STANFORD

Lecture 2

Channel Partitioning: Vector Coding & DMT

April 4, 2024

JOHN M. CIOFFI

Hitachi Professor Emeritus (recalled) of Engineering

Instructor EE379B – Spring 2024

Announcements & Agenda

- Announcements
  - Problem Set 1 due Wednesday, April 12 @ 17:00
  - Most relevant reading – Sections 2.5, 4.4-4.7
  - Education Foundation?
  - HWH 1 is at web site.

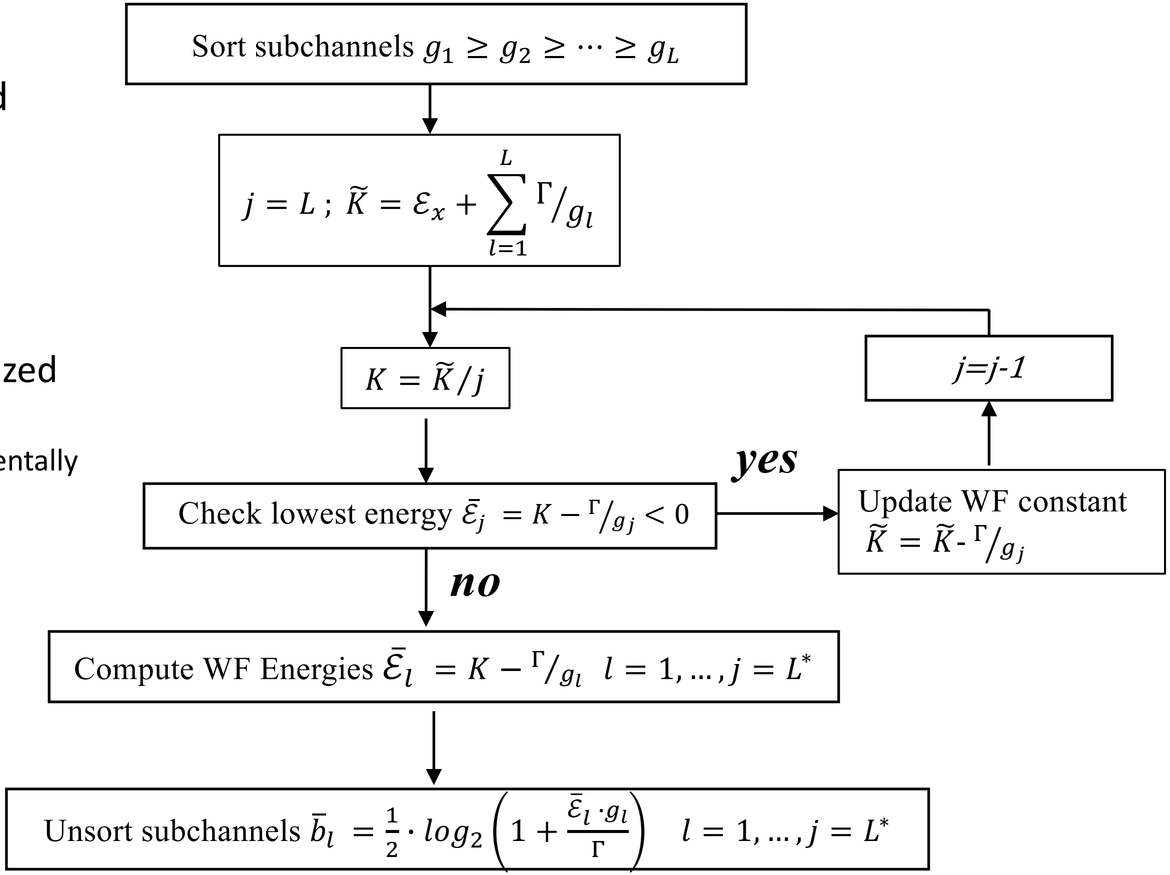
- Agenda
  - RA/MA water-fill flow charts (finish L1)
  - Vector Coding in Time-Frequency
    - $1+.9D^{-1}$ Vector-Code Example
  - DMT/OFDM partitioning
    - DMT Waterfilling Software
    - Vector DMT/OFDM partitioning

RA Water-Fill Flow Chart

- Can start with all channels energized
  - Compute K, test lowest energy
  - Reduce number of dimensions incrementally

- Can also start with 1 channel energized
  - Compute K, test lowest energy
  - Increase number of dimensions incrementally

- The sort is most complex part
  - Can use pivots and bi-section
  - Avoids sort



Margin Adaptive Flowchart

Sort subchannels $g_1 \geq g_2 \geq \cdots \geq g_L$

$$i = L \; ; \; \log_2(\tilde{K}) = 2 \cdot b - \sum_{l=1}^{L} \log_2(g_l)$$

Compute WF constant: $\log_2(K) = \log_2(\Gamma) + \frac{\log_2(\tilde{K})}{i}$

Check lowest energy $\bar{\mathcal{E}}_i = K - {}^{\Gamma}/_{g_i} < 0$

yes

$i = i - 1$

$\log_2(\tilde{K}) = \log_2(\tilde{K}) + \log_2(g_i)$

no

Compute WF Energies $\bar{\mathcal{E}}_l = K - {}^{\Gamma}/_{g_l} \quad l = 1, \ldots, j = L^*$

