T1E1.4/98-173

Project: **T1E1.4:VDSL**

Title: **PAR reduction with minimal or zero bandwidth loss and low complexity (98-173)**

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Date: **June 1-5, 1998**

Distribution: **T1E1.4**

Abstract

This contribution investigates no-rate-loss PAR reduction methods in terms of complexity and performance. In particular, a new method that achieves greater PAR reduction for less complexity than all other no-rate-loss PAR reduction schemes. Even with less complexity than the others (including Motorola's recent ITU/UAWG proposal in [1]), this new method has complexity feasible only for upstream ADSL, where more than 5-6 dB of PAR reduction is achieved. We maintain that our previous methods are best for downstream PAR reductions of 6-8 dB.

The new methods do (as do all no-rate-loss methods, including [1]) require some level of understood specification for interoperability unlike our previous methods that simply don't use some subset of selected tones that are not otherwise used for data transmission.

1 Introduction

Our previous contributions [2, 3, 4] describe a method for reducing PAR in DMT transmission, and show that reserving a small fraction of tones leads to large reductions in PAR (as much as 10 dB in realistic situations) depending on the complexity of implementation that can be tolerated. These previous contributions also present some efficient methods, requiring only order N operations per DMT symbol, that readily achieve 4-5 dB reduction in PAR with minimal (less than 5%) and 8 dB reduction in PAR with a maximum of 20% rate loss. This previously reported reduction in PAR is achieved without need of handshake, and with no additional complexity at the receiver. These methods constitute the most efficient methods we know for the levels of PAR achieved and lead to negligible rate loss for DMT systems with 128 or more tones, as in downstream-ADSL, downstream-lite, or VDSL. Our earlier contributions did not present some alternative methods of which we knew, that achieve comparable PAR reduction with no rate loss, but with greater complexity. A subset of those more general methods has been recently introduced by Motorola in [1], but without addressing the complexity issue.

When the number of tones N, is small, the set of tones reserved for PAR reduction might represent a significant fraction of the available bandwidth that could result in a small reduction in data rate, motivating the use of no-rate-loss PAR reduction methods perhaps in upstream ADSL. Moreover, when N is small we can increase the computational complexity per tone since the symbol rate is constant. This contribution presents these alternative methods for PAR reduction. We differ from the particular choice made by Motorola in [1], and instead use a more general method that has greater PAR reduction and has less complexity. This method reduces PAR with no bandwidth loss through duplication of the signal constellation points, such that the constellations will have several points with the same information content. If these duplicate signal points are spaced by $D = 2M + c$, where M is the constellation size, and c a positive constant[1] the BER will not increase and the only addition to the standard receiver is a *modulo-D* after the FFT. The selection of the constant is shown to be very important to PAR reduction and complexity, and this constant was not used in [1]. These methods require no handshake, but like [1], require specification of transmitter and receiver implementation for interoperability.

The complexity for no-rate-loss PAR reduction is higher because the choice of the duplicate points that minimize the PAR has exponential complexity where the complexity increases as $(NL)^{\gamma\sqrt{N}}$ where L is the number of duplicate points, N is the number of tones and $\gamma > 1/2$. Such huge "exponential-hard" complexity is not a feasible. A suboptimal iterative algorithm with maximum complexity order $\alpha N^2 + N\sqrt{N}$, where $\alpha << 1$ is presented. Bounds on the maximum reduction that can be achieved with the optimal solution are derived and it is shown that the iterative method is very close to this bound. It is also shown that choosing a larger separation between duplicate points leads to faster convergence and thus less complexity. For $N < 100$, the complexity of this new proposed iterative algorithm is low and comparable to our original reserved tone method. Therefore it is a viable option for the upstream direction of a xDSL DMT modem.

[1] The importance of c is described later

2 PAR Reduction using generalized constellations

The transmitter of a DMT modulator must perform an IFFT of the N-tone complex data vector of QAM constellation values, $\mathbf{X} = X[k] = [g_0X_0,\ g_1X_1, \ldots,\ g_{N-1}X_{N-1}]^T$, to get a time vector of length N, $\mathbf{x} = x[n] = [x_0 \cdots x_{N-1}]^T = IFFT(\mathbf{X})$, where g_k is the scaling factor or gain of the *k-th* tone [2]. It's well known that for large N, the output time vector distribution can be well approximated by a Gaussian distribution and has a relatively large Peak to Average power Ratio (PAR).

