Project: **T1E1.4: VDSL**

Title: Construction of Modulated Signals From Filter-Bank Elements (99-)

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Abstract:

This contribution illustrates the concept of convergence theory for QAM and Multitone signals by constructing both from a set of filter-bank elements, furthering some concepts posed recently in contribution 99-329. Simple examples are given to motivate the study of VDSL transmission technique design that combines the features of DMT and CAP/QAM transmission, based on the filter-bank concept of 99-339. Specific study of such methods as an alternative transmission method for a single choice of VDSL line-code is suggested for the VDSL activity in both T1E1.4 and ITU.

NOTICE

Construction of Modulated Signals From Filter-Bank Elements (99-)

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1. Introduction

Contribution 99-329 [1]describes a transmission method that is intermediate to multitone and QAM/CAP transmisssion methods in that both types of signals can be constructed using elements in that contribution. This contribution contain an examples that illustrates the construction of either type of signal. Further, such construction of transmit signals is suggested for serious study as a method for a single VDSL standardized transmission method.

Section 2 simplifies the basics of so-called "convergence theory" [2] that outlines the conditions for equivalent performance of QAM/CAP methods and multichannel (or DMT) transmission methods. An example is provided to motivate further study of the methods in [1] for VDSL. Section 3 concludes. An annex provides a second example called the "Bandwidth Optimization Paradox" in convergence theory that illustrates the effect of incorrectly satisfying the convergence conditions.

2. Convergence

References [2] and [3] are sometimes cited as noting performance equivalence of various types of modulation methods, including, but not limited to, DMT and CAP/QAM. The methods in [1] are generally within the main consideration in [2] with finite-delay packet realizations considered in [3]. The conditions for equivalence are sometimes also not well understood or translated. To simplify understanding of the conditions, this section illustrates equivalent transmitters and receivers for infinite-length filters. After viewing these examples, this section progresses to stating the convergence conditions for generally, but in simple non-mathematical terms.

2.1 Transmit Convergence

Figure 1 illustrates equivalent transmitters based on a bank [1] of 8 equal bandwidth filters for modulation. Channel characteristics are such in this system that the optimum bandwidth use (which can be found through a procedure known as ``water-filling" [2]) uses only 6 of the filter bands. 4 of the used bands (set A) are adjacent and the other 2 of the bands (set B) are also adjacent. Sets A and B, however, are separated by an unused band.[1]

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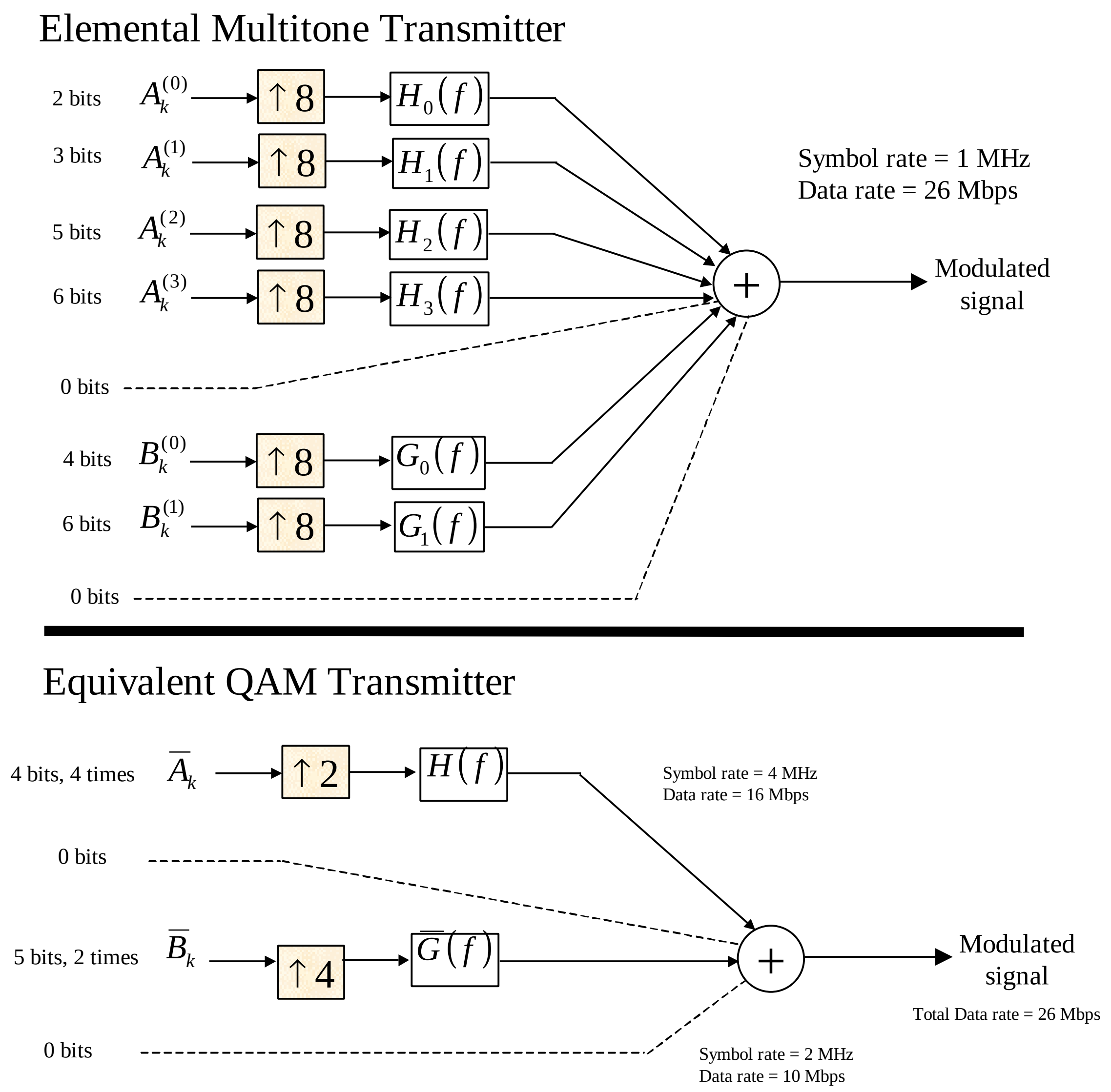
[1] An unused band corresponds to high noise, perhaps RF or Crosstalk; or can correspond to excessive channel attenuation, for instance called by bridged taps or just high-frequency attenuation on a twisted pair.