Unsort subchannels $\bar{b}_l = \frac{1}{2} \cdot log_2\left(1 + \frac{\mathcal{E}_l \cdot g_l}{\Gamma}\right) \quad l = 1, \ldots, j = L^*$

$$\gamma_{max} = \frac{\mathcal{E}_x}{\sum_{l=1}^{L^*} \bar{\mathcal{E}}_l}$$

Section 4.3.2

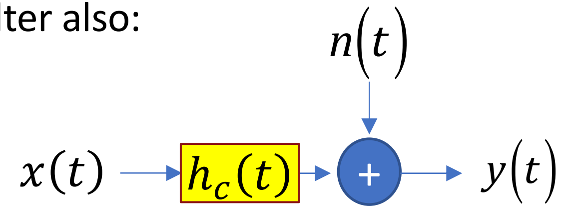
Vector Coding in Time/Frequency

Section 4.6.1

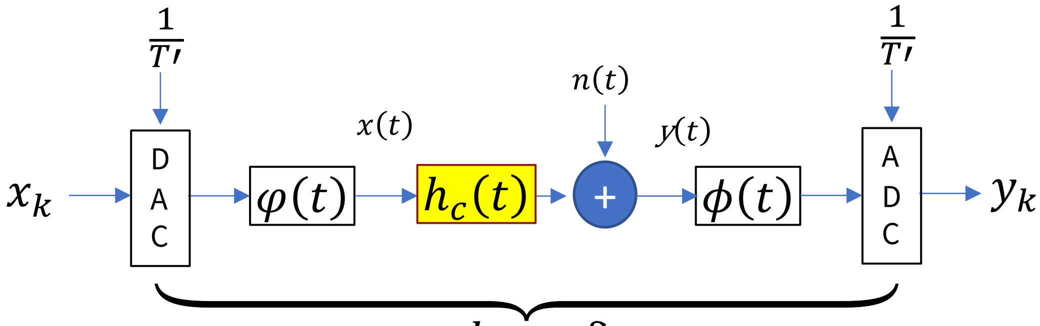
See PS1.5 (Prob 4.25) (matrix AWGN and vector coding)

Scalar Time/Frequency (filtered) AWGN Channel

- Ideal AWGN channels just have noise, but often filter also:
  - Attenuation,
  - band-limits, &
  - spectrally shaped noise (See Sec 1.3.7).



- The $h_c(t)$ causes interference between successive transmissions – complicates and changes performance;
  - see Chapter 3 in EE379A.
- Sampled equivalent has $T' < T$ is the **sample period** – generalizes 379A's "fractional spacing."



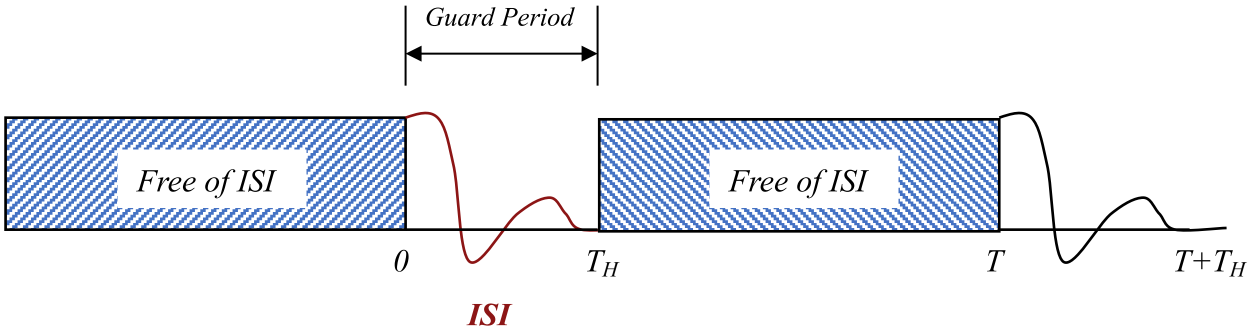
$h_k \neq \delta_k$

sampled pulse response

$$y_k = x_k * h_k + n_k$$

Guard Periods for frequency-time

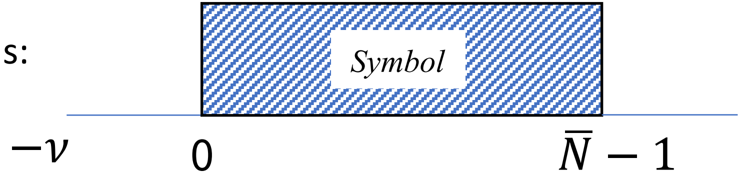
- The **guard period (GP)** T_H
  - lets ISI abate before next symbol.



