STANFORD

Lecture 11

Interference and Other MU Channels

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Announcements & Agenda

- Announcements
  - PS #5 due May 17
  - REMINDER – Class for 5/15 → 5/19 3:00 @ **200-003**

- Agenda
  - Vector WCN-BC Design
  - Scalar Duality (BC and MAC)
  - Continuous-time scalar BC
  - MAC-set approach to IC
    - Examples
  - IC maximum rate sum and energy sums
  - Nesting, DAS, cellfree, & relay

		Multi-User Fundamentals		
7	4/24	Multi-User Channels and the Capacity Region	2.6	-/-
8	4/26	Multiple Access Channels	2.7	4/3
9	5/1	Broadcast Channels	2.8	-/-
--	5/3	**Midterm Exam** (open bk) hmwk, **5pm Tues**		-/4
10	5/8	Broadcast Channels continued	2.8	5/-
11	5/10	Interference and Other MU Channels	2.9-11	-/-
		GDFE Foundation		
		no class Monday 5/15 - make up Friday 5/19, 3:00 **200-003**		
12	5/17	GDFE Basics	5.1-3	6/5
13	5/19	GDFE Input Optimization and Forms	5.3	-/-

Vector WCN-BC Design

PS5.3 - 2.30 Vector BC Design

WCN Design focuses on primary users

- Any secondary-user components"freeload" on the dimensions best used by primary-user components
- Delete the secondary-user components' rows from $\tilde{H}$ for initial WCN design
- Nontrivial precoder coefficients depend only upon these primary components
  - Energized secondary components "dimension-share" those primary dimensions (reducing overall rate sum)
- WCN design provides BC insight, but is less direct than the earlier BC Design with mu_bc.m
- Chapter 5 will find a way for any desired $\boldsymbol{b}' \in \mathcal{C}(\boldsymbol{b})$ to derive the $\{R_{\boldsymbol{xx}}(u)\}$, but the choice of $\boldsymbol{b}'$ (scheduling) may want to know about primary/secondary components

Only for primary users (→ nonsingular WCN)

- Use R_{wcn} directly
  - General S_{wcn} is block diagonal
  - Find A

$$R_{wcn}^{-1} - [\underbrace{H \cdot A \cdot A^* \cdot H^* + R_{wcn}}_{R_{\boldsymbol{yy}}}]^{-1} = \mathcal{S}_{wcn}$$

- Indeed, that is the backward MMSE channel in there!
  - Q_{wcn} is also block diagonal

$$\mathcal{S}_{wcn} = R_{wcn}^{-1} - \left[R_{wcn}^{-1} - R_{wcn}^{-1} \cdot H \cdot A \left(I + A^* \cdot H^* \cdot R_{wcn}^{-1} \cdot H \cdot A\right)^{-1} A^* \cdot H^* \cdot R_{wcn}^{-1}\right]$$

$$Q_{wcn}^* \cdot \mathcal{S}'_{wcn} \cdot Q_{wcn} = R_{wcn}^{-1} \cdot H \cdot A \underbrace{\left(I + A^* \cdot H^* \cdot R_{wcn}^{-1} \cdot H \cdot A\right)^{-1}}_{R_b \triangleq G^{-1} \cdot S_0^{-1} \cdot G^{-*}} A^* \cdot H^* \cdot R_{wcn}^{-1} \quad (2.432)$$

$$\underbrace{\mathcal{S}'_{wcn}}_{\text{diagonal}} = \underbrace{Q_{wcn} \cdot R_{wcn}^{-1} \cdot H \cdot A}_{\text{triangular inverse}} \cdot R_b \cdot \underbrace{A^* \cdot H^* \cdot R_{wcn}^{-1} \cdot Q_{wcn}^*}_{\text{triangular inverse}} \ , \quad (2.433)$$

- QR factorization (primaries' channel)
  - Extract $A = R_{\boldsymbol{xx}}^{1/2}$ from tri inverse,
  - which is the forward channel

$$Q_{wcn} \cdot R_{wcn}^{-1} \cdot H = \left[\underbrace{\mathbf{0}}_{(L_x - U^o) \times U^o} \quad \underbrace{R}_{U^o \times U^o}\right] \left[\begin{array}{c} \boldsymbol{q} \\ \underbrace{Q^*}_{U^o \times L_x} \end{array}\right] = R \cdot Q^*$$

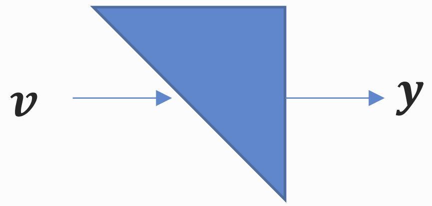
- Cholesky factorization (input)

$$\Phi \cdot \Phi^* = Q^* \cdot R_{\boldsymbol{xx}} \cdot Q \quad .$$

- A special square root!
  - "pre-triangularizes" the channel,
  - which becomes $R \cdot \Phi$

$$R_{\boldsymbol{xx}}^{1/2} = A = Q \cdot \Phi$$

Goal!



The precoder

- Want monic G for precoder

$$D_A \triangleq \text{Diag}\{R \cdot \Phi\}$$

Find diagonal values

$$\begin{aligned} G &= D_A^{-1} \cdot R \cdot \Phi \\ S_0 &= D_A \cdot (S')_{wcn}^{-1} \cdot D_A \end{aligned}$$

Monic Equivalent

- Check = R_b for G and S_0

$$\begin{aligned} G^{-1} \cdot S_0^{-1} \cdot G^{-*} &= \left(\Phi^{-1} \cdot R^{-1} \cdot D_A\right) \cdot \left(D_A^{-1} \cdot S_{wcn} \cdot D_A^{-1}\right) \cdot \left(D_A \cdot R^{-*} \cdot G^{-*}\right) && (2.441) \\ &= \Phi^{-1} \cdot R^{-1} \cdot S_{wcn} \cdot R^{-*} \cdot \Phi^{-*} && (2.442) \\ &= \Phi^{-1} \cdot R^{-1} \cdot \left[Q_{wcn} \cdot R_{wcn}^{-1} \cdot H \cdot A \cdot R_b \cdot A^* \cdot H^* \cdot R_{wcn}^{-1} \cdot Q_{wcn}^*\right] \cdot R^{-*} \cdot \Phi^{-*} \\ &= \Phi^{-1} \cdot R^{-1} \cdot R \cdot Q^* \cdot Q \cdot \Phi \cdot R_b \cdot \Phi^* \cdot R^* \cdot R^{-*} \cdot A \cdot Q^* \cdot \Phi^{-*} && (2.443) \\ &= R_b \ . && (2.444) \end{aligned}$$

- Check SNR and mutual-info

$$\begin{aligned} 2^{\mathcal{I}_{wcn}(\boldsymbol{x};\boldsymbol{y})} &= \frac{\left| H \cdot R_{\boldsymbol{xx}} \cdot H^* + R_{wcn} \right|}{\left| R_{wcn} \right|} \\ &= \left| R_{wcn}^{-1/2} \cdot H \cdot R_{\boldsymbol{xx}} \cdot H^* \cdot R_{wcn}^{-*/2} + I \right| \\ &= \left| R_{wcn}^{-1/2} \cdot H \cdot A \cdot A^* \cdot H^* \cdot R_{wcn}^{-*/2} + I \right| \\ &= \left| A^* \cdot H^* \cdot R_{wcn}^{-1} \cdot H \cdot A + I \right| \quad \text{follows from SVD of } R_{wcn}^{-1/2} \cdot H \cdot A \\ &= \left| R_b^{-1} \right| \\ &= \left| S_0 \right| \\ \mathcal{I}_{wcn}(\boldsymbol{x};\boldsymbol{y}) &= \log_2\left(|S_0|\right) \quad \text{bits/complex subsymbol.} \end{aligned}$$

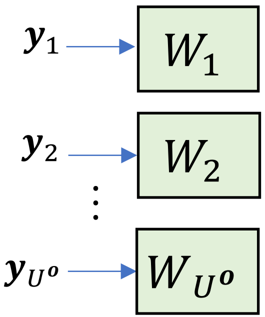
Works!

