Homework Help - Problem Set 3
Solutions

Coded OFDM Data-rate computation multiplies the (sub)symbol-rate by the total number of bits per (sub)symbol. For instance, if the sampling rate (for each of inphase and quadrature) of a digital-video broadcast system has $T' =$ 109.375 ns with $\bar{N} = 8192$ and a cyclic prefix that is 1/4 the FFT size, then the symbol period is

$$T = 8192(1 + \frac{1}{4}) \cdot T' = 10240 \cdot 109.375 \times 10^{-9} = 1.12 \times 10^{-3}$$

so the symbol rate is then 893 Hz. Thus, a 64 QAM system with no code ($r = 1$) would carry, with for example 6000 used tones,

$$R = 893 \cdot 6 \cdot 6000 \approx 32 \text{ Mbps}$$

Coded-OFDM usually has a limited set of "MCS" (Modulation Coding Scheme) options that basically are the code rates allowed for a set of constellations. Thus, the data rates R are basically found by multiplying r (code rate) by $\log_2(C)$ (number of bits in constellation) times the number of used tones times the symbol rate. If there were 4 codes and 5 constellations allowed, there would be 20 possible data rates.

With the MCS, the code will have certain free distance improvements that multiply the SNR so $SNR \to d_{free} \cdot SNR$ at the expense of a lower r for higher d_{free}. The MCS-style loading knows the distances for each of the codes, and has measured the SNR. Loading amounts to finding the code and d_{free} at this particular SNR that provides largest reliable data rate. Because there are only a few choices, simple trial-and-error over the MCS sets is usually sufficient. So for instance, if a Wi-Fi device (like a smart phone) moves closer to the Wi-Fi router, the channel gain presumably would improve. Improvement may then lead to a higher-data-rate MCS choice. For each channel gain g, there will thus be a best MCS choice. So at design time, a simple table of code/rate versus measured channel gain may suffice as long as the channel-gain distribution remains constant. In reality, the channel-gain distribution may vary and so a simple table is not sufficient and thus calculation of $< P_e >$ and/or P_{out} with selection of MCS to meet both criteria improves with respect to single fixed table.

For Problem 4.14(b), a portion of the solution with a different channel input energy/noise ratio of 2000 (33 dB) is provided here to help students progress:

The MA K_{ma} is given by

$$K_{ma} = \Gamma \cdot \frac{\left(2^{<b>}\right)^{\left[\frac{1}{\sum_{g \in \mathcal{G}^*} p_g}\right]}}{\prod_{g \in \mathcal{G}^*} (g)^{\left[\frac{p_g}{\sum_{g \in \mathcal{G}^*} p_g}\right]}} \quad ,$$

which simplifies for this problem's constant $p_g = 0.1$ (so initially $|\mathcal{G}^*| = 10$) to:

$$\begin{aligned} K_{ma} &= \Gamma \cdot \left[\frac{2^{\frac{10 \cdot <b>}{|\mathcal{G}^*|}}}{\prod_{g \in \mathcal{G}^*} g^{\frac{1}{|\mathcal{G}^*|}}} \right] \\ &= \Gamma \cdot \left[\frac{2^{10 \cdot <b>}}{\prod_{g \in \mathcal{G}^*} g} \right]^{\frac{1}{|\mathcal{G}^*|}} \end{aligned}$$

Ergodic water-filling starts with that $|\mathcal{G}^*| = 10$ and includes all g's in the calculation of K_{ma} for any of the permitted constellations. The energy corresponding to the lowest g value is checked and if negative, then this range of g is discarded and erased by the receiver with the set $\mathcal{G}^*$ then being reduced in cardinality by deletion of this smallest g value and its corresponding interval. A new K_{ma} is then computed. Once all energies are positive and the average energy is not exceeded, the solution is feasible. For 4-QAM, the first K_{ma} try is thus

4QAM:

```
Kma =10^(0.9)*(4^10 / (prod(2000*g(1:10))))^(1/10) =  0.2708

e1=Kma-10^(0.9)/(2000*g(1)) = -0.4857 < 0

e2=Kma-10^(0.9)/(2000*g(2)) =  0.0287 > 0
```

So delete the smallest g value of .0053, and try $|\mathcal{G}^*| = 9$.

```
Kma =10^(0.9)*(4^10 / (prod(2000*g(2:10))))^(1/9) =   0.2416
>> e2=Kma-10^(0.9)/(2000*g(2)) =0 ( so g(2) is not used)
```

so, MA ergodic water-filling for 4QAM will not use g in the first tier that has value of .0053. However, the average energy also should be checked, which corresponds to ensuring the margin is positive.

```
e1=Kma-10^(0.9)/(2000*g(1));
e2=Kma-10^(0.9)/(2000*g(2));
e3=Kma-10^(0.9)/(2000*g(3));
e4=Kma-10^(0.9)/(2000*g(4));
e5=Kma-10^(0.9)/(2000*g(5));
e6=Kma-10^(0.9)/(2000*g(6));
```

```
e7=Kma-10^(0.9)/(2000*g(7));
e8=Kma-10^(0.9)/(2000*g(8));
e9=Kma-10^(0.9)/(2000*g(9));
e10=Kma-10^(0.9)/(2000*g(10));

>> e=[0,e2,e3,e4,e5,e6,e7,e8,e9,e10];
>>  0.1*sum(e) = 0.1491 < 1
```

Thus 4 QAM is feasible on this channel with full constellation and gap $\Gamma = 9$ dB. The same process is followed for 16-QAM and 64- QAM.

