STANFORD

Supplementary Lecture 7

MAC Cap Region with continuous channels (exchanging frequency for time)

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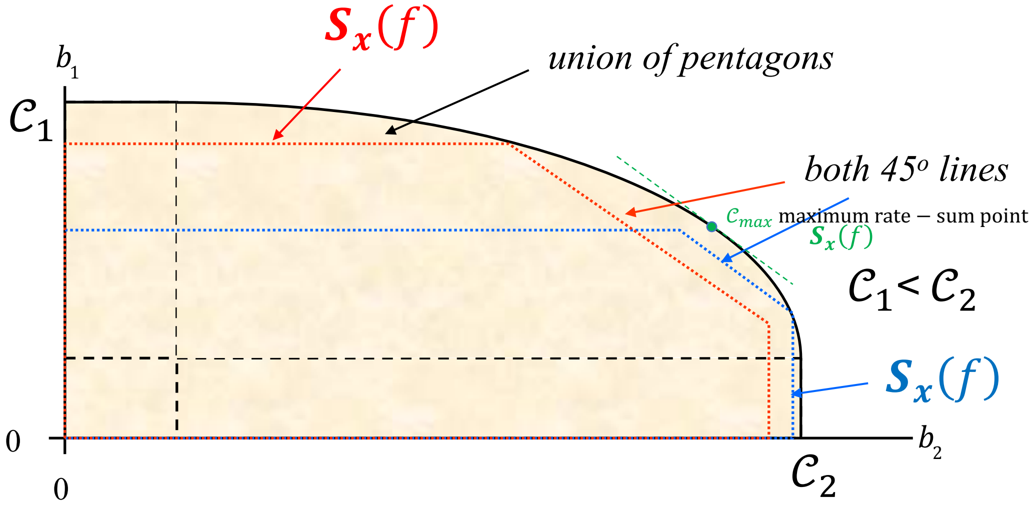
$\mathcal{C}(b)$ is Union of $S_x(f)$-indexed Pentagons



$$\bar{b} = \sum_{u=1}^{U} \bar{b}_u \leq \bar{\mathcal{I}}(\boldsymbol{x};\boldsymbol{y}) = \int_{-\infty}^{\infty} \frac{1}{2} \cdot \log_2 \left[1 + \frac{\sum_{u=1}^{U} S_{x,u}(f) \cdot |H_u(f)|^2}{S_n(f)}\right] df$$

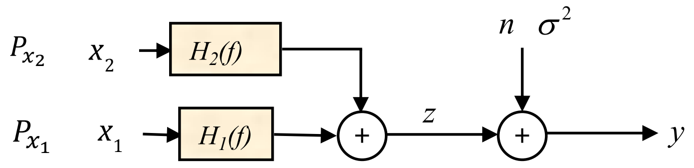
Calculus of variations again, decomposes into U water-fills.

$$S_{x,u}(f) + \frac{\sigma^2 + \sum_{i \neq u} S_{x,i}(f) + |H_i(f)|^2}{|H_u(f)|^2} = K_u$$