The Receiver

- The MMSE receiver is block diagonal!
  - For WCN only
  - Just what the BC needs

$$
\begin{aligned}
W &= \underbrace{S_0^{-1} \cdot G^{-*}}_{1-to-1} \cdot \underbrace{A^* \cdot H^* \cdot R_{wcn}^{-1}}_{\text{noise-white-match}} \cdot \underbrace{Q_{wcn}^* \cdot Q_{wcn}}_{I} \\
&= S_0^{-1} \cdot G^{-*} \cdot \Phi^* \cdot Q^* \cdot Q \cdot R^* \cdot Q_{wcn} \\
&= S_0^{-1} \cdot G^{-*} \cdot \Phi^* \cdot R^* \cdot Q_{wcn} \\
&= S_0^{-1} \cdot G^{-*} \cdot G^{-1} \cdot D_A \cdot Q_{wcn} \\
&= S_0^{-1} \cdot D_A \cdot Q_{wcn} \quad ,
\end{aligned}
$$

- Same bias removal as with all MMSE



BC WCN-Design Steps Summary (2.8.3.3)

Special Square Root

- Find R_{wcn} - this step also finds $\mathcal{S}_{wcn}$ and also the primary/secondary users and $b_{max}(R_{\boldsymbol{xx}})$
  - Delete rows/columns (secondary sub user dimensions) with zeros from $\mathcal{S}_{wcn}$, and correspondingly then in R_{wcn}
- If $\mathcal{S}_{wcn}$ is non-trivial (block diagonal), form $\mathcal{S}_{wcn}$=$Q^*_{wcn} \cdot \mathcal{S}'_{wcn} \cdot Q_{wcn}$ (eigen decomp)
- Perform QR factorization on $Q_{wcn} \cdot R^{-1}_{wcn} \cdot H = R \cdot Q^*$ where R is upper triangular, and Q is unitary
- Perform Cholesky Factorization on $Q^* \cdot R_{\boldsymbol{xx}} \cdot Q$ = $\Phi \cdot \Phi^*$ where Φ is also upper triangular
- And now, the special square root is $R^{1/2}_{\boldsymbol{xx}} = Q \cdot \Phi$ (see diagram last page = A)

Precoder and Diagonal Receiver

- Find the diagonal matrix D_A =Diag$\{R \cdot \Phi\}$
- Find the (primary sub-user) precoder $G = D_A^{-1} \cdot R \cdot \Phi$ (monic upper triangular)
- Find the backward MMSE (block) diagonal matrix $S_0 = D_A \cdot (S')^{-1}_{wcn} \cdot D_A$ (note, $R_b^{-1} = G^* \cdot S_0 \cdot G$)
- Block diagonal (unbiased) receiver is $W_{unb} = (S_0 - I)^{-1} \cdot D_A \cdot Q_{wcn}$
- Can check but $b_{max}(R_{\boldsymbol{xx}})$ from WCN will be $\mathcal{I}_{wcn}(\boldsymbol{x};\boldsymbol{y})$=$\log_2|S_0| = \sum_{u=1}^{U^o} \log_2\left(1 + SNR_{BC,wcn,u}\right)$

Secondary users then share this system

Example – all primary

- Energy $\mathcal{E}_x$ =2 , $L_x = 2$

```
>> H = [   80    70
           50    60 ];
>> Rxx=[1 .8
        .8 1];
```

```
>> [Rwcn,b]=wcnoise(Rxx,H,1)
Rwcn =
    1.0000    0.0232
    0.0232    1.0000
b = 9.6430
>> Swcn = inv(Rwcn)-inv(H*Rxx*H'+Rwcn) =
    0.9835    0.0000
    0.0000    0.9688
>> Htilde=inv(Rwcn)*H =
   78.8817   68.6440
   48.1687   58.4064
>> [R,Q,P]=rq(Htilde)
R =
  -12.4389  -74.6780
         0  -104.5673
Q =
    0.6565   -0.7544
   -0.7544   -0.6565
P =     2     1
```

ORDER IS REVERSED SO SWITCH USERS!

Nonsingular Rwcn

```
>> Rxxrot=Q'*Rxx*Q;
>> Phi=lohc(Rxxrot) =
    0.4482    0.0825
         0    1.3388
>> DA=diag(diag(R*Phi));
>> G=inv(DA)*R*Phi  =
    1.0000   18.1182
         0    1.0000
>> A=Q*inv(R)*DA*G =
    0.2942   -0.9557
   -0.3381   -0.9411
>> S0=DA*inv(Swcn)*DA =   1.0e+04 *
    0.0032   -0.0000
   -0.0000    2.0229
>> Wunb=(inv(S0)-eye(2))*DA =
    5.3983   -0.0000
   -0.0000  139.9857
```

Indeed Diagonal!

```
>> Gunb=eye(2)+S0*inv(S0-eye(2))*(G-eye(2)) =
    1.0000   18.7103
         0    1.0000
>> b=0.5*log2(diag(S0))' =   2.4909    7.1521
>> sum(b) =    9.6430 (checks
```

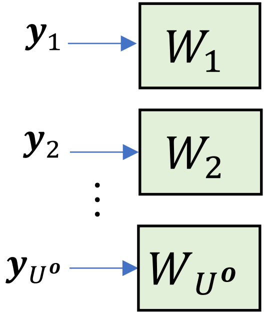
```
Wunb*H*A*inv(G) =
  -0.6987 -755.7235
 -780.3821 -454.9625
```

Try different Input Rxx, See text

Return to Design

- The design can allocate R_{xx} energy to secondary and primary users as

- The receivers are easy



Another example – singular 3x3 BC (Ex 2.8.8)

```
>> H=[80 60 40
60 45 30
20 20 20];
>> rank(H)  =     2
>> Rxx=diag([3 4 2]);
>> [Rwcn1, b]=wcnoise(Rxx, H, 1, 1e-5 , 1e-4);
>> Rwcn1
    1.0000    0.7500    0.0016
    0.7500    1.0000    0.0012
    0.0016    0.0012    1.0000
>> b =    11.3777
>> Swcn=inv(Rwcn1)-inv(H*Rxx*H'+Rwcn) =
    0.9995    0.0000    0.0000
    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.9948
```

User 2 is secondary – remove for now

```
>> H1=[H(1,1:3)
H(3,1:3)] =
   80   60   40
   20   20   20
>> [Rwcn, b]=wcnoise(Rxx, H1, 1, 1e-5 , 1e-4);
>> Rwcn =
   1.0000   0.0016
   0.0016   1.0000
>> b =   11.3777
>> Swcn=inv(Rwcn)-inv(H1*Rxx*H1'+Rwcn) =
   0.9995   0.0000
   0.0000   0.9948
```

Primary/Secondary

```
>> [R,Q,P]=rq(inv(Rwcn)*H1)
R =
       0   9.1016  -33.2537
       0        0 -107.6507
Q =
   0.4082  -0.5306  -0.7429
  -0.8165   0.1517  -0.5571
   0.4082   0.8340  -0.3713
P =    2    1
```

ORDER IS REVERSED (Here it is order of users 1 and 3 since 2 was eliminated)

```
>> R1=R(1:2,2:3);
>> Q1=Q(1:3,2:3);
>> Rxxrot=Q1'*Rxx*Q1  =
   2.3275   0.2251
   0.2251   3.1725
>> Phi=lohc(Rxxrot);
>> DA=diag(diag(R1*Phi)) =
  13.8379        0
        0 -191.7414
>> G=inv(DA)*R1*Phi  =
   1.0000  -4.1971
        0   1.0000
>> A=Q1*inv(R1)*DA*G =
  -0.8067  -1.3902
   0.2306  -0.9730
   1.2679  -0.5559
>> A*A'  =
   2.5833   1.1667  -0.2500
   1.1667   1.0000   0.8333
  -0.2500   0.8333   1.9167
```

Sq Root & Precoder

Not equal to Rxx
Energy not inserted into null space (same on part that is in pass space)

Rcvr & Data Rate

```
>> S0=DA*inv(Swcn)*DA =   1.0e+04 *
   0.0192   0.0000
   0.0000   3.6957
>> MSWMFunb=(inv(S0)-eye(2))*DA  =
  -13.7657   0.0000
   -0.0000  191.7362
>> Gunb=eye(2)+S0*inv(S0-eye(2))*(G-eye(2))  =
   1.0000  -4.2191
  -0.0000   1.0000
>> b=0.5*log2(diag(S0))'  =
   3.7909   7.5868
>> sum(b) =   11.3777 checks
>> H*A =
   0.0219 -191.8333
   0.0164 -143.8749
  13.8379  -58.3825
```

See Example 2.8.8 or details of below

Assign 1 energy unit to User 1, 1/3 to user 3, and now squeeze in 2/3 energy on user 2

```
>> b=0.5*log2(diag([ 1 1/3]) *diag(S0))  =
   3.7909
   6.7943
```

Crosstalk is >> ct=1/3*143.9^2 = 6.8928e+03

```
>> b2=0.5*log2(1+(2/3)*60^2/6892.8) =    0.2155
>> b2+sum(b) = 10.8007 < 11.3777
```

Energy on secondary reduces rate sum

System Diagram for this WCN design

$$v_1 = \sqrt{\mathcal{E}_1} \cdot v_1^o + \sqrt{\mathcal{E}_{1,2}} \cdot v_2$$
$$v_3 = \sqrt{\mathcal{E}_3} \cdot v_3^o + \sqrt{\mathcal{E}_{3,2}} \cdot v_2$$

