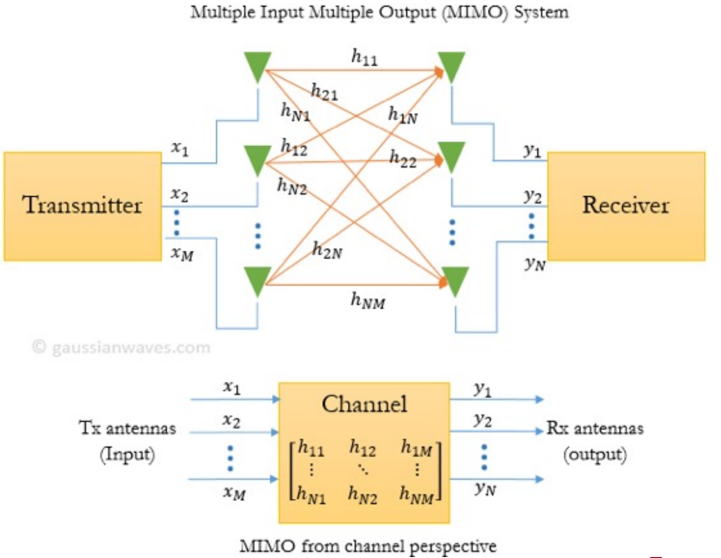
- How do we learn and adjust either or both of energy/bits per dimension?
  - Dynamically
- Some of very first AI methods in communication (from Stanford)
  - “bit-swapping”
  - #3 Stanford patent on value/royalty in Engineering

Multiple directions in Space

Best energies will also be water-fill over the channel's spatial singular vectors

Essentially matrix form of machine learning
From earlier





End Lecture 1