Assuming the *k-th* QAM value, X_k has $2b$ bits, where b is the number of bits per dimension, then X_k could have one of 2^{2b} discrete values. Let's assume for simplicity a square constellation with a minimum distance between constellation points of $d = 2$. Then, the real part of X_k, R_k and the imaginary part, I_k can take values $\{\pm1, \pm3, \ldots, \pm(M-1)\}$, where $M = 2^b$ is the number of levels per dimension. Fig. 1 shows a $16QAM$ constellation, for which $b = 2$, and $M = 4$.

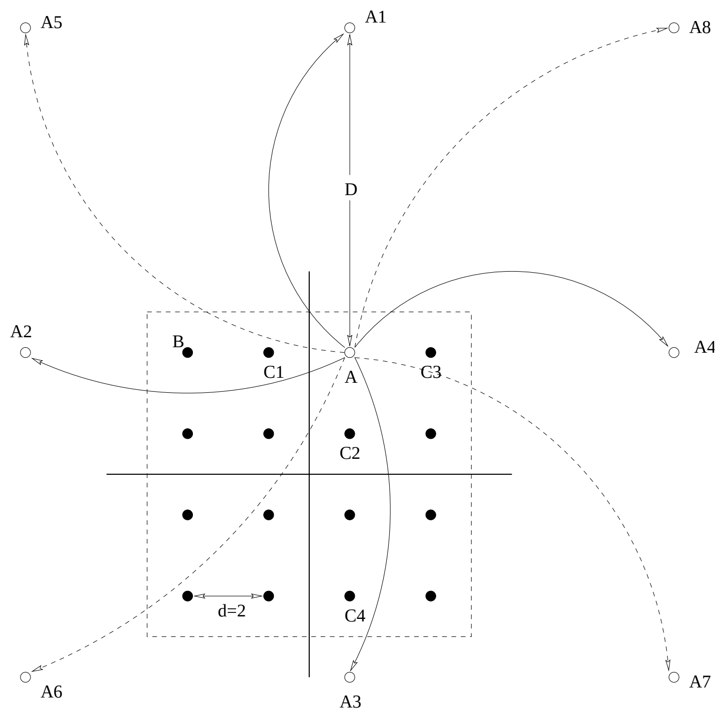

Figure 1: *The constellation value A is the minimum energy point of the equivalent set* $A_i = A + p_iD + jq_iD$

Assume the *k-th* data element, X_k is the point $1 + j3$ or $(1, 3)$, denoted by A in Fig. 1. By modifying the real and/or imaginary part of A we could reduce the PAR of the transmitted vector, but if we want the the receiver to decode A correctly, without sending any side information, we must change A by an amount that can be estimated at the receiver. A very simple case would be to transmit $\hat{A} = A + pD + jqD$ where p and q are any integer values, j is $\sqrt{-1}$ and D is a positive

[2]The values $\{g_k\}$ have a dual role. First, they will allow us to use integer values for the components X_k and simplify the discussion, and second they will be used to take into account the gains from the loading algorithm

real number known at the receiver. The most obvious choice, $p = q = 0$ would give us the original A. The choice $p = -1$ and $q = 0$ will map A into $\hat{A} = A2$.

If we do not want to increase the complexity at the receiver we must choose the value of D carefully. For example, if $D = M$, $A2$ would overlap with B, and the receiver would decode $\hat{A}$ erroneously. Other choices, such as $D = M + 1/2$ would not overlap points in the transmitted constellation, but the minimum distance between possible transmit points would be only 1/2 instead of the original $d = 2$. On the other hand, if we choose $D \geq 2M$ then the probability of decoding $\hat{A}$ erroneously at the receiver for an uncoded system is roughly the same as the the probability of decoding A erroneously and therefore the Symbol Error Rate (SER) will not change. The only extra complexity at the receiver is a generalized modulo operation. This operation requires subtracting/adding multiples of D from $Real\{\hat{X}_k\}$ and $Imag\{\hat{X}_k\}$ until they both lie in the interval $[-M, M]$. If we restrict the values p and q such that $|p| \leq 1$ and $|q| \leq 1$ the new generalized constellation is the one in Fig. 2, where the horizontal and vertical translations between 16QAM sub-constellations is D. If the transmitter chooses the special case where $D = 2M$ then the generalized constellation in Fig. 2 becomes a lattice (equally spaced points). We have therefore expanded the original 16QAM constellation to a bigger constellation where we can choose one of several values that carry the same information. These extra degrees of freedom can be used to generate lower DMT symbols with lower PAR.