The bands in Set A can carry 2, 3, 5, and 6 bits respectively corresponding to increasing signal-to-noise ratios in the corresponding channel, while the two bands in Set B can carry 4 and 6 bits respectively. The unused bands carry 0 bits. The average number of bits per band in Set A is 4 while the average number in Set B is 5. If the symbol rate for the system is 1 MHz, then the data rate is (2+3+5+6+4+6) bits transmitted one million times per second for a total data rate of 26 Mbps.



Elemental Multitone Transmitter

Equivalent QAM Transmitter

Figure 1 - Transmitter convergence example.

For equivalent performance, a QAM system with two bands of different width may be used instead as also shown in Figure 1. The first A band has a symbol rate of 4 MHz and carries 4 bits on all symbols for a data rate of 16 Mbps. The B band carries 10 Mbps at a symbol rate of 2 MHz with 5 bits on all symbols. The total data rate is 26 Mbps. With the correct choice of receiver for each system (see next Subsection, Figure 2), the performance of the two systems is identical.

Note that the carrier frequencies symbol rates for the CAP/QAM system have been chosen so that the used bands exactly correspond to those in the multitone transmission system. Further, the average number of bits in any set of multitone bands must equal the fixed number of bits used in the corresponding QAM/CAP band. The example could be extended to any number of sets of bands in a straightforward manner following the same rules of using the same bandwidth and average bits/band for all separate bands.

2.2 Receiver Convergence

Figure 2 illustrates the corresponding receivers. The multitone system chooses an appropriate set of matched filters and directly decodes the outputs of each band. No equalizer is necessary as the system is free of intersymbol and intraband interference. The QAM/CAP system uses a decision-feedback equalizer (DFE)for each band. These two systems will be equivalent if the transmit signals are the same as in Figure 1. Reference [2] shows that the MMSE-DFE converts each band into the equivalent of an intersymbol-interference-free additive white Gaussian noise channel and the signal-to-noise ratio of this channel is computed according to

$$SNR_{dfe} = 2^{C} - 1$$

where *C*, the capacity in bits per Hz for the particular band and DFE under study, is a function of the channel. An equivalent SNR for the multitone system is well known to follow the same expression. The formula does not mean that either system operates at capacity unless extremely power codes with no extra margin are used. However, as long as the same code of the same power is used in both systems, then the performance is the same in each band.