Simultaneous water-filling
→ Maximum rate sum

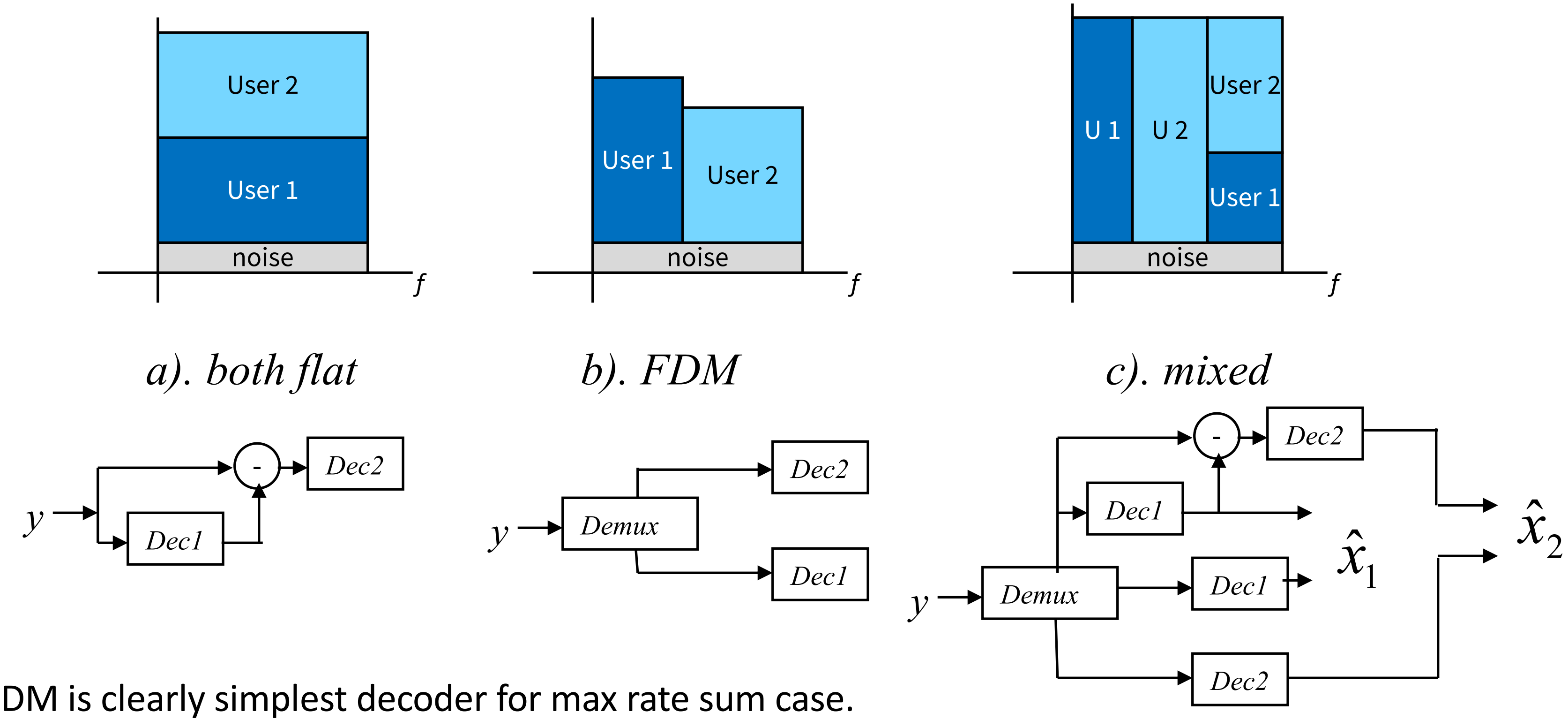
- Each pentagon corresponds to an $S_x(f)$ choice.
  - The pentagons become triangles for the sum-energy MAC.
- The union (convex hull is union when inputs are Gaussian) can dimension-share in frequency as $N \to \infty$.

MT MAC



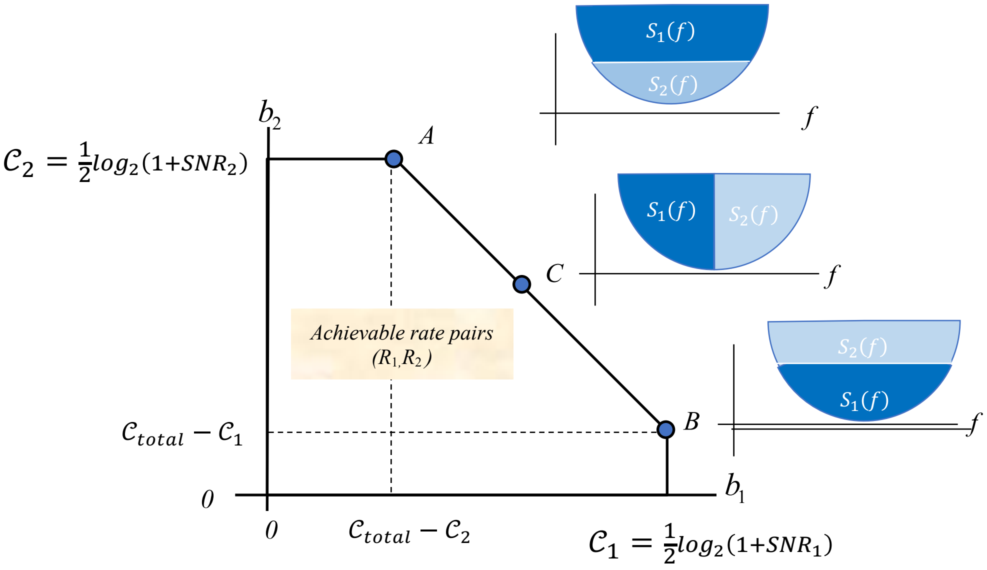
- The users have continuous-time/frequency channels → use MT on each, theoretically.
- This really means dimensionality is infinite (or very large) so "dimension-sharing" may be inherent.
- SWF applies, but with some interpretation (like power instead of energy, etc and power per dimension instead of power-spectral density, etc.)

Decoders and SWF



- FDM is clearly simplest decoder for max rate sum case.

Symmetric 2-user channel and SWF



- Symmetric means $H_1(f) = H_2(f)$ (noise is one-dimensional and added to sum)
- Each of points A, B, and C have different SWF spectra – all have same (max) rate sum

End Supplementary Lecture 7B