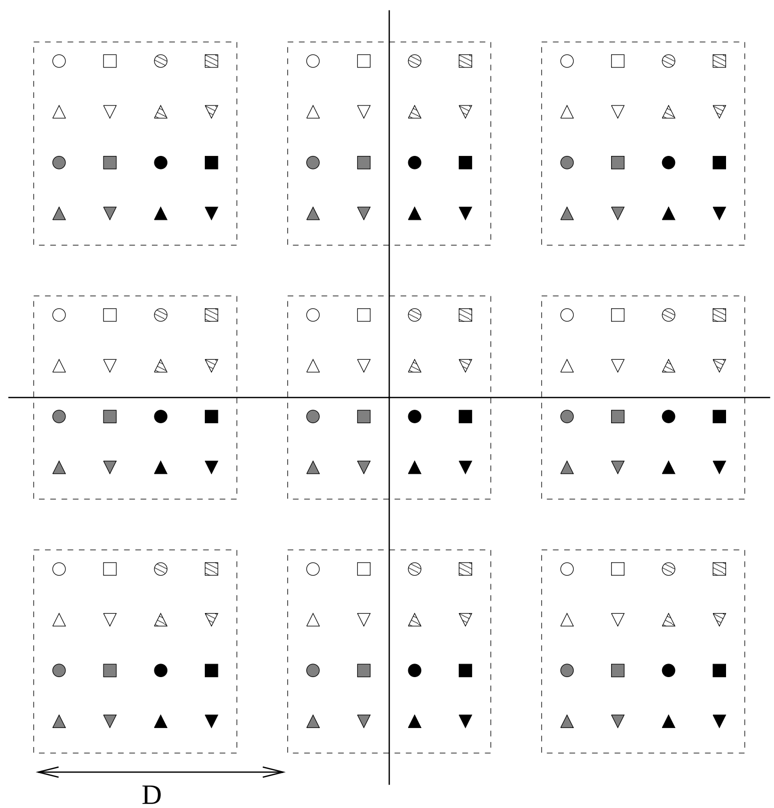

Figure 2: *Generalized constellation for 16QAM for a given value D. Only first ring is shown.*

From Fig. 1 it is clear that for when $p \neq 0$ or $q \neq 0$, $\hat{A}$ has more energy than A, and therefore the new transmit vector will have more power. Fortunately, good choices of $\hat{A}$ can reduce the PAR by over $5dB$ with a power increase of less than 2%. Fig. 1 only plots the nine values $\hat{A}$ with the lowest power.

As mentioned above, any choice $D \geq 2M$ will not significantly increase the SER for an *uncoded system*, but the new nearest neighbors added due to the modulo operation can increase the probability of error significantly for a coded system. For the standard 16 QAM constellation, the nearest neighbors for A are $\{C1, C2, C3\}$, but the nearest neighbors for the generalized constellation are $\{C1, C2, C3, C4\}$. Most codes would be able to correct for the error $\{C1, C2, C3\}$, but probably not $C4$. Thus, for coded systems it might be beneficial to choose $D = 2M + c$ where c is a positive real number that will reduce the error in decoding $\{p, q\}$, that is the *modulo* error, to a desired value such that the new neighbor $C4$ does not become an issue. An additional benefit from choosing $D > 2M$ is that the transmitter can reduce the PAR with a smaller number of tone changes as explained later. Other solutions are possible, such as not duplicating the outermost points of the original QAM constellation. If this algorithm only modifies the *interior points*, such as $C2$, the minimum distance for the *modulo* error is $2d$ instead of d, but as described later this will increase the transmit power. Another solution would be to integrate the generalized constellation and the code. For example with Trellis codes, the receiver would need to consider all the points in the generalized constellation when computing the metrics of the Viterbi decoder. This alternative will not increase the SER, but the number of points in each coset will increase and so will the receiver complexity.