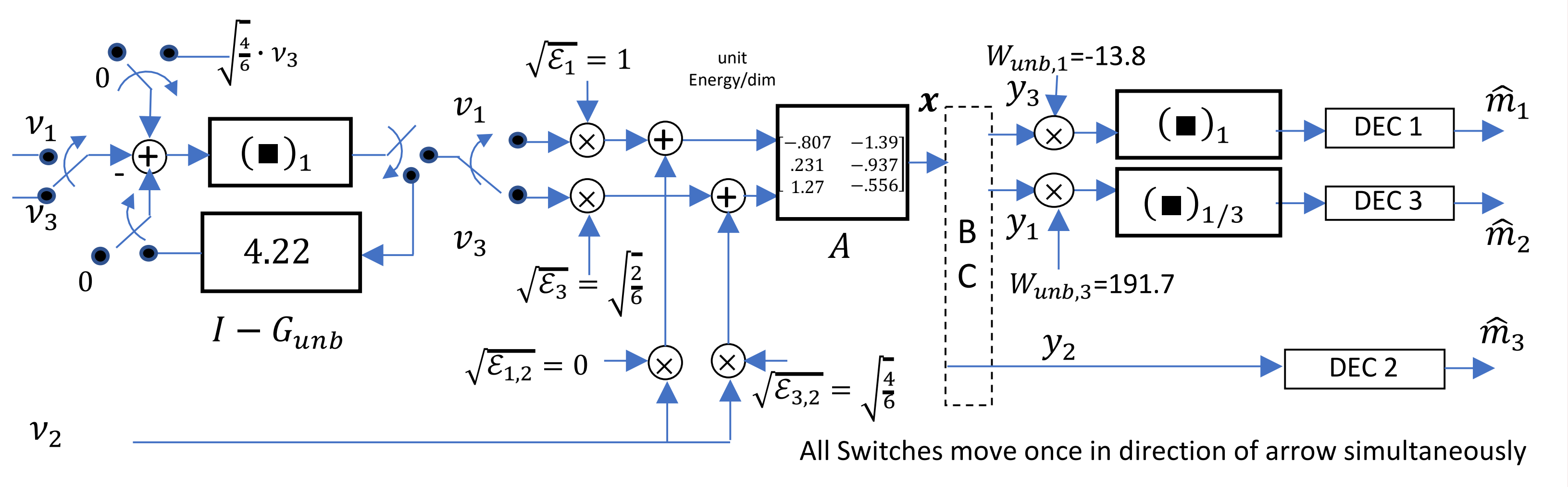
```
GU=   1.0000  -4.2191
     -0.0000   1.0000

MSWMFU=
 -13.7657   0.0000
  -0.0000 191.7362
```

See Ex 2.8.8

$$\boldsymbol{x} = \underbrace{\begin{bmatrix} -0.8067 & -1.3902 \\ 0.2306 & -0.9370 \\ 1.2679 & -0.5559 \end{bmatrix}}_{A} \cdot \begin{bmatrix} \sqrt{\mathcal{E}_1} & \sqrt{\mathcal{E}_{12}} & 0 \\ 0 & \sqrt{\mathcal{E}_{23}} & \sqrt{\mathcal{E}_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Try: $\mathcal{E}_1 = 1 \;\; and \;\; \mathcal{E}_{12} = 0$

$\mathcal{E}_3 = \frac{2}{6} \;\; and \;\; \mathcal{E}_{32} = \frac{4}{6}$



Scalar Duality

PS5.2 - 2.29 scalar BC region

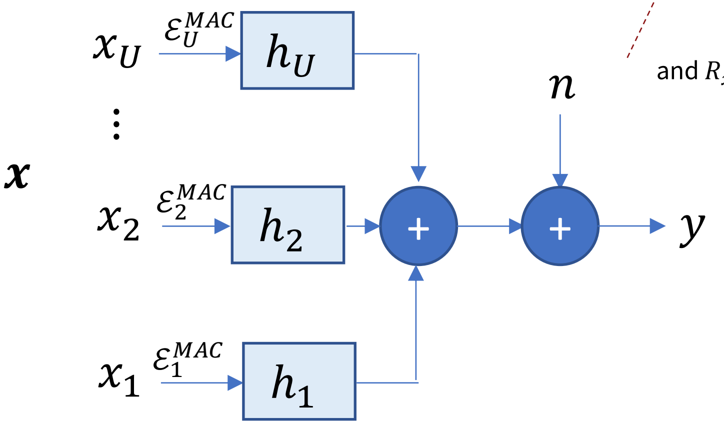
General Dual Channels – Same $\mathrm{I}(x; y)$

- Dual Channels
  - H_{MAC} for MAC
  - H_{BC} = $H_{MAC}^* \cdot J$ for dual-of-MAC as a BC

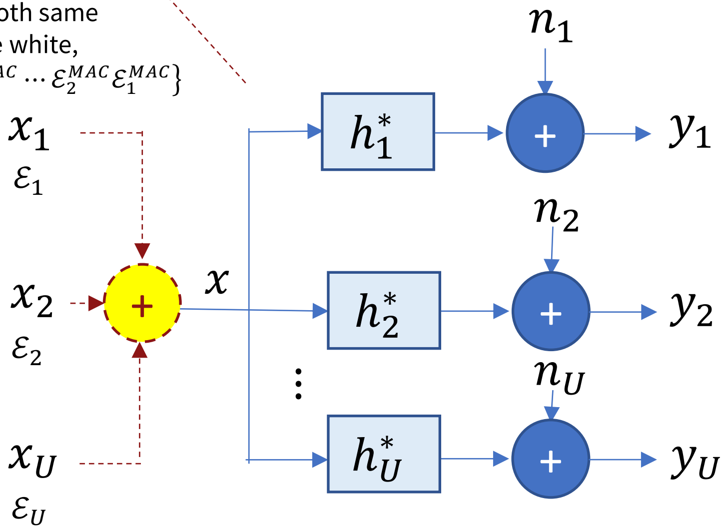
$$\mathcal{I}(x;y) = \mathcal{I}_{MAC} = \log_2 \left| H_{MAC} \cdot diag\{\mathcal{E}_U^{MAC} \cdots \mathcal{E}_2^{MAC} \mathcal{E}_1^{MAC}\} \cdot H_{MAC}^* + I \right|$$

$$\mathcal{I}_{BC}(x;y) = \sum_{u=1}^{U} \log_2 \left(\frac{\mathcal{E}_u^{BC} \cdot |h_u|^2}{1 + |h_u|^2 \cdot \sum_{i=u+1}^{U} \mathcal{E}_i^{BC}} + 1 \right)$$

$$H_{MAC} = [h_U \ \cdots h_2 \ h_1]$$



Would like both same
Let R_{nn} be white,
and $R_{xx} = diag\{\mathcal{E}_U^{MAC} \cdots \mathcal{E}_2^{MAC} \mathcal{E}_1^{MAC}\}$



$$H_{BC} = \begin{bmatrix} h_1^* \\ h_2^* \\ \vdots \\ h_U^* \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{E}_U^{MAC} \\ \vdots \\ \mathcal{E}_1^{MAC} \end{bmatrix} \preccurlyeq \boldsymbol{\mathcal{E}}$$

$$\sum_{u=1}^{U} \mathcal{E}_u^{BC} = \mathcal{E}_x = \sum_{u=1}^{U} \mathcal{E}_u^{MAC}$$ (scalar case)

Scalar Duality

- Set data rates equal and solve for $\mathcal{E}_u^{MAC/BC}$

MAC	BC
$b_1 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_1^{MAC}\cdot g_1}{1+\mathcal{E}_2^{MAC}\cdot g_2+\ldots+\mathcal{E}_U^{MAC}\cdot g_U}\right)$	$b_1 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_1^{BC}\cdot g_1}{1}\right)$ 1 ND user
$b_2 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_2^{MAC}\cdot g_2}{1+\mathcal{E}_3^{MAC}\cdot g_3+\ldots+\mathcal{E}_U^{MAC}\cdot g_U}\right)$	$b_2 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_2^{BC}\cdot g_2}{1+\mathcal{E}_1^{BC}\cdot g_2}\right)$
$\vdots$	$\vdots$
$b_U = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_U^{MAC}\cdot g_U}{1}\right)$ 1 ND user	$b_U = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_U^{BC}\cdot g_U}{1+\left[\mathcal{E}_1^{BC}+\ldots+\mathcal{E}_{U-1}^{BC}\right]\cdot g_U}\right)$

order intentionally reversed

Corresponding Energies

$$
\begin{aligned}
\mathcal{E}_1^{BC} &= \mathcal{E}_1^{MAC} \cdot \frac{1}{1 + \mathcal{E}_2^{MAC} \cdot g_2 + \ldots + \mathcal{E}_U^{MAC} \cdot g_U} \\
\mathcal{E}_2^{BC} &= \mathcal{E}_2^{MAC} \cdot \frac{1 + \mathcal{E}_1^{BC} \cdot g_2}{1 + \mathcal{E}_3^{MAC} \cdot g_3 + \ldots + \mathcal{E}_U^{MAC} \cdot g_U} \\
\vdots &= \vdots \\
\mathcal{E}_U^{BC} &= \mathcal{E}_U^{MAC} \cdot \left(1 + \left[\mathcal{E}_1^{BC} + \ldots + \mathcal{E}_{U-1}^{BC}\right] \cdot g_U\right)
\end{aligned}
$$

- By selecting these energies, all user rates are the same (with the order reversal) and running through all such energies that sum to total produces the SAME capacity region
- See proof in notes (Theorem 2.8.2 in Section 2.8.4)

Revisit Scalar Example

- Total energy is 1 , instead use dual MAC to investigate BC with
  - $\mathcal{E}_2^{BC} = 0.25$ (bottom of BC)
  - $\mathcal{E}_1^{BC} = 0.75$ (top BC)
  - reversing order $g_1 = 6400$ and $g_2 = 2500$

$$\mathcal{E}_2^{MAC} = \frac{\mathcal{E}_2^{BC}}{1+\mathcal{E}_1^{BC}\cdot g_2} = \frac{.25}{1+2500\cdot(.75)} = \frac{1}{7504} = 1.3326 \times 10^{-4} \text{ (top MAC)}$$

$$\mathcal{E}_1^{MAC} = \mathcal{E}_1^{BC} \cdot \left(1 + g_2 \cdot \mathcal{E}_2^{MAC}\right) = .75 \cdot (1 + 2500/7504) = \frac{7503}{7504} = .9999 = 1 - \mathcal{E}_2^{MAC} \text{ (bottom MAC)}$$