Similarly for part c, but with just the water-filling constant given, here but proceeds in similar fashion:

The RA K_{ra} simplifies with $p_g = 0.1$ to:

$$K_{ra} = \frac{1 + \Gamma \cdot \sum_{g \in \mathcal{G}^*} \frac{1}{10g}}{\sum_{g \in \mathcal{G}^*} 0.1} \quad ,$$

or:

$$K_{ra} = \frac{10 + \Gamma \cdot \sum_{g \in \mathcal{G}^*} \frac{1}{g}}{|\mathcal{G}^*|} \tag{1}$$

The corresponding maximized average bit rate is

$$< b >= \sum_{g \in \mathcal{G}^*} 0.1 \cdot \log_2 \left(\frac{K_{ra}}{\Gamma} g \right) \quad . \tag{2}$$

Execution of water-filling for $\Gamma = 9$ dB yields:

```
>> Kra=0.1*(10 + (10^.9)*sum(ones(1,10)./(2000*g))) =  1.1441
```

Discrete Loading Best Discrete loading adds an extra unit of information, which for SQ QAM restriction means two bits/tone (one bit in each dimension), simply adds bits in the positions of next least energy. This is the "greedy algorithm" that is used by Levin Campello. Thus, the incremental-energy table is just a way to enumerate how much energy needed for each additional unit on each tone. The better tones have smaller incremental energy until they get heavily loaded and each additional bit on them looks less attractive that putting a few bits on some of the lower-SNR tones.

SQ QAM is pretty easy and basically has (Γ in dB)

$$\mathcal{E}_1(2) = 2 \cdot \frac{10^{\Gamma/10}}{g_n} \left(2^2 - 1 \right) \quad ,$$

and thus

$$\begin{aligned} e_1(4) &= 4 \cdot \mathcal{E}_1(2) \\ e_1(6) &= 4 \cdot e_1(4) \\ e_1(8) &= 4 \cdot e_1(6) \quad , \end{aligned}$$

and so on. Tables can be created from this and greedy applied.

Binning The binning problem 4.16 attempts to be self-explanatory by taking the student through a series of steps that lead to comparison of the sampled/learned distribution to random samples generated from a known distribution. Of course in practice, the distribution won't be known for comparison.

The inverse $P_e = 10^{-5}$ expression (any $\bar{P}_e$ could be substituted in and for instance, 10^{-6} produces $10^{1.37}$) is

$$g = \frac{|C|-1}{3 \cdot d_{free}} \cdot \left(\underbrace{Q^{-1}(10^{-5})}_{10^{0.63}} \right)^2 \quad .$$

so one can generate g values for potential bin interval endpoints from such an expression, substituting in the possible constellation sizes. For instance the matlab commands run through a set of values:

```
>> dfree=[10 6 5 4 3];
>> r=[1/2 2/3 3/4 5/6 7/8];
>> M=[4 16 64 256];
>> arg=kron((ones(1,5)./(3*dfree))',M-ones(1,4))  =
    0.1000    0.5000    2.1000     8.5000
    0.1667    0.8333    3.5000    14.1667
    0.2000    1.0000    4.2000    17.0000
    0.2500    1.2500    5.2500    21.2500
    0.3333    1.6667    7.0000    28.3333
```

The data rates form from $r \cdot |C|$, running through r values and constellation sizes.

Evaluation of the set of g's generated can form a distribution, for instance exponential distribution with

```
pg=[exp(-.1*([0,g]))]-[exp(-.1*(g)),0]
```

that should sum to 1. From basic probability, the cumulative distribution is formed by cumulative sums, or the matlab cumsum command. However, the homework problem can easily directly compute the cumulative distribution in simple math, and then generate N samples according to

```
>> samples=F(rand([1,N]);
```

Wi-FI Loading The analog front-end noise of any wireless receiver simply adds the noise figure to the ambient psd (which at room temperature is -174 dBm/Hz). To find the power over a frequency range, simply add $10 \cdot \log_{10} W$ to the psd. Wider range (larger W) is larger power. The transmit power is 2 times the energy/real-dimension for complex signals, of course reduced by any channel attenuation to be used at the channel output.

The rest of Problem 4.15 uses similar calculations to the examples in the notes and in Problem 4.22.