Given these more general constellations, where each data sub-symbol X_k can be substituted for an equivalent $\hat{X}_k$, we can now choose the transmitted vector from the following set:

$$\hat{x}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (X_k + p_k D + j q_k D) e^{j2\pi kn/N}, \quad n = 0, \ldots, N-1, \tag{1}$$

where p_k, q_k are integers and $D \geq 2M$.

At the receiver end, decoding X_k from $\hat{X}_k$ is very simple. Since $D \geq 2M$ the receiver can decode X_k by performing a generalized modulo operation, $X_k = mod_{[-M,M],D}\{\hat{X}_k\}$. This operation requires subtracting/adding multiples of D from $Real\{\hat{X}_k\}$ and $Imag\{\hat{X}_k\}$ until they both lie in the interval $[-M, M]$. If we restrict $|p| \leq 1$ and $|q| \leq 1$ the receiver must perform at most 2N compares/adds per DMT vector symbol. Usually the number of nonzero values for p_k, q_k is only a small fraction of N. For the special case where $D = 2M$ this generalized modulo operation becomes the standard modulo operator.

3 Power increase

As described in the previous section, $\hat{X}_k = X_k + p_k D + j q_k D$ has more energy than X_k, whenever $p_k \neq 0$ or $q_k \neq 0$ and $D \geq 2M$. This section will show that by choosing the values p_k, q_k and the modified tone values X_k correctly, we can greatly reduce this power increase. The importance of reducing this power increase is twofold. First, any power increase results in a reduction in SNR margin. Second, unnecessary power increases can lead to higher peaks, which will complicate iterative algorithms for computing $\hat{X}_k$.

The 16QAM in Fig. 1 will be used for the description, but QAM constellations of any size can be studied following the same argument. Also, we will only consider the first *ring* of the generalized constellation, i.e. the points for which $|p| \leq 1$ and $|q| \leq 1$. Farther points could be used, but the energy increase for that tone will be very large, and thus these points probably would not be used in practice.

Consider first the point $A = (1, 3)$. Its set of equivalent points, sorted in ascending power (for $2M \leq D \leq 2M + 2$) is: $\{A, A3, A2, A6, A4, A7, A1, A5, A8\}$. The power increase for these points is a function of the choice of D, and the constellation size M. For example if $M = 4$ and $D = 8.5 > 2M$, $A3$ has 3 times more power than A, but $A8$ has 20 times the power in A. If we consider the point $C2(1, 1)$, its minimum power equivalent has 30 times the energy of $C2$. The reason for this is that $C2$ is close to the origin, and thus all its equivalent points are rather far away from the origin. Thus, if our target is to minimize the power increase, we rather find equivalent values for tones with values close to the boundaries of the original constellation. That is, we rather modify X_k values equal to A or $C3$ but probably not $C2$.

Since each value X_k is scaled by g_k prior to taking the IFFT, the amount of energy in $x[n]$ from *k-th* tone is $g_k^2|X_k|^2$. Calling $\mathcal{E}_k = \mathcal{E}[g_k^2|X_k|^2]$ the average energy of the *k-th* tone, g_k must satisfy:

$$g_k = \sqrt{\frac{3\mathcal{E}_k/2}{M^2 - 1}} \tag{2}$$

As we increase the constellation size, the minimum power increase for the outer points in the constellation becomes smaller. For example, if we change the $R_k = Real\{X_k\}$ from $M - 1$ to $M - 1 - D$ $(p = -1, q = 0)$ and $D = 2M$ the power increase is

$$g_k(|\hat{X}_k|^2 - |X_k|^2) = \frac{3\mathcal{E}_k}{2} \frac{((M+1)^2 - (M-1)^2)}{M^2 - 1} \tag{3}$$

which decreases as the constellation size increases.

These power increases might seem rather large, but only a small fraction of tones need to be adjusted. By selecting the tones for which $X_k \neq \hat{X}_k$ using the argument above, the total power increase of the DMT symbol can be below 2% on the average.