- This wastes T_H/T of resources (time dimensions):
  - If $T >> T_H$, then the guard period may be worth it.
  - GP may be zeroed or anything the receiver ignores.

excess bandwidth $\alpha = \dfrac{T_H}{T - T_H}$

- A (scalar / SISO) symbol with $\bar{N}$ dimensions (samples) has:
  - $T_H = \nu \cdot T'$ so up to ν+1 non-zero samples/dimensions in h_k .
  - $\bar{N}$ works for real ($\tilde{N} = 1$) or complex ($\tilde{N} = 2$).



reindex time in sample periods

SISO (time-dimension) Case

- Simple scalar convolutional matrix channel with guard band

$$
\begin{bmatrix} y_{\overline{N}-1} \\ y_{\overline{N}-2} \\ \vdots \\ y_0 \end{bmatrix}
=
\begin{bmatrix}
h_0 & h_1 & \dots & h_\nu & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & h_0 & \ddots & h_{\nu-1} & h_\nu & \ddots & 0 & 0 & 0 & 0 \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & 0 & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \dots & 0 & 0 & 0 & 0 & h_0 & h_1 & \dots & h_\nu
\end{bmatrix}
\begin{bmatrix} x_{\overline{N}-1} \\ \vdots \\ x_0 \\ x_{-1} \\ \vdots \\ x_{-\nu} \end{bmatrix}
+
\begin{bmatrix} n_{\overline{N}-1} \\ n_{\overline{N}-2} \\ \vdots \\ n_0 \end{bmatrix}
$$

Ignore -1:-ν

guard period could be anything, including 0 or cyclic

$$\boldsymbol{y} = H \boldsymbol{x} + \boldsymbol{n}$$

- Non-square shift “Toeplitz” matrix for convolution
  - More inputs than outputs when $\nu \neq 0$

SVD Again for the time-dimension case

- Singular Value Decomposition performs: $H = F \cdot \left[\underbrace{\Lambda}_{\bar{N}\times\bar{N}} \;\vdots\; \underbrace{\mathbf{0}_{-1:-\nu}}_{N\times\nu} \right] \cdot M^*$

$N = \bar{N}$ if real subsymbols and $\tilde{N} = 1$
$N = 2 \cdot \bar{N}$ if complex subsymbols and $\tilde{N} = 2$

$$\underbrace{FF^* = F^*F = I}_{\bar{N}\times\bar{N}} \qquad \Lambda = \begin{bmatrix} \lambda_{\bar{N}-1} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \lambda_1 & 0 \\ 0 & \cdots & 0 & \lambda_0 \end{bmatrix} \qquad \underbrace{MM^* = M^*M = I}_{(\bar{N}+\nu)\times(\bar{N}+\nu)}$$

unique (real) singular values ≥ 0

- The vector-coding input construction is:

$$\boldsymbol{x} = M \begin{bmatrix} \boldsymbol{X} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \underbrace{[\boldsymbol{m}_{\bar{N}-1}\ \boldsymbol{m}_{\bar{N}-2} \ldots \boldsymbol{m}_1 \boldsymbol{m}_0}_{(\bar{N}+\nu)\times\bar{N}} \underbrace{\ldots\ \boldsymbol{m}_{-\nu}]}_{zeroed} \begin{bmatrix} X_{\bar{N}-1} \\ X_{\bar{N}-2} \\ \vdots \\ X_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \sum_{n=0}^{\bar{N}-1} X_n \cdot \boldsymbol{m}_n$$

Vector-Coded Time Dimensions Only

- Channel output processed by matched vectors: $\boldsymbol{Y} = F^* \cdot \boldsymbol{y} = \begin{bmatrix} \boldsymbol{f}^*_{\bar{N}-1} \cdot \boldsymbol{y} \\ \vdots \\ \boldsymbol{f}^*_0 \cdot \boldsymbol{y} \end{bmatrix}$

$\mathbb{E}[\boldsymbol{n} \cdot \boldsymbol{n}^*] = R_{\boldsymbol{nn}} = R^{1/2}_{\boldsymbol{nn}} \cdot R^{*/2}_{\boldsymbol{nn}}$