- User data rates for this combination are (and were in earlier table found directly for BC)

$$b_1 = \frac{1}{2} \cdot \log_2\left(1 + \frac{\mathcal{E}_1^{MAC} \cdot g_1}{1 + \mathcal{E}_2^{MAC} \cdot g_2}\right) = 6.1144$$

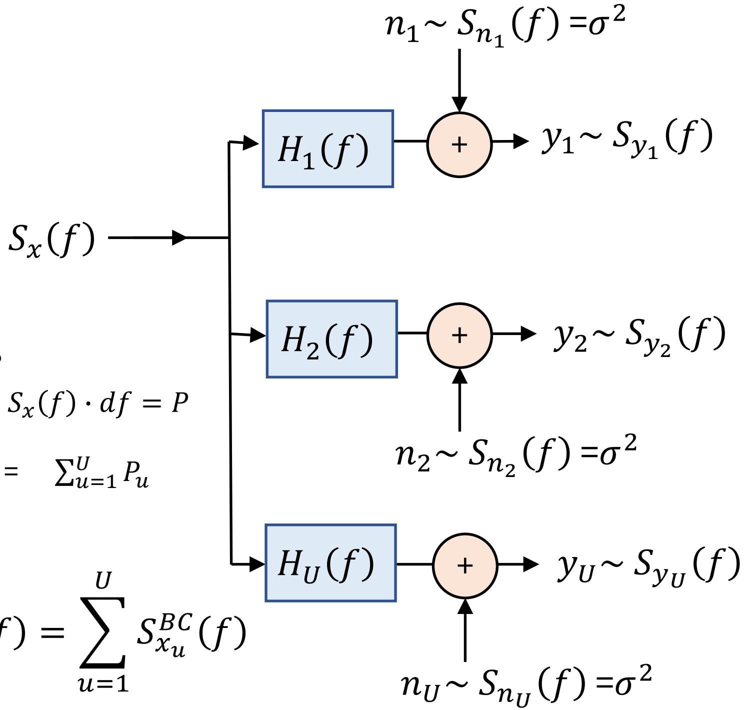
$$b_2 = \frac{1}{2} \cdot \log_2\left(1 + \frac{\mathcal{E}_2^{MAC} \cdot g_2}{1}\right) = .2074$$

- Can use the easier MAC developments to analyze the BC through duality

Continuous-time Scalar BC

Will skip this short theoretical section to save time this year
Design over vector DMT systems will reappear later for any number of tones

Section 2.8.5

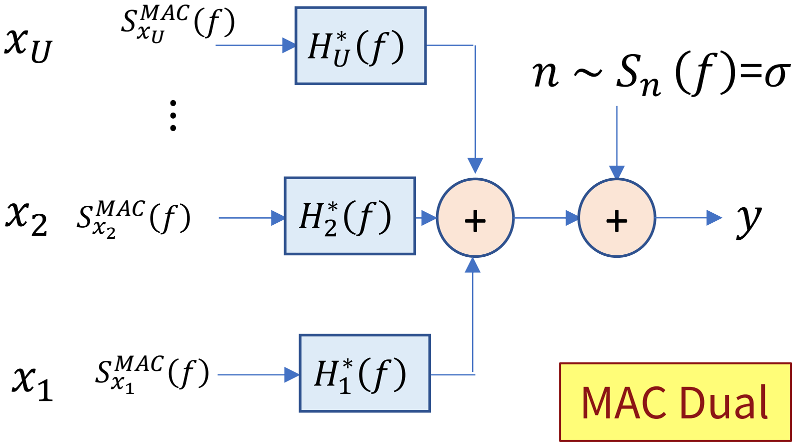


$$\int_{-\infty}^{\infty} S_x(f) \cdot df = P = \sum_{u=1}^{U} P_u$$

$$S_x(f) = \sum_{u=1}^{U} S_{x_u}^{BC}(f)$$

$$\ln S_{0,u} = \int_{-\infty}^{\infty} \ln\left[1 + \frac{|H_u(f)|^2}{S_{n_u}(f)}\right] \cdot df \;\; ; \;\; (if < \infty)$$

$$SNR_{geo,u} = P_u \cdot S_{0,u}$$



$\boldsymbol{x}$

$$\sum_{u=1}^{U} \int_{-\infty}^{\infty} S_{x_u}^{MAC}(f) \cdot df = P$$

Design for this MAC, and then find dual

Scalar Duality

- Replace with integrals and $\mathcal{E}_u^{MAC/BC} \rightarrow S_u^{MAC/Bc}(f)$

order intentionally reversed

MAC	BC
$b_1 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_1^{MAC}\cdot g_1}{1+\mathcal{E}_2^{MAC}\cdot g_2+\ldots+\mathcal{E}_U^{MAC}\cdot g_U}\right)$	$b_1 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_1^{BC}\cdot g_1}{1}\right)$ 1 ND user
$b_2 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_2^{MAC}\cdot g_2}{1+\mathcal{E}_3^{MAC}\cdot g_3+\ldots+\mathcal{E}_U^{MAC}\cdot g_U}\right)$	$b_2 = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_2^{BC}\cdot g_2}{1+\mathcal{E}_1^{BC}\cdot g_2}\right)$
$\vdots$	$\vdots$
$b_U = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_U^{MAC}\cdot g_U}{1}\right)$ 1 ND user	$b_U = \frac{1}{2}\log_2\left(1 + \frac{\mathcal{E}_U^{BC}\cdot g_U}{1+\left[\mathcal{E}_1^{BC}+\ldots+\mathcal{E}_{U-1}^{BC}\right]\cdot g_U}\right)$

Corresponding PSD's

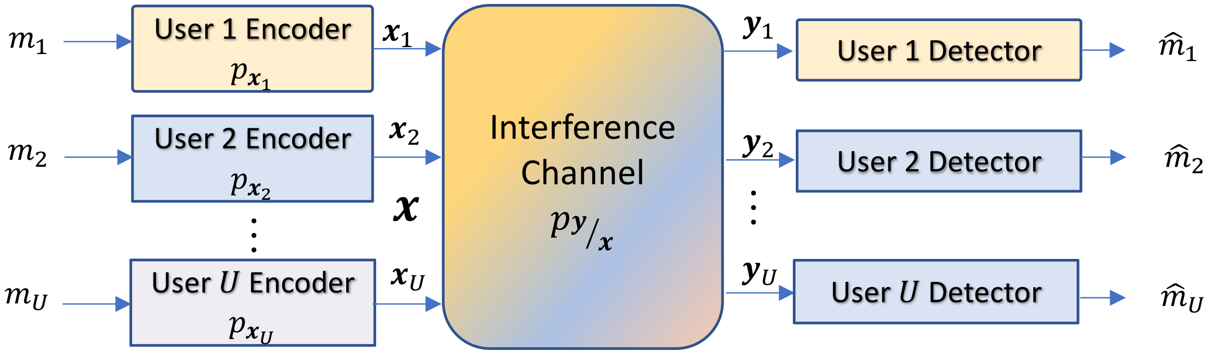
- $\mathcal{E}_u^{MAC/BC} \rightarrow S_u^{MAC/BC}(f)$
- See proof in notes (Theorem 2.8.2 in Section 2.8.4), but execute with PSD's $S_u^{MAC/BC}(f)$

$$\begin{aligned}
\mathcal{E}_1^{BC} &= \mathcal{E}_1^{MAC} \cdot \frac{1}{1+\mathcal{E}_2^{MAC}\cdot g_2+\ldots+\mathcal{E}_U^{MAC}\cdot g_U} \\
\mathcal{E}_2^{BC} &= \mathcal{E}_2^{MAC} \cdot \frac{1+\mathcal{E}_2^{BC}\cdot g_2}{1+\mathcal{E}_3^{MAC}\cdot g_3+\ldots+\mathcal{E}_U^{MAC}\cdot g_U} \\
\vdots &= \vdots \\
\mathcal{E}_U^{BC} &= \mathcal{E}_U^{MAC} \cdot \left(1+\left[\mathcal{E}_1^{BC}+\ldots+\mathcal{E}_{U-1}^{BC}\right]\cdot g_U\right)
\end{aligned}$$