4 Maximum PAR reduction per dimension translation

The following sections will focus on baseband multicarrier systems, but these results can be easily generalized to the passband case. This section quantifies the maximum PAR reduction this algorithm can achieve per dimension translation. Afterwards we describe a simple algorithm that can achieve most of this promised PAR reduction and is specially useful if the transmitter desires to minimize the power increase. We then describe some more advanced algorithms that reduce the PAR even further. There are many improvements/variations of these algorithms that trade complexity, PAR reduction and power increase.

For the baseband case, assuming N is even, the transmitter IFFT output vector is:

$$x[n] = \frac{2}{\sqrt{N}} \sum_{k=1}^{N/2-1} g_k \left[R_k cos(2\pi kn/N) - I_k sin(2\pi kn/N)\right] + g_0 \frac{R_0 + I_0 cos(n\pi)}{\sqrt{N}}, \quad n = 0, \ldots, N-1, \tag{4}$$

where $X_k = R_k + jI_k$ and g_k is the scaling factor for tone k. If the PAR of this DMT symbol is high, there must be at least one value of $|x[n]|$ that is large. Assuming the location of the maximum value is n_0, we can replace $n = n_0$ in Eq. (4):

$$x[n_0] = \frac{2}{\sqrt{N}} \sum_{k=1}^{N/2-1} g_k \left[R_k cos(2\pi kn_0/N) - I_k sin(2\pi kn_0/N)\right] + g_0 \frac{R_0 + I_0 cos(n_0\pi)}{\sqrt{N}}, \tag{5}$$

Let's assume that $x[n_0] > 0$ and that $cos(2\pi k_0 n_0/N) = l > 0$ for some frequency bin k_0. If we subtract[3] D from R_{k_0} the new transmit DMT symbol vector $\hat{x}[n]$ can be computed without repeating the IFFT because the algorithm has only modified one tone. The new symbol is simply

$$\hat{x}[n] = x[n] + \frac{2}{\sqrt{N}}(-g_{k_0} D) cos(2\pi k_0 n/N), \quad n = 0, \ldots, N-1, \tag{6}$$

After replacing R_{k_0} for $R_{k_0} - D$ the reduced peak at $\hat{x}[n_0]$ will satisfy the relationship:

$$\hat{x}[n_0] = x[n_0] - g_{k_0} \frac{2lD}{\sqrt{N}} \tag{7}$$

Assuming all tones have unit energy, we can substitute g_{k_0} from Eq. (2). The maximum peak reduction of sample $x[n_0]$ that can be achieved after replacing one dimension occurs when $l = 1$. Thus the new peak will satisfy:

$$\hat{x}[n_0] \geq x[n_0] - \sqrt{\frac{3/2}{M_{k_0}^2 - 1}} \frac{2D}{\sqrt{N}} \tag{8}$$

Since other secondary peaks might appear at other locations $n_1 \neq n_0$, the new DMT symbol will satisfy:

$$max|\hat{x}[n]| \geq max|x[n]| - \sqrt{\frac{3/2}{M_{k_0}^2 - 1}} \frac{2D}{\sqrt{N}} \tag{9}$$

A similar argument would follow for all other permutations. If $cos(2\pi k_0 n_0/N) = l < 0$ we must substitute R_{k_0} for $R_{k_0} + D$. These ideas can be also extended to the $I_k sin(2\pi kn_0/N)$ terms. In general, the single dimension *D-shift* update, Eq. (6) could take any of the following 4 options:

$$\hat{x}[n] = x[n] + \frac{2}{\sqrt{N}}(\pm g_{k_0} D)\{cos, sin\}(2\pi k_0 n/N), \quad n = 0, \ldots, N-1, \tag{10}$$

From Eq. (9) the maximum peak reduction per tone shift is:

[3] The values of D can be different from tone to tone. Moreover different values of D for the real and imaginary part can be used.

$$\delta = \sqrt{\frac{3/2}{M_{k_0}^2 - 1}}\frac{2D}{\sqrt{N}} = \sqrt{\frac{6}{M_{k_0}^2 - 1}}\frac{2M_{k_0} + c}{\sqrt{N}} \tag{11}$$