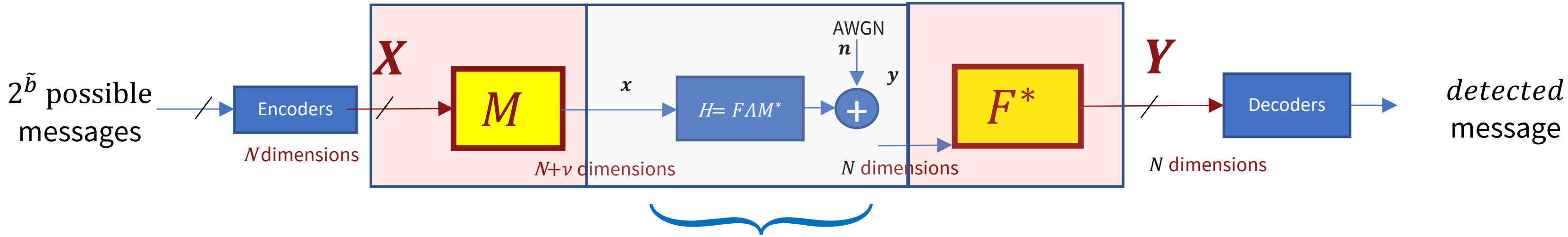
Noise-Equivalent Channel

$\boldsymbol{y} \leftarrow R^{-1/2}_{nn}\boldsymbol{y} = \left(R^{-1/2}_{nn} \cdot H\right) \cdot \boldsymbol{x} + \tilde{\boldsymbol{n}}$

Replaces H

Section 4.6.1.2

- Vector-coded channel partitioning is :



- Parallel Channels are then:

$$Y_n = \lambda_n \cdot X_n + N_n$$

$$SNR_n = \frac{\lambda_n^2 \cdot \bar{\mathcal{E}}_n}{\sigma^2}$$

Data Rates and Mutual Information

- For any energies and consequent SNR's
  - **If complex baseband, replace ν with** 2ν for $\bar{b}$ calculation.

$$\bar{b} = \frac{b}{N+\nu} = \frac{1}{2}\cdot \log_2\left(1+\frac{SNR_{vc}}{\Gamma}\right)$$

- When the gap = 0 dB, this is the "mutual information" , reliable $b \le \bar{\mathcal{I}}(\boldsymbol{x};\boldsymbol{y})$.
  - The **mutual information** bounds data rate, for (Gaussian) input with given autocorrelation $R_{\boldsymbol{xx}}$, or any energy distribution (possibly not WF).
  - Good scalar-AWGN code applies "outside" the parallel channel set.

$$SNR_{VC} = 2^{2\cdot\bar{\mathcal{I}}} - 1$$

- Maximized SNR, and thus mutual information, occur when energy is water-filling → Capacity.

$$\boxed{SNR_{VC,water-fill} = 2^{2\cdot\bar{\mathcal{C}}} - 1}$$

- Highest reliable data rate that can be transmitted (Shannon 1948):
  - **for the given block size $\bar{N}$ and guard period ν.**

$1+.9D^{-1}$ Vector-Code Example

Section 4.6

Matlab $1+.9D^{-1}$ example returns:

- Form H and do SVD:

```
>> C=[.9
zeros(7,1)];
>> R=[.9 1 zeros(1,7)];
```

```
>> H=toeplitz(C,R)

H =

   0.9000   1.0000        0        0        0        0        0        0        0
        0   0.9000   1.0000        0        0        0        0        0        0
        0        0   0.9000   1.0000        0        0        0        0        0
        0        0        0   0.9000   1.0000        0        0        0        0
        0        0        0        0   0.9000   1.0000        0        0        0
        0        0        0        0        0   0.9000   1.0000        0        0
        0        0        0        0        0        0   0.9000   1.0000        0
        0        0        0        0        0        0        0   0.9000   1.0000
```

```
>> [F,L,M]=svd(H)
F =
  -0.1612   0.3030  -0.4082   0.4642  -0.4642   0.4082   0.3030  -0.1612
  -0.3030   0.4642  -0.4082   0.1612   0.1612  -0.4082  -0.4642   0.3030
  -0.4082   0.4082   0.0000  -0.4082   0.4082   0.0000   0.4082  -0.4082
  -0.4642   0.1612   0.4082  -0.3030  -0.3030   0.4082  -0.1612   0.4642
  -0.4642  -0.1612   0.4082   0.3030  -0.3030  -0.4082  -0.1612  -0.4642
  -0.4082  -0.4082  -0.0000   0.4082   0.4082   0.0000   0.4082   0.4082
  -0.3030  -0.4642  -0.4082  -0.1612   0.1612   0.4082  -0.4642  -0.3030
  -0.1612  -0.3030  -0.4082  -0.4642  -0.4642  -0.4082   0.3030   0.1612
```