MAC-set Approach to IC

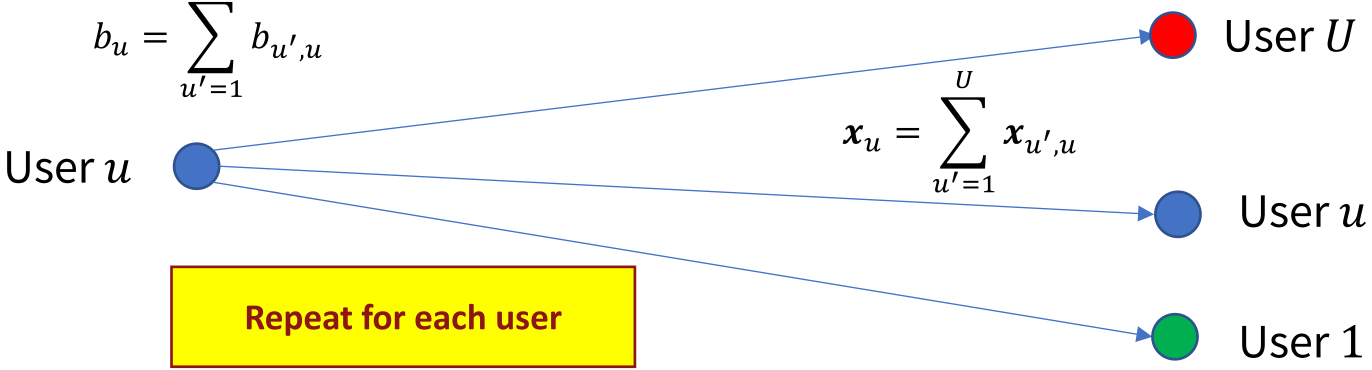
Sec 2.9

The Interference Channel (MAC)



Refine Capacity Region Def: with order-approach: Assume each user must use a single-user $\Gamma = 0\ dB$ code

$$b_u = \sum_{u'=1}^{U} b_{u',u}$$



$$\boldsymbol{x}_u = \sum_{u'=1}^{U} \boldsymbol{x}_{u',u}$$

U^2 potential sub-user channels/ decodes

Repeat for each user

Prior-User Set (repeat from L7)

- Order vector and inverse
  - Permutation (permutation matrix J), etc

$$\boldsymbol{\pi}_u = \begin{bmatrix} \pi(U') \\ \vdots \\ \pi(1) \end{bmatrix} \quad \boldsymbol{\pi}_u^{-1} = \begin{bmatrix} U' \\ \vdots \\ 1 \end{bmatrix} \qquad j = \pi(i) \;\rightarrow i = \pi^{-1}(j)$$

- Prior-User Set is $\mathbb{P}_u(\boldsymbol{\pi}) = \{j \mid \boldsymbol{\pi}^{-1}(j) < \boldsymbol{\pi}^{-1}(u)\}$
  - That is "all the users before the desired user u in the given order $\boldsymbol{\pi}$.
  - Receiver u best decodes these "prior" users and removes them, while "post" users are noise
  - $\boldsymbol{\pi}$ can be any order in $\mathbb{P}_u(\boldsymbol{\pi})$, but the most interesting is usually $\boldsymbol{\pi}_u$ (receiver u's order)

rcvr/ User i	$\boldsymbol{\pi}_4(i)$	$\boldsymbol{\pi}_3(i)$	$\boldsymbol{\pi}_2(i)$	$\boldsymbol{\pi}_1(i)$
$i = 4$	3	3	4	3
$i = 3$	4	2	3	2
$i = 2$	1	4	2	1
$i = 1$	2	1	1	4
$\mathbb{P}_u(\boldsymbol{\pi}_u)$	{1,2}	{2,4,1}	{1}	{4}

$$\boldsymbol{\Pi} = \begin{bmatrix} 3 & 3 & 4 & 3 \\ 4 & 2 & 3 & 2 \\ 1 & 4 & 2 & 1 \\ 2 & 1 & 1 & 4 \end{bmatrix}$$

really should be 16 with full subusers, but simplify here to 4

- Data rates (mutual information bounds average out only those users who are nog cancelled as noise)

$\mathfrak{I}$	$\mathfrak{I}_4$	$\mathfrak{I}_3$	$\mathfrak{I}_2$	$\mathfrak{I}_1$
top	∞	$\mathcal{I}_3(3/1,2,4)$ 20	∞	∞
	$\mathcal{I}_4(4/1,2)$ 10	$\mathcal{I}_3(2/1,4)$ 9	∞	∞
	$\mathcal{I}_4(1/2)$ 5	$\mathcal{I}_3(4/1)$ 4	$\mathcal{I}_2(2/1)$ 4	$\mathcal{I}_1(1/4)$ 2
bottom	$\mathcal{I}_4(2)$ 1	$\mathcal{I}(1)$ 2	$\mathcal{I}_2(1)$ 2	$\mathcal{I}_1(4)$ 5

$$\mathcal{I}_{min}(\boldsymbol{\Pi}, p_{\boldsymbol{xy}}) = \begin{bmatrix} 4 \\ 20 \\ 1 \\ 2 \end{bmatrix}$$

Maximum number of subusers U^2

- User u has maximum bit rate, when all other users are given (cancelled): $b_u \leq \mathcal{I}(\boldsymbol{x}_u; \boldsymbol{y}_u / \boldsymbol{x}_{U\backslash u})$

- Although -- this may not be good for the other users $i \neq u$.
  - $\mathcal{I}_{min}(\boldsymbol{\Pi}, p_{\boldsymbol{xy}})$ calculation, given a $\boldsymbol{\Pi}$, precedes a subsequent convex-hull search over all $\boldsymbol{\Pi}$ to obtain the achievable region $\mathcal{A}(\boldsymbol{b}, p_{\boldsymbol{xy}})$

$$\boldsymbol{b} = \sum_{u'=1}^{U} \alpha_{u'} \cdot \boldsymbol{b}_{u'} \quad 1 = \sum_{u'=1}^{U} \alpha_{u'}$$

No more than U terms needed for U-dimensional $\boldsymbol{b}$

$$\boldsymbol{b} = \begin{bmatrix} b_U \\ \vdots \\ b_1 \end{bmatrix} \qquad b_u = \sum_{u'=1}^{U} b_{u',u}$$

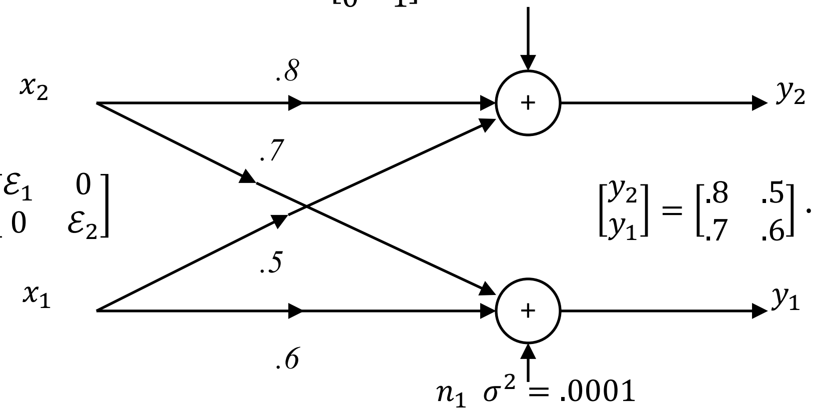
Will do this for $\mathcal{I}_{min}(\boldsymbol{\Pi}, p_{\boldsymbol{xy}})$ values over possible $\boldsymbol{\Pi}$'s

- For $U \geq 2$, the other users' subuser components may be desirable to decode, but not all → $U'! \leq (U^2)!$ for **each** receiver's order search
- $\boldsymbol{\Pi}$ maximally has $(U'!)^U$ possible choices (in most general case)

At any receiver u , the subuser components separate into two groups for any given order $\boldsymbol{\pi}_u$:
(1) those cancelled (or generally conditional probability has specific given values for those components), and
(2) those not cancelled, which are averaged out generally in marginal distributions
If that choice is made for each user for each receiver, there are thus maximally U choices across all receivers into (1) or (2), so $U' \leq U^2$

Example channel – Scalar Gaussian IC

$R_{nn} = .0001 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



$R_{xx} = \begin{bmatrix} \mathcal{E}_1 & 0 \\ 0 & \mathcal{E}_2 \end{bmatrix}$

$$\begin{bmatrix} y_2 \\ y_1 \end{bmatrix} = \begin{bmatrix} .8 & .5 \\ .7 & .6 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} + \begin{bmatrix} n_2 \\ n_1 \end{bmatrix}$$

$\frac{\log_2\left(1+\frac{.64}{.0001}\right)}{2}=6.3220$	$\frac{\log_2\left(1+\frac{.36}{.0001}\right)}{2}=5.9071$	$\frac{\log_2\left(1+\frac{.64}{.2501}\right)}{2}=0.9157$	$\frac{\log_2\left(1+\frac{.25}{.6401}\right)}{2}=0.2378$
$\frac{\log_2\left(1+\frac{.36}{.4901}\right)}{2}=0.3973$	$\frac{\log_2\left(1+\frac{.49}{.3601}\right)}{2}=0.6196$	$\frac{\log_2\left(1+\frac{(.5).64}{.0001}\right)}{2}=5.8222$	$\frac{\log_2\left(1+\frac{(.5).36}{.0001}\right)}{2}=5.4073$
$\frac{\log_2\left(1+\frac{.64}{.1251}\right)}{2}=1.3063$	$\frac{\log_2\left(1+\frac{.18}{.4901}\right)}{2}=0.2257$	$\frac{\log_2\left(1+\frac{.49}{.1801}\right)}{2}=0.9478$	$\frac{\log_2\left(1+\frac{.36}{.2451}\right)}{2}=0.6519$
$\frac{\log_2\left(1+\frac{.5(.64)}{.2501}\right)}{2}=0.5944$	$\frac{\log_2\left(1+\frac{.5(.49)}{.3601}\right)}{2}=0.3744$	$\frac{\log_2\left(1+\frac{.5(.25)}{.6401}\right)}{2}=0.1287$	$\frac{\log_2\left(1+\frac{.25}{.3201}\right)}{2}=0.4163$