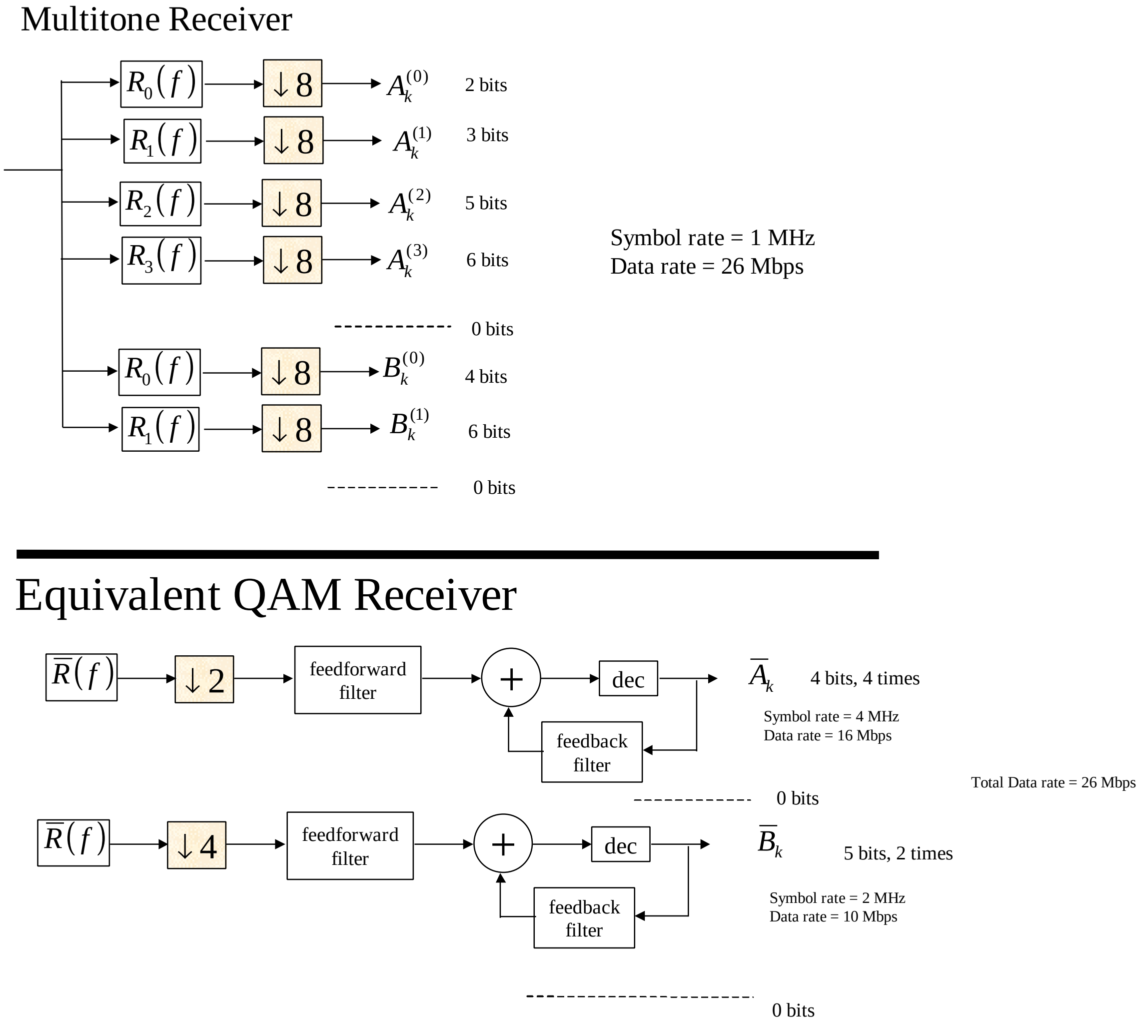


Figure 2 - Receiver Convergence Example

For the system in Figures 1 and 2, assuming uncoded QAM on all subchannels, the capacities are approximately 3 bits per Hz higher than transmitted or 7 and 8 bits/Hz respectively. Thus the DFE and MT SNRs are

$$SNR_{dfe} = 2^7 - 1 = 127 \qquad (21\,\mathrm{dB})$$

for band A, and

$$SNR_{dfe} = 2^8 - 1 = 255 \qquad (24\,\mathrm{dB})$$

for band B.

An overall SNR can be computed as in [3], but is not necessary for the present discussion.

2.3 Conditions for Convergence

Simply stated and paraphrased from [2] and [3], the QAM/CAP systems and multitone system have the same performance when:

1. Each continuous set of adjacent used bands in the multitone system can be replaced by a single band in the QAM system whose symbol rate is equal to the width of the set of used adjacent multitone bands and whose carrier/center frequency is exactly in the middle.