$\boldsymbol{f}_0$ ... $\boldsymbol{f}_7$

```
L =
   1.8712        0        0        0        0        0        0        0        0
        0   1.7857        0        0        0        0        0        0        0
        0        0   1.6462        0        0        0        0        0        0
        0        0        0   1.4569        0        0        0        0        0
        0        0        0        0   1.2237        0        0        0        0
        0        0        0        0        0   0.9539        0        0        0
        0        0        0        0        0        0   0.6566        0        0
        0        0        0        0        0        0        0   0.3443        0

M =
  -0.0775   0.1527  -0.2232   0.2868  -0.3414   0.3852   0.4153  -0.4214   0.4728
  -0.2319   0.4037  -0.4712   0.4182  -0.2608   0.0428  -0.1748   0.3238  -0.4255
  -0.3583   0.4657  -0.2480  -0.1415   0.4320  -0.4280  -0.1475  -0.1871   0.3830
  -0.4415   0.3099   0.2232  -0.4674   0.1108   0.3852   0.4008   0.0278  -0.3447
  -0.4714   0.0090   0.4712  -0.0208  -0.4705   0.0428  -0.4666   0.1348   0.3102
  -0.4445  -0.2960   0.2480   0.4602   0.0526  -0.4280   0.3140  -0.2812  -0.2792
  -0.3639  -0.4626  -0.2232   0.1806   0.4522   0.3852  -0.0146   0.3936   0.2513
  -0.2395  -0.4127  -0.4712  -0.3975  -0.2097   0.0428  -0.2917  -0.4586  -0.2261
  -0.0862  -0.1697  -0.2480  -0.3187  -0.3794  -0.4280   0.4615   0.4683   0.2035
```

$\boldsymbol{m}_0$ ... $\boldsymbol{m}_7$

Matlab continued

- Use singular values[2] for channel gains: SVs = [1.87 1.78 1.64 1.45 1.22 .95 .66 .34]
- These then are: $$g_n = \frac{\lambda_n^2}{\sigma^2(=.181)} = [19.3\ \ 17.6\ \ 15.0\ \ 11.7\ \ 8.3\ \ 5.0\ \ 2.4\ \ 0.66]$$
- Water-filling (RA) with 0 dB gap $$K = \frac{1}{7} \cdot \left(9 + \sum_{n=0}^{6} \frac{\Gamma}{g_n} \right) = 1.43$$
- Energies [1.38 1.37 1.36 1.34 1.30 1.23 1.01 0] $$\mathcal{E}_n = K - \frac{\Gamma}{g_n}$$
- SNRs [26.2 24.2 20.4 15.8 10.6 6.2 2.4 0] $$\mathcal{E}_n \cdot g_n$$

Not assigned, but might find Prob 4.17 interesting for look

Overall performance and rate

- Product SNR

$$SNR_{VC} = \left[\prod_{n=0}^{6}(SNR_n + 1)\right]^{1/9} - 1 = 6.46 = 8.1\ dB$$

- Rate = capacity

$$\bar{\mathcal{C}} = \frac{1}{9} \cdot \sum_{n=0}^{6} \frac{1}{2} \cdot \log_2(1 + SNR_n) = 1.45 \text{ bits/dimension}$$

```
>> R=[.9 1 zeros(1,99)];
>> C=[.9 zeros(1,99)];
>> H=(1/sqrt(.181))*toeplitz(C,R);
>> [F,L,M]=svd(H);
>> g=diag(L).*diag(L);
```

```
>> K=(1/89)*(101+sum(ones(1,89)./g(1:89)'))
K = 1.3294
>> K-1/g(89) =  -0.0219

>> K=(1/88)*(101+sum(ones(1,88)./g(1:88)'))
K =  1.3292
>> K-1/g(88) = 0.1627

So N* = 88
```

```
>> E=K*ones(1,88)-ones(1,88)./g(1:88)';

>> snr=E.*g(1:88)';
>> b=(0.5/101)*(1/log(2))*sum(log(ones(1,88)+snr))

b =

   1.5360
```

$\bar{N} \to \infty$, then $\dfrac{\bar{N}+\nu}{\bar{N}} \to 1$, so no loss (max is 1.55)

DMT/OFDM Partitioning

Section 4.7.1-5

See PS1.3 (Prob 4.18)

Cyclic Extension

- Remember this convolution from vectoring coding?

$$\begin{bmatrix} y_{\bar{N}-1} \\ y_{\bar{N}-2} \\ \vdots \\ y_0 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \cdots & h_\nu & 0 & \cdots & 0 \\ 0 & h_0 & \ddots & h_{\nu-1} & h_\nu & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_\nu \end{bmatrix} \begin{bmatrix} x_{\bar{N}-1} \\ \vdots \\ x_0 \\ \boxed{\begin{matrix} x_{-1} \\ \vdots \\ x_{-\nu} \end{matrix}} \end{bmatrix} + \begin{bmatrix} n_{\bar{N}-1} \\ n_{\bar{N}-2} \\ \vdots \\ n_0 \end{bmatrix}$$