Table 2.6: Some useful calculations for the upcoming example

Convex combos of These 2 vertices Probably includes Other sub user combos (because they are big)

- Earlier H, but this time as an IC
  - Not complete set of orders (4 instead of (4!)^2 = 576)
- Shaded points are interior to line formed by unshaded points

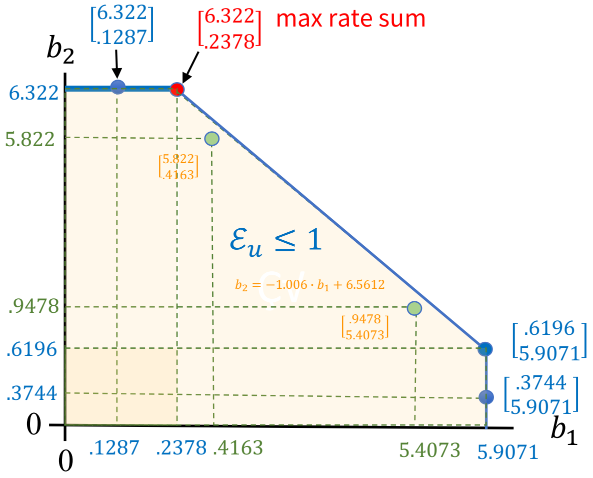
$\mathbf{\Pi}$	u	$\mathcal{E}_u$	$\mathbb{P}_u(\boldsymbol{\pi}_u)$	$\mathscr{I}_u(x_u; y_u/\mathbb{P}_u(\boldsymbol{\pi}_u))$	$\mathscr{I}_u(x_{\neg u}; y_{\neg u}/\mathbb{P}_{\neg u}(\boldsymbol{\pi}_{\neg u}))$	$\mathcal{I}_{min,u}(\mathbf{\Pi}, \boldsymbol{\mathcal{E}})$
$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$	2	1	$\{1\}$	6.322	∞	6.322
	1	1	$\emptyset$	.3973	.2378	.2378
$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$	2	1	$\emptyset$	.9157	.6196	.6196
	1	1	$\{2\}$	5.9071	∞	5.9071
$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$	2	1	$\emptyset$	.9157	∞	.9157
	1	1	$\emptyset$	.3973	∞	.3973
$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$	2	1	$\{1\}$	6.322	.2378	.2378
	1	1	$\{2\}$	5.9071	.6196	.6196

Table 2.7: Evaluation of $\mathcal{I}_{min}$ for different orders.

Example continued

$\mathbf{\Pi}$	u	$\mathcal{E}_u$	$\mathbb{P}_u(\boldsymbol{\pi}_u)$	$\mathscr{I}_u(x_u; y_u/\mathbb{P}_u(\boldsymbol{\pi}_u))$	$\mathscr{I}_u(x_{\neg u}; y_{\neg u}/\mathbb{P}_{\neg u}(\boldsymbol{\pi}_{\neg u}))$	$\mathcal{I}_{min,u}(\mathbf{\Pi}, \boldsymbol{\mathcal{E}})$
$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$	2 1	1 0.5	$\{1\}$ $\emptyset$	6.322 .2257	∞ .1287	6.322 .1287
$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$	2 1	1 0.5	$\emptyset$ $\{2\}$	1.3063 5.4073	.9478 ∞	.9478 5.4073
$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$	2 1	0.5 1	$\{1\}$ $\emptyset$	5.822 .6519	∞ .4163	5.822 .4163
$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$	2 1	0.5 1	$\emptyset$ $\{1\}$	5.822 5.9071	.3744 ∞	.3744 5.9071
$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$	2 1	1 0.95	$\{1,2\}$ $\{1\}$	6.322 .3818	∞ .2276	6.322 .2276
$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$	2 1	1 0.95	$\{2\}$ $\{1,2\}$	.9425 5.8701	.6412 ∞	.6412 5.8701

Table 2.8: More example points with the best orders



- Note the dimension-sharing of the large-b_u points dominates the other points on the interior
- Also – check of vertices' derivatives relative to the dimension-sharing line (-1.006) – try upper

$$\ln(2) \cdot \frac{db_2}{d\mathcal{E}_2} = \frac{3200}{6400 \cdot \mathcal{E}_2 + 1} = 0.4999$$

$$\ln(2) \cdot \frac{db_1}{d\mathcal{E}_2} = -\frac{3200 \cdot 2500 \cdot \mathcal{E}_1}{6400 \cdot (\mathcal{E}_2 + 2500 \cdot \mathcal{E}_1 + 1) \cdot (6400 \cdot \mathcal{E}_2 + 1)} = -.1404$$

$$\frac{db_2}{db_1} = -3.56$$

Vertices all (○) inside pentagon for this example

- For 2 vertices if magnitude of slope is less than 1, then upper point and otherwise lower point (or whole line)
  - Could check other vertex also, but if curvature is within the line already (convex), then no need

IC Rate Region Examples

Sec 2.9.1

So-called "weak" symmetric IC

$$\begin{bmatrix} y_2 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} + \begin{bmatrix} n_2 \\ n_1 \end{bmatrix}$$

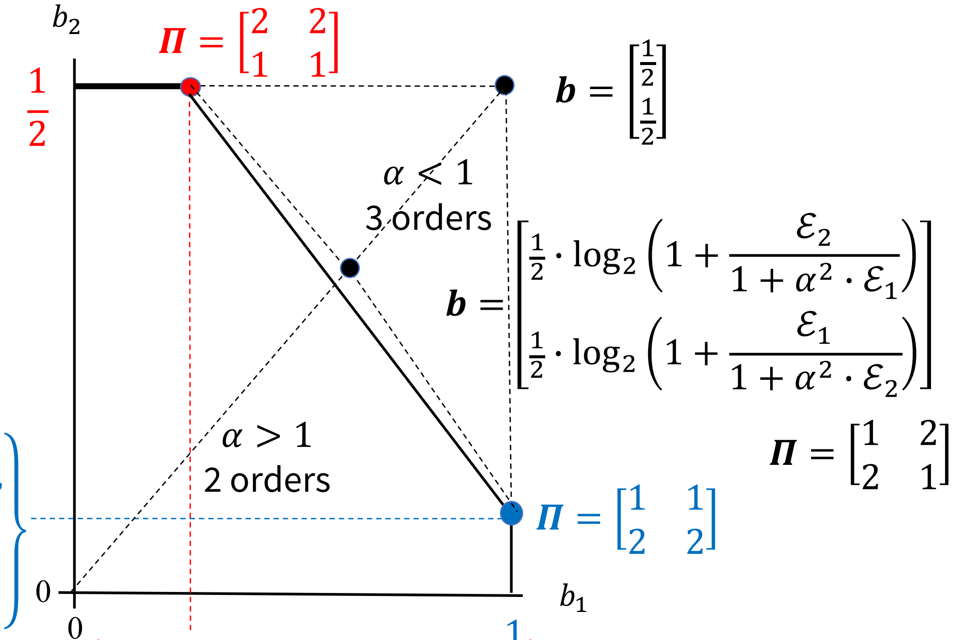
$R_{\boldsymbol{nn}} = I$ and $\boldsymbol{\mathcal{E}} \preccurlyeq \mathbf{1}$

- When $\alpha \to 0$, there is no crosstalk and so $\mathcal{C}_{IC}(\boldsymbol{b})$ is a square.
- When $\alpha > 1$, $\mathcal{C}_{IC}(\boldsymbol{b})$ is a pentagon.
- When $0 < \alpha < 1$, $\mathcal{C}_{IC}(\boldsymbol{b})$ is a hexagon.

$$\min\left\{ \frac{1}{2}\cdot\log_2\left(1+\frac{\mathcal{E}_2}{1+\alpha^2\cdot\mathcal{E}_1}\right),\ \frac{1}{2}\cdot\log_2\left(1+\frac{\alpha^2\cdot\mathcal{E}_2}{1+\mathcal{E}_1}\right) \right\}$$

These two are same for $\alpha = 1$ and equal energy, and determine Imin vector possibilities

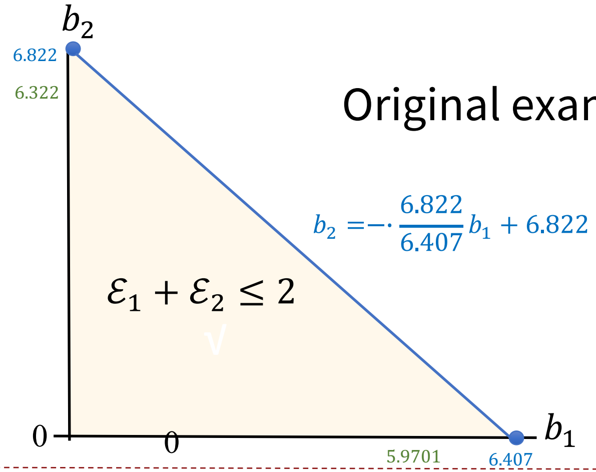
Achievable Region when $\mathcal{E}_1 = \mathcal{E}_2 = 1$



$$\min\left\{ \frac{1}{2}\cdot\log_2\left(1+\frac{\mathcal{E}_1}{1+\alpha^2\cdot\mathcal{E}_2}\right),\ \frac{1}{2}\cdot\log_2\left(1+\frac{\alpha^2\cdot\mathcal{E}_1}{1+\mathcal{E}_2}\right) \right\}$$