2. The average number of bits/Hz must be the same in the two systems in each used band.

It should be noted that a single used band replacing several separated bands does not perform as well no matter what receiver structure is used. There must be a separate modulated signal and separate DFE for each separate band. Intuitively this means that when the best spectrum has gaps that a transmit band-reject or "notch" filter is not sufficient to reach best performance - multiple QAM signals with optimized center and carrier frequency are then necessary. The appendix gives an example of performance loss when this rule is not observed.

3. Conclusion/Action

The authors suggest that serious consideration be given to the types of transmission technique described in [1] for VDSL standardization.

4. References:

[1] G. Cherubini, E. Eleftheriou, and S. Oelcer (IBM) et al., "Some Thoughts on Modulation," *ANSI T1E1.4 Contribution 99-329*, Ottawa, Ontario, June 7, 1999.

[2] J. Cioffi, G. Dudevoir, M. V. Eyuboglu and G.D. Forney, "MMSE-DFEs and Coding: Parts I and II," *IEEE Transactions on Communications,* October 1995, pp. 2582-2604.

[3] J. Cioffi and G.D. Forney, "Generalized DFE for Packet Transmission with ISI and Gaussian Noise," Chapter 4 of Communications, Computation, and Control, A. Paulraj, Editor, Kluwer: Boston, 1997, see also the web site: http://www.stanford.edu/class/ee379c/reader.html , Chapter 11 for update and examples.

A. Appendix - A Bandwidth-Optimization Paradox

Let us return to the example of Figures 1 and 2, but suppose that only the first 3 subchannels are to be used because the channel SNR is too poor on the other 5. Let us also suppose that each subchannel carries 4 bits (or 4 bits/Hz) for a total data rate of 12 Mbps. An optimum QAM system with a DFE would also carry 4 bits at a symbol rate of 3 MHz, again for a total of 12 Mbps at the same performance level.

Let us suppose further that the transmission system uses a code (like forward error correction for instance) that improves performance by 3 dB, but that a 6 dB margin must also be ensured at the desired error probability of 10^{-6}. A simple method for quantifying bit rate for a coded QAM system is to use the "gap" [2] approximation:

$$b = \log_2\left(1 + \frac{SNR}{\Gamma}\right)$$

where $\Gamma = 6$ dB for QAM with FEC at symbol error probability of 10^{-6}. Thus, the number of bits transmitted at this error rate is computed as the capacity for a channel with 6 dB less SNR. When margin is included, the formula generalizes to

$$b = \log_2\left(1 + \frac{SNR}{\Gamma \cdot g_m}\right)$$

where g_m is the margin. This means that achievable data rate for 6 dB margin corresponds to a channel with 12 dB less SNR, or equivalently a 4 bit loss using the well-known 3 dB/bit rule in QAM transmission. A DFE forms an equivalent AWGN with SNR=SNR_{dfe} and [2] shows that the gap formulae still holds in this DFE case (when MMSE optimization is used). Thus, the capacity for the example with DFE receiver is is then 4+4=8 bits per Hz (or 24 Mbps) and thus the SNR is

$$SNR_{dfe} = 2^8 - 1 = 255 \quad (24\,\text{dB}) \ .$$

Let us suppose instead that the QAM system instead uses a 4 MHz symbol rate, and thus transmits instead at 3 bits/Hz for the same data rate of 12 Mbps. The capacity in bits per Hz is now 24 Mbps/4 MHz = 6 bits/Hz and the DFE SNR for the wider bandwidth system is then

$$SNR_{dfe,4MHz} = 2^6 - 1 = 63 \quad (18\,\text{dB}) .$$

The new margin of this system operating at the same probability of error and same data rate of 12 Mbps has a gap-plus-margin of

$$\text{gap} + \text{margin (dB)} = 10 \cdot \log\left(\frac{\text{SNR}_{\text{dfe,4MHz}}}{2^3 - 1}\right) = 9\,\text{dB} .$$

Since the gap for the QAM+FEC is fixed at 6 dB, the margin has reduced to 3 dB. The greater the mismatch of DFE symbol rate to correct bandwidth, the greater the loss in margin. Thus, it is essential to use the correct symbol rate, or truly set of symbol rates, to get convergence of the two methods. For VDSL, this often means several DFEs are necessary because waterfilling optimized spectra often consist of several disjoint frequency bands, especially when RF emissions or ingress are of concern. While the DFE feedforward section may notch the RF (or match to an egress filters notch), it corresponds to using 4 MHz instead of the correct 3 MHz in the simple example here. Even the best DFE with infinite length filters will perform worse than a set of DFEs corresponding to the used bands (and which are probably simpler to implement anyway).