If we follow this procedure on K real/imaginary dimensions simultaneously the maximum peak reduction is $K\delta$. From Eq. (11) the peak reduction factor δ decreases as N increases, which increases the number of iterations needed to reduce the PAR to the target value. For example, if a peak reduction of Δ is desired, the number of iterations needed is $K = \Delta/\delta \propto \sqrt{N}$. Thus, for larger values of N increasing c is a good choice. Table 1 lists the maximum PAR reduction from multiple tone modifications for three different values of D. An IFFT size of 64 with 6bits/tone is considered and the original symbol worst case PAR is 15 dB (clipping rate 10^{-7}).

Iterations	D=2M	D=2M+2	D=2M+4
1	1(dB)	1.1(dB)	1.3(dB)
2	2.2	2.5	2.8
3	3.5	4	4.6
4	5	6	7

Table 1: *Maximum PAR reduction (in dB) vs. number of iterations. All tones carry 6bits/QAM symbol and the IFFT size is 64.*

Table 2 assumes an IFFT size of 512 with 4bits/tone and a original symbol worst case PAR of 15.5 dB. This table shows that the case $c = 0$ requires 50% more iterations/complexity than $c = 4$.

Iterations	D=2M	D=2M+2	D=2M+4
1	.3(dB)	.4(dB)	.5(dB)
2	.7	.9	1
3	1	1.3	1.6
4	1.4	1.8	2.2
6	2.2	2.9	3.6
8	3.1	4.1	5.2
10	4.1	5.5	7.2

Table 2: *Maximum PAR reduction (in dB) vs. number of iterations. All tones carry 4bits/QAM symbol and the IFFT size is 512.*

5 Simple algorithms for computing $\hat{X}_k$ and $\hat{x}[n]$

The previous section provided upper bounds on the PAR reduction that is possible from these generalized constellation methods. Here we will describe some simple algorithms that are close to achieving these upper bounds with small transmit power increase and low complexity.

Rewriting Eq. (1) for the baseband case we get[4]:

[4]The DC term (R_0) and Nyquist term (I_0) have set to zero to simplify the discussion

$$\hat{x}[n] = \frac{2}{\sqrt{N}} \sum_{k=1}^{N/2-1} g_k \left[(R_k + p_k D_k) cos(2\pi kn/N) - (I_k + q_k D_k) sin(2\pi kn/N) \right], \quad n = 0, \ldots, N-1, \tag{12}$$

Finding the values of p_k and q_k that produce the lowest PAR for $\hat{x}[n]$ requires solving a integer programming problem, which have exponential complexity. Assuming L duplicate points per constellation, if K dimensions are to be modified there are

$$\begin{pmatrix} N \\ K \end{pmatrix} L^K \approx \frac{N^K}{K!} L^K \approx (NL)^K \tag{13}$$

combinations for the vectors $[p_0 \ldots p_N]$ and $[q_0 \ldots q_N]$. From the argument above, a peak reduction of amplitude Δ requires $K = \gamma\sqrt{N}$ dimension shifts, where γ is a function of M and c as given by Eq. (11) and the desired peak reduction Δ. Therefore the number of combinations required for a fixed PAR reduction is:

$$\begin{pmatrix} N \\ \gamma\sqrt{N} \end{pmatrix} L^{\gamma\sqrt{N}} \approx (NL)^{\gamma\sqrt{N}} \tag{14}$$

which explains the exponential complexity of the optimal solution. For a PAR reduction of more than $5dB$, $M \geq 4$ and $c = 0$ we have $\gamma \approx 1/2$. Fortunately good approximations to the optimal solution are possible with low complexity with an iterative algorithm that utilizes the principles described in the previous sections.

First, since a nonzero value for p_k or q_k increases the symbol energy, although the algorithm should reduce the larger peaks it will inevitably increase other samples of the DMT symbol. To facilitate subsequent iterations choosing the equivalent points that reduce the power increase is important. For example if all tones transmit 16QAM constellations, by choosing the terms where $|R_k| = 3$ or $|I_k| = 3$ we minimize the transmitter power increase by selecting $sign(p_k) = -sign(R_k)$ or $sign(q_k) = -sign(I_k)$. Second, by choosing the tones where the sinusoid value, l in Eq. (7), is large at the peak locations, gets larger PAR reductions per step. Similarly larger values of D will cancel the peak faster but at the expense of larger power increases.