guard period

could be anything, including 0 or cyclic

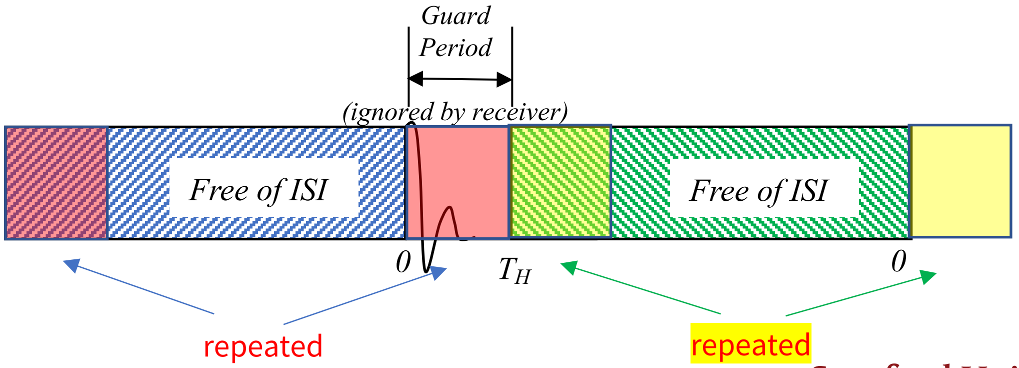
$$\boldsymbol{y} = \tilde{H} \boldsymbol{x} + \boldsymbol{n}$$

- Let the guard period be such that $x_{-i} = x_{\bar{N}-i}$ for *i=1,…,* ν → **CYCLIC PREFIX** or **CYCLIC EXTENSION**.
  - **The channel now appears periodic!**

- In fact, "Toeplitz-distribution" limiting results are based on this type of cyclic-extension concept.

Ideally $\bar{N} >> \nu$

So small loss of dimensions



Cyclic Convolution

- The matrix expression now uses an *N x N* circulant matrix:

$$\begin{bmatrix} y_{\overline{N}-1} \\ y_{\overline{N}-2} \\ \vdots \\ y_0 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \dots & h_\nu & 0 & \dots & 0 \\ 0 & h_0 & \ddots & h_{\nu-1} & h_\nu & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & h_0 & h_1 & \dots & h_\nu \\ h_\nu & 0 & \dots & 0 & h_0 & \dots & h_{\nu-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_1 & \dots & h_\nu & 0 & \dots & 0 & h_0 \end{bmatrix} \begin{bmatrix} x_{\overline{N}-1} \\ \vdots \\ x_0 \end{bmatrix} + \begin{bmatrix} n_{\overline{N}-1} \\ n_{\overline{N}-2} \\ \vdots \\ n_0 \end{bmatrix}$$

$\widetilde{H}$ is circulant $\overline{N} \times \overline{N}$

$$\boldsymbol{y} = \widetilde{H} \cdot \boldsymbol{x} + \boldsymbol{n}$$

- As far as output $\boldsymbol{y}$ is concerned, the input is periodic with the same period *N* as the output.
- The cyclic prefix is added for each and every symbol.
- ν dimensions are lost (both in terms of energy lost and no new information).

Cyclic Convolution and the DFT

- DFT = Discrete Fourier Transform
  - Normalized (maintains squared norm, energy from time ←→ frequency)
  - N or $\bar{N}$

$$X_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{-\jmath \frac{2\pi}{N} kn} \quad \forall \ n \in [0, N-1]$$

$$\boxed{\sum_{n=0}^{N-1} |X_n|^2 = \sum_{n=0}^{N-1} |x_k|^2}$$

- IDFT = Inverse Discrete Fourier Transform
  - Symmetrical form

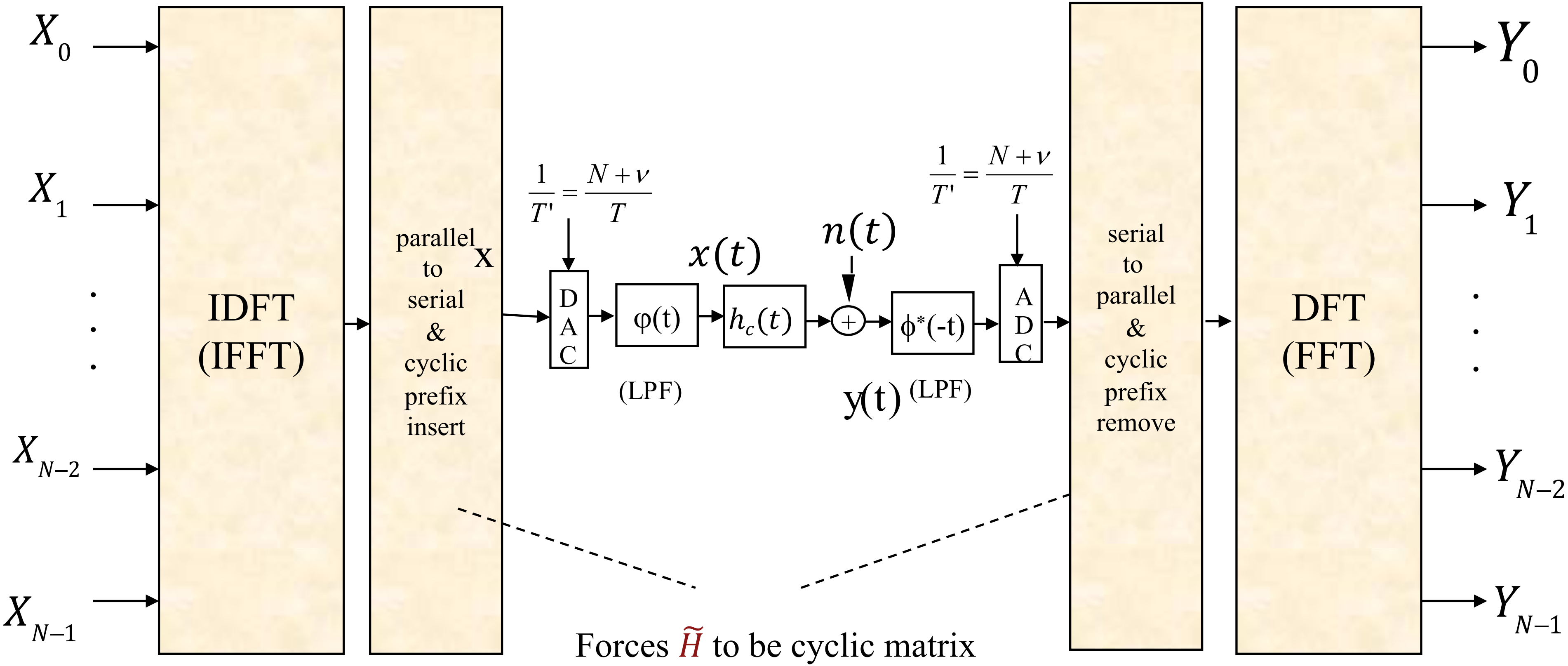
$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{\jmath \frac{2\pi}{N} kn} \quad \forall \ k \in [0, N-1]$$