Energy-sum IC extension

- Same channel with only energy-sum constraint:
- Same derivative test shows dimension-sharing line is boundary

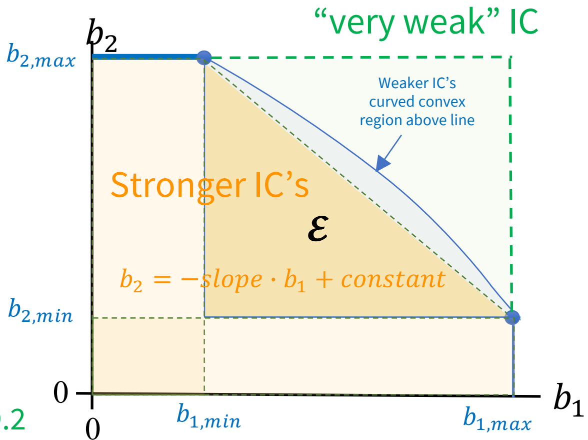


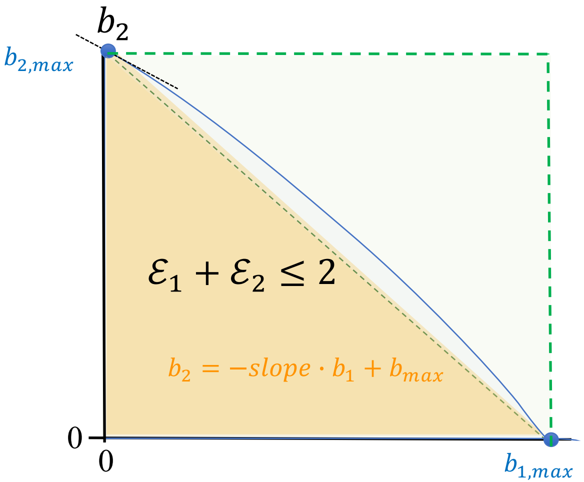
- General derivative test

$$\ln(2) \cdot \frac{db_2}{d\mathcal{E}_2} = \frac{g_{22}/2}{g_{22} \cdot \mathcal{E}_2 + 1}$$

$$\ln(2) \cdot \frac{db_1}{d\mathcal{E}_2} = -\frac{g_{22}/2 \cdot g_{12} \cdot \mathcal{E}_1}{g_{22} \cdot (\mathcal{E}_2 + g_{12} \cdot \mathcal{E}_1 + 1) \cdot (g_{22} \cdot \mathcal{E}_2 + 1)} \quad \textbf{or}$$

$$\ln(2) \cdot \frac{db_1}{d\mathcal{E}_2} = -\frac{g_{21}/2 \cdot g_{11} \cdot \mathcal{E}_1}{g_{21} \cdot (\mathcal{E}_2 + g_{11} \cdot \mathcal{E}_1 + 1) \cdot (g_{21} \cdot \mathcal{E}_2 + 1)},$$





Vector Gaussian IC Example

- 2 users and H is 4 x 2 $y = \begin{bmatrix} \boldsymbol{y}_2 \\ \boldsymbol{y}_1 \end{bmatrix} = \begin{bmatrix} H_2 \\ H_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_2 \\ \boldsymbol{n}_1 \end{bmatrix}$

$$H_2 = [\boldsymbol{h}_{22} \quad \boldsymbol{h}_{21}] = \begin{bmatrix} .9 & .3 \\ .3 & .8 \end{bmatrix}$$

$R_{\boldsymbol{nn}} = .01 \cdot I$

$$H_1 = [\boldsymbol{h}_{12} \quad \boldsymbol{h}_{11}] = \begin{bmatrix} .8 & .7 \\ .6 & .5 \end{bmatrix}$$

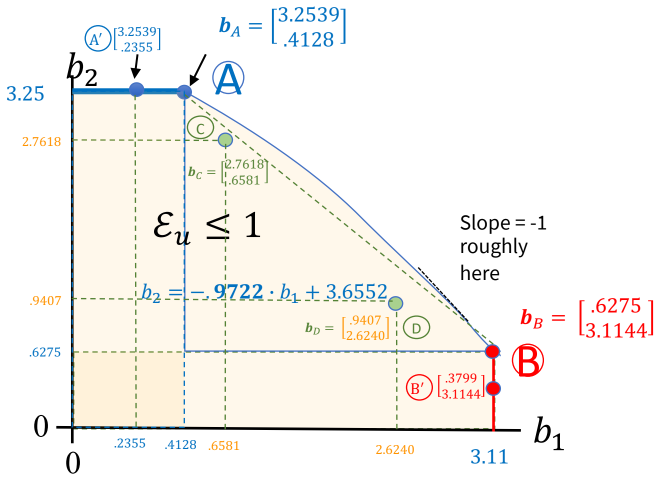
```
>> H2 = [9     3
        3     8 ];
>> Rb2inv=H2'*H2+diag([1 1]);
>> Gbar2=chol(Rb2inv);
>> G2=inv(diag(diag(Gbar2)))*Gbar2;
>> S02=diag(diag(Gbar2))*diag(diag(Gbar2));
>>  0.5*log2(diag(S02)) =
b2 =  3.2539
b1 =  2.7526
>>  H1 = [8     7
         6     5 ];
>> Rb1inv=H1'*H1+diag([1 1]);
>> Gbar1=chol(Rb1inv);
>> S01=diag(diag(Gbar1))*diag(diag(Gbar1));
>> 0.5*log2(diag(S01)) =
b2 =  3.3291
b1 =  0.4128
```

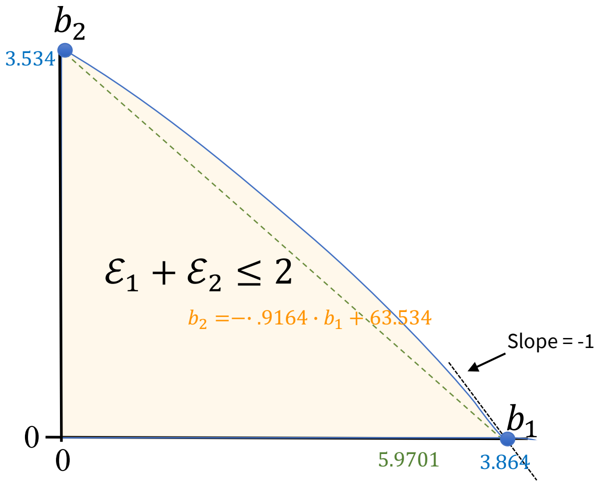
```
>> J2=hankel([0 1]);
>> Rb2inv=J2*H2'*H2*J2+diag([1 1]);
    74    51
    51    91
>> Gbar2=chol(Rb2inv);
>> S02=diag(diag(Gbar2))*diag(diag(Gbar2));
>> 0.5*log2(diag(S02)) =
b1 =  3.1047
b2 =  2.9018

>> Rb1inv=J2*H1'*H1*J2+diag([1 1]);
>> Gbar1=chol(Rb1inv);
>> S01=diag(diag(Gbar1))*diag(diag(Gbar1));
>> 0.5*log2(diag(S01)) =
b1 =  3.1144
b2 =  0.6275
```

Try various energy points
With the same orders

Potential student Project mu_IC.m

4x2 IC Example continued





- Vary energies near max points to see if local points above or below dimension-sharing, so curved boundary or flat
- Since the line has slope magnitude less than 1, then the curvature is above this line with max rate sum at magnitude 1

PS 5.4 (2.31) – IC channel has mix, one 2x2 user and one scalar user

IC Maximum Rate Sum
(& Minimum Energy Sum)

Max IC Sum Rate by Iterative Water-filling?