The algorithm would start with the original DMT symbol ($p_k = 0$ and $q_k = 0$). After finding the maximum, n_0 we find a tone k_0 such that either $|R_{k_0}|$ or $|I_{k_0}|$ is large and such that the sinusoid is of the opposite sign and close to one in magnitude at n_0. After the desired tone, k_0, is found $\hat{x}[n]$ is updated using Eq. (10). If more than one value of $x[n]$ is large we must find a tone k_0 that reduces as many peaks as possible. This procedure can be repeated several times until the desired PAR is achieved or the maximum number of iterations or maximum transmit power increase has been reached. This procedure was used to generate Fig. 3 and Fig. 4.

Fig. 3 compares the DMT symbol clipping rate at different clipping levels PAR_0. The simulation parameters for these results are: DFT of size 64, $M = 4$ (16QAM), $D = 2M$ and the maximum number of single tone iterations, $K = 4$, or equivalently, a maximum of 4 values of R_k and I_k combined have changed. The PAR reduction is about $5dB$ at a clipping rate of 10^{-6} which is the maximum reduction for 4 iterations from Table 1. The average power increase is only 1.3%.

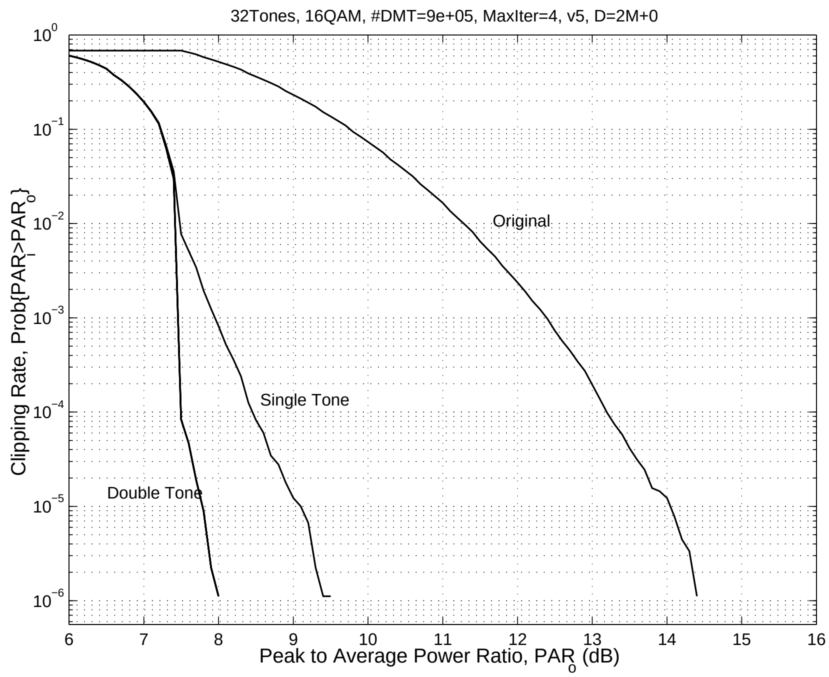

Figure 3:

Fig. 3 also includes PAR results for a more complicated double tone algorithm. This algorithm reduces the PAR by $6.5dB$ at the expense of a larger power increase.

Fig. 4 plots the PAR reduction for the 10 first iterations for a DFT of size 512, 16QAM in each tone and $D = 2M + 2$. The proposed algorithm reduces PAR by more than $5dB$ at a clipping rate of 10^{-6}. This is achieved with a maximum of 10 nonzero values of p_k, q_k, or equivalently, a maximum of 10 values of R_k and I_k combined have changed. This is only $.5dB$ away from the maximum reduction that the exponentially hard algorithm could get as shown in Table 2. The average power increase is only 1.6%

The main focus of these algorithms was to minimize the transmit power increase with low complexity instead of achieving the absolute minimum PAR. Different algorithms will be presented at a later date. Larger reduction in PAR are possible by relaxing the constraint on average power increase, increasing the number of iterations or by using more complex algorithms.