- Subsymbol channel $Y_n = \widetilde{H}_n \cdot X_n \ (+ N_n)$
  - Vector coding with $M = F^* = Q$, but diagonal can be complex
  - *And the guard period must be cyclic*
  - Still parallel set of subchannels

$$\boxed{\widetilde{H} = Q \cdot \Lambda \cdot Q^*}$$

$$\Lambda = \begin{bmatrix} \widetilde{H}_{N-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \widetilde{H}_0 \end{bmatrix}$$ = DFT values on diagonal

DMT / OFDM Transmission



Forces $\widetilde{H}$ to be cyclic matrix

$X_{N-i} = X_i^*$ if $x(t)$ is real

If complex, use size $\bar{N}$

Heavily used, wireline & wireless

Product SNR

- DMT uses 7 subchannels, DC plus 3 two-dimensional QAM subchannels, of a total of 9 dimensions.

$$\mathrm{SNR}_{DMT} = \left[\prod_{n=0}^{6}(1+\mathrm{SNR}_n)\right]^{1/9} - 1 = 7.6\ \mathrm{dB} \quad < \ SNR_{vc}$$

- But **no** channel-dependent partitioning, and much easier to implement (N log (N) vs N^2).

- How can we exploit this??

INCREASE N !

- DMTra and DMTma will help.

$1+.9D^{-1}$ revisited

- Circulant Channel is 8 x 8 (but wastes 1 dimension in cyclic extension, and loses its energy).

$$\widetilde{H} = \begin{bmatrix} .9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .9 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .9 & 1 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & .9 \end{bmatrix}$$

- Channel FFT (size 8) leads to:

Water-fill (Γ=0 dB) with $\mathcal{E}_x = 8$

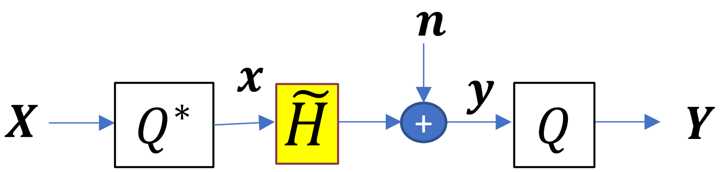
n	$\lambda_n = \lvert P_n \rvert$	$g_n = \frac{\lvert P_n \rvert^2}{.181}$	$\mathcal{E}_n$	SNR_n	b_n
0	1.90	20	1.24	24.8	2.34
1	1.76	17	1.23	20.9	2.23
6	1.76	17	1.23	20.9	2.23
2	1.35	9.8	1.19	11.7	1.85
5	1.35	9.8	1.19	11.7	1.85
3	.733	3	.96	2.9	.969
4	.733	3	.96	2.9	.969
7	.100	.05525	0	0	0

- $\bar{b} = 1.38$

See PS1.4 (Prob 4.7)

Partitioning with DFT

- DFT Partitioning is:



- DFT creates a set of parallel channels – A "Discrete MultiTone" partitioning.
  - Some call it OFDM, but there is a difference in loading (DMT optimizes loaded energy, OFDM fixes equal energy).

$$\boxed{SNR_n = \frac{\mathcal{E}_n \cdot \left|\widetilde{H}_n\right|^2}{\sigma^2}}$$

Noise-Equivalent Channel

$$E\left[\boldsymbol{n}\boldsymbol{n}^*\right] = R_{nn}\sigma^2 = R_{nn}^{1/2} R_{nn}^{-1/2}\sigma^2$$

$$\boldsymbol{y} \leftarrow R_{nn}^{-1/2}\boldsymbol{y} = \left(R_{nn}^{-1/2} H\right)\boldsymbol{x} + \tilde{\boldsymbol{n}}$$

- Receiver is DFT (and noise remains white).
- Transmitter is IDFT (no power increase).