Posed Project/Research "maxRIC" or "minPIC"

$$\max \sum_{u=1}^{U} b_u$$

IC Sum Rate

ST: $$b_{u,i} \geq \begin{cases} \log_2 \left\{ \frac{1+\sum_{j=i}^{U'} \tilde{H}_{u,\pi_u^{-1}(j)} \cdot \mathcal{E}_{\pi_u^{-1}(j)} \cdot \tilde{H}^*_{u,\pi_u^{-1}(j)}}{1+\sum_{j=i+1}^{U'} \tilde{H}_{u,\pi_u^{-1}(j)} \cdot \mathcal{E}_{\pi_u^{-1}(j)} \cdot \tilde{H}^*_{u,\pi_u^{-1}(j)}} \right\} & \pi_u(i) \leq \pi_u(u) \\ 0 & \pi_u(i) > \pi_u(u) \end{cases} \quad \forall \begin{matrix} u = 1, \dots, U \\ i = 1, \dots, U \end{matrix}$$

Users' $\mathcal{A}_u(\boldsymbol{b})$ constraints

$$\sum_{i=1}^{U} b_{u,i} = b_u$$

subusers' sum constraints

$$\boldsymbol{\mathcal{E}}_x \succcurlyeq \boldsymbol{\mathcal{E}}$$

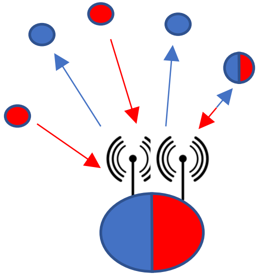
users' energy constraints

- For an order $\boldsymbol{\Pi}$ and each user u
  - RA water-fill $\mathcal{E}_u$ with $\pi_u^{-1}(i) \geq \pi_u^{-1}(u)$ as xtalk noise, others cancelled (uncancelled users' energies not updated)
  - There are $U' = U^2$ user rate constraints for the U receivers' achievable regions, which transform into convex constraints, $\boldsymbol{\theta}_u$ (will show how later in Sections 5.4 and 5.6)
  - Along with the energy-vector constraint $\boldsymbol{w}_u$ where each user's energy is sum of its subusers' energies
- Between water-fills, descent update $\boldsymbol{\Pi}$ for all u
  - The successive values within $\boldsymbol{\theta}_u$ must be ordered in value within the sum constraint to keep constraint convex
  - Each other user has a $\Delta \boldsymbol{b}_u$error for $\boldsymbol{\theta}_u$ that updates the constraint
  - Each similarly adds over subusers and has $\Delta \boldsymbol{\mathcal{E}}_u$ for $\boldsymbol{w}_u$

May need Ellipsoid descent for Lagrange multipliers, See Chapter 5

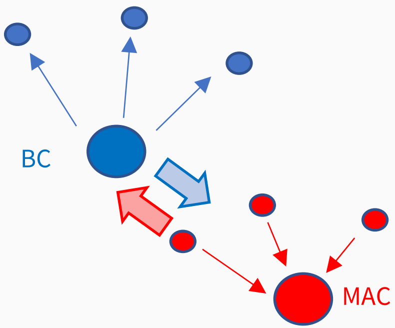
Nesting, DAS, cellfree & Relay

Multiuser Nesting



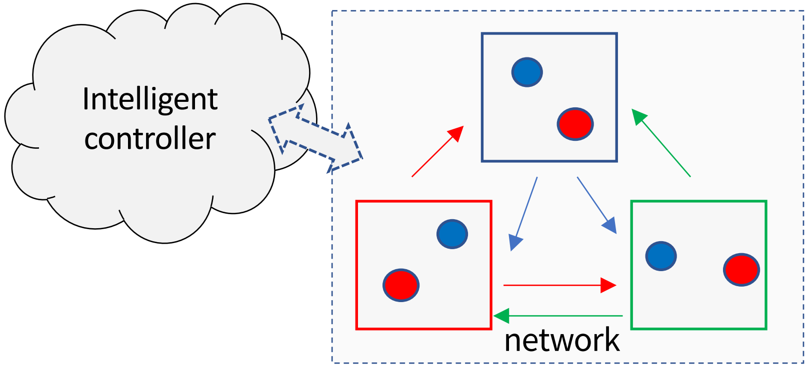
a) radio node edge
(base station or
Access point)

~



b) two subnetworks
Down (BC) and up (MAC)
IC network between them

Macro User

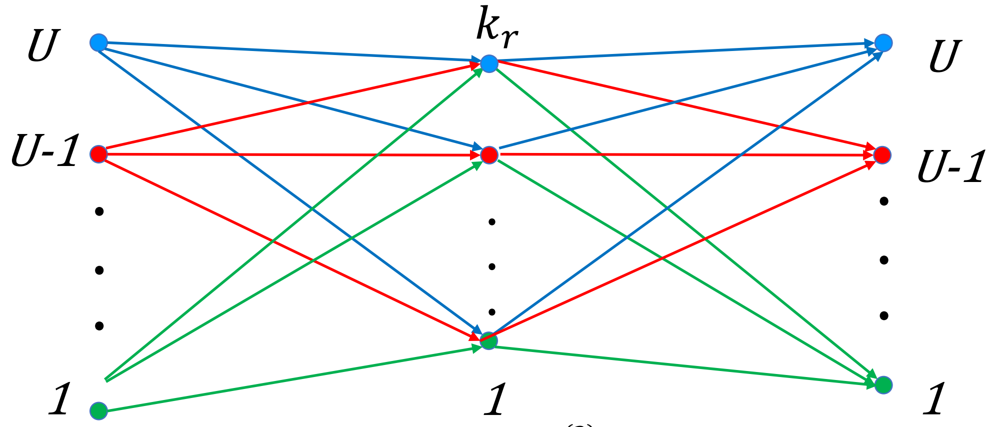


c) three nested IC: (BC, MAC)
Like those in b), nested into
3x3 IC network

**Will be suboptimal,
But easier
Section 5.6.2.3 later**

Single-Stage Relay Channel

- Conceptually uses what we know already and introduces sub-users at k_r relay points



PS 5.5 (2.32) – simple relay channel

$\boldsymbol{B}^{(1)}$ is $U \times k_r$ MAC user set

$\boldsymbol{B}^{(2)}$ is $k_r \times U$ BC user set

$$\mathcal{A}^{(1)}\left(\boldsymbol{B}^{(1)}, R_{\boldsymbol{xx}}^{(1)}\right) = \cup_{\boldsymbol{\Pi}^{(1)}}^{conv}\left\{\boldsymbol{B}_{min}^{(1)}\left(\boldsymbol{\Pi}^{(1)}, R_{\boldsymbol{xx}}^{(1)}\right)\right\}$$

$$\mathcal{A}^{(2)}\left(\boldsymbol{B}^{(2)}, R_{\boldsymbol{xx}}^{(2)}\right) = \cup_{\boldsymbol{\Pi}^{(2)}}^{conv}\left\{\boldsymbol{B}_{min}^{(2)}\left(\boldsymbol{\Pi}^{(2)}, R_{\boldsymbol{xx}}^{(2)}\right)\right\}$$

$\beta_k^{(1)}$ $\quad$ $\beta_u^{(2)}$

$$b_u\left(R_{\boldsymbol{xx}}^{(1)}, R_{\boldsymbol{xx}}^{(2)}\right) = \left\{ b_u \mid b_u \in \sum_{i=1}^{k_r} \min_{\left\{\beta_k^{(1)} \in \boldsymbol{B}_k^{(1)} \wedge \beta_k^{(2)} \in \boldsymbol{B}_k^{(2)}\right\}} \left[\beta_k^{(1)}\left(u, R_{\boldsymbol{xx}}^{(1)}\right) ; \beta_u^{(2)}\left(k, R_{\boldsymbol{xx}}^{(2)}\right)\right]\right\}$$

- Searches can be very complex

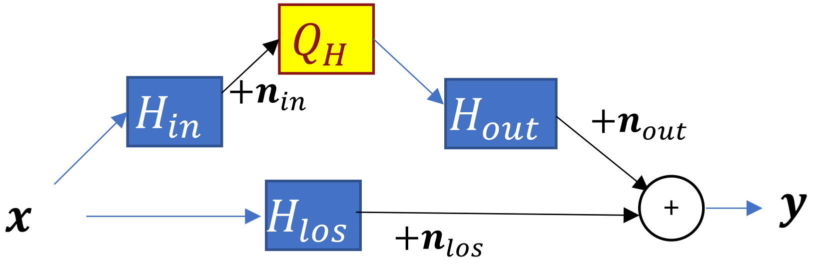
$$\mathcal{C}(\boldsymbol{b}) = \cup_{\left[R_{\boldsymbol{xx}}^{(1)} R_{\boldsymbol{xx}}^{(2)}\right]}^{hull} \begin{bmatrix} b_U \\ \vdots \\ b_1 \end{bmatrix}$$

- Multi-stage tedious, but same principles apply recursively

Reflective Intelligent Surfaces (RIS)

Posed Project/Research
“maxRIS” or “minRIS”

$$\boldsymbol{y} = \underbrace{\begin{bmatrix} H_{los} \\ H_{out} \cdot Q_H \cdot H_{in} \end{bmatrix}}_{H_{RIS}} \cdot \boldsymbol{x} + \underbrace{\begin{bmatrix} \boldsymbol{n}_{los} \\ \boldsymbol{n}_{out} + Q_H \cdot \boldsymbol{n}_{in} \end{bmatrix}}_{\boldsymbol{n}_{RIS}}$$



- The RIS matrix Q_H satisfies $\|Q_H\|_F^2 \leq G_H$, the RIS gain – it may also satisfy
  - Q_H is unitary matrix (preserves energy)
  - Q_H is diagonal, and usually also unitary, to be phase/gain-only adjustment on each antenna port (in-to-out)
  - Q_H has individual elements restricted

- For a given $R_{\boldsymbol{xx}}$, maximize over Q_H

$$\mathcal{I}(\boldsymbol{y};\boldsymbol{x}) = \log_2 \left| R_{n,RIS} + H_{RIS} \cdot R_{\boldsymbol{xx}} \cdot H_{RIS}^* \right|$$

- For a given Q_H , maximize the same over $R_{\boldsymbol{xx}}$

$$R_{\boldsymbol{nn},RIS} = \begin{bmatrix} R_{\boldsymbol{nn}} & 0 \\ 0 & R_{\boldsymbol{nn},out} + Q_H \cdot R_{\boldsymbol{nn}in} \cdot Q_H^* \end{bmatrix}$$

- Iterate

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End Lecture 11