6 Further Enhancements

Eq. (6) shows that the time domain DMT symbol is changed by a sinusoid of amplitude $2g_k D/\sqrt{N}$ when the real or imaginary part of a given tone is changed by D. Since $D \geq 2M$, if the PAR of a given symbol is only slightly above the target PAR, this fixed step size can be too large, specially if the DFT size, N is small. The constraint $D \geq 2M$ was needed to preserve the minimum distance between constellation points for any p_k and q_k. With the restriction $sign(p_k) = sign(R_k)$ or $sign(q_k) = sign(I_k)$ the constraint on D can be relaxed to $D \geq M$. With this idea the algorithm

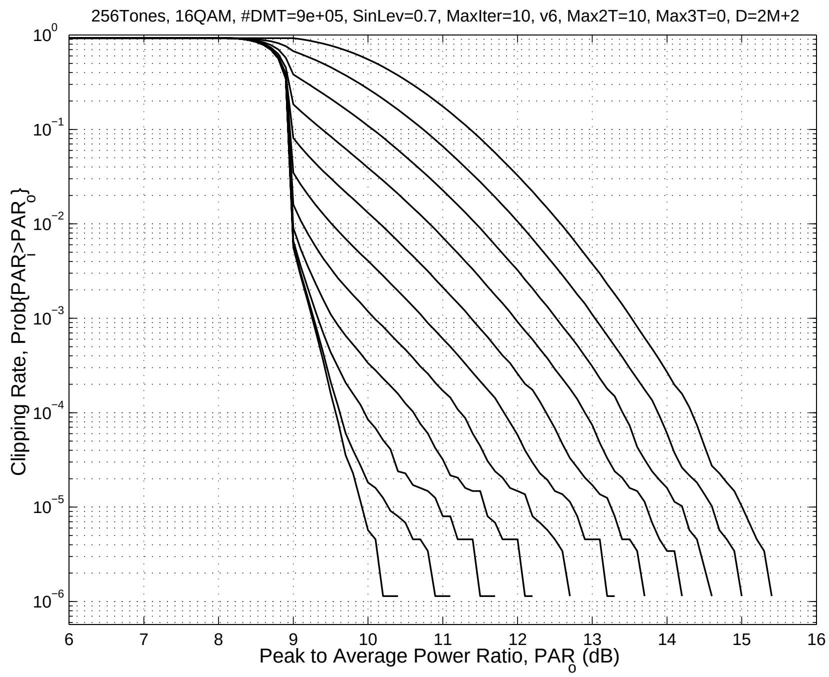

Figure 4:

can decrease the granularity by a factor of two. Smaller granularities are possible if we add the constraints $|R_k| > P$ or $|I_k| > P$, since now $D \geq M - P$. If the transmitter and the receiver agree on the value of D used for each tone, we can get several granularities to speed up convergence and further reductions in PAR. For a DFT size of 64 and two different levels $D = 2M$ and $D = M$ the results in Fig. 3 can be improved up to 1 dB.

This contribution has only studied *D-shifts* of the real or imaginary value of the DMT tones. More general mappings from the standard (minimum energy) constellation to any point outside it can be used as long as the receiver can map these values back to their original value. These mappings can be pre-agreed or adapted during transmission with a small amount of side information.

The number of iterations can be reduced at the expense of extra memory. For example, we can pre-generate PAR reduction kernels, which have desirable time domain properties, and such that the frequency domain values are $\{0, \pm D, \pm jD, \pm D \pm jD\}$. This can provide PAR reductions of more than δ per iteration.

The method described here can also be combined with the methods in [5, 2, 3, 4] which utilize reserved/unused tones to to get further reductions in PAR.

7 Conclusions

A method for reducing PAR is presented with no data rate loss. This method is based on expanding the transmit constellation by multiples of some fixed value. These generalized constellations can help us reshape the original time domain DMT symbol and easily reduce its peaks by more than

5 dB with low complexity and negligible transmit power increase. These generalized constellations can be easily mapped into the original constellation with a simple generalized modulo operation at the receiver. This algorithm can be applied with different levels of complexity and performance depending on our application constraints.

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