Neither is a function of the channel

Can use efficient "FFT"

DMT Water-Filling Software

Subsection 4.7.4

See PS1.4 (Prob 4.7) and PS1.5 (Prob 4.9)

Rate Adaptive DMT

```
function [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra(H,NoisePSD,Ex_bar,N,gap)
```

OUTPUTS

gn	channel SNRs
en_bar	energy distribution
bn_bar	bit distribution
Nstar	# of Used dim
b_bar	Total bits
SNRdmt	Prod SNR

INPUTS

H	sampled channel response
NoisePSD	AWGN variance
Ex_bar	energy/ Dimension or cpx sample
N	FFT size
gap	Γ dB

- >> [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,8,0)

- gn =   19.9448   17.0320   10.0000   2.9680   0.0552   2.9680   10.0000   17.0320
- en_bar =   1.2415   1.2329   1.1916   0.9547   0   0.9547   1.1916   1.2329
- bn_bar =   2.3436   2.2297   1.8456   0.9693   0   0.9693   1.8456   2.2297
- Nstar =   7
- b_bar =   1.3814
- SNRdmt =   7.6247 dB

In using the program, know if your channel is truly baseband or complex baseband equivalent.

In real case, each dimension shown is a real dimension.

Increase N

- [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,16,0);
  - >> SNRdmtSNRdmt =   8.1152
- [gn,en_bar,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,1024,0);
  - >> SNRdmtSNRdmt =   8.7437
- Feel free to experiment, PS1.4 goes better if you use this.

How about non-zero gap?

- SNR can look higher, but bit rate is overall lower

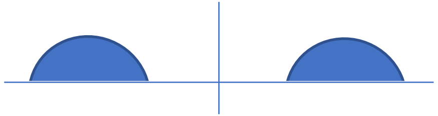
```
>> [gn,En,bn_bar,Nstar,b_bar,SNRdmt]=DMTra([.9 1],.181,1,8,8.8)

gn = 19.9448   17.0320   10.0000    2.9680    0.0552    2.9680   10.0000   17.0320

En = 1.7773    1.7123    1.3991         0         0         0    1.3991    1.7123

bn_bar = 1.2521    1.1382    0.7540         0         0         0    0.7540    1.1382

Nstar = 5

b_bar = 0.5596

SNRdmt = 9.4904 dB  (remember this gets divided by the gap)

>> En.*gn = 35.4481   29.1634   13.9907         0         0         0      13.9907  29.1634
            24.7613   20.9991   11.9163    2.8336     0    2.8336   11.9163   20.9991 (Γ=0)

>> 10*log10(ans)  = 15.4959   14.6484   11.4584      -Inf      -Inf      -Inf   11.4584   14.6484
   (these subchannel SNR's also get divided by gap)
```

Data rate is roughly 1/3 of before.

Note SNRdmt increase for nonzero gap.

Suppose $1+.9D^{-1}$ were complex baseband?

- The channel effectively has twice as many dimensions.
  - The same results would be for 8 complex dimensions.
  - So bbar, ebar are really bits/tone and energy/tone.
  - Same values, but the constellations on each tone are two dimensional.

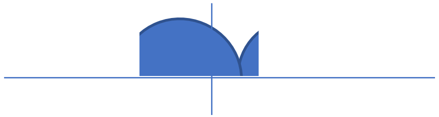


```
>> [gn,En,bn,Nstar,b,SNRdmt]=DMTra([.9 1],.181,2,8,0)

gn =  19.9448   17.0320   10.0000    2.9680    0.0552    2.9680   10.0000   17.0320

En = 2.3843    2.3758    2.3345    2.0976         0    2.0976    2.3345    2.3758

bn = 2.8008    2.6869    2.3028    1.4266         0    1.4266    2.3028    2.6869

Nstar = 7

b = 1.7370

SNRdmt = 10.0484 dB

>> sum(En) = 16.0000      ;     >> sum(bn) =   15.6332
```

- The bits/tone though is slightly larger.
  - 1.73 > 1.38
- What happened?
- The "DC" tone is now complex and has an additional good dimension.

Suppose $1+.9jD^{-1}$ - *must be* complex baseband

- The channel definitely has twice as many real dimensions



This channel rotates the earlier one in time, so circular shift in frequency

```
>> [gn,En,bn,Nstar,b,SNRdmt]=DMTra([.9*i 1],.181,2,8,0)

gn = 10.0000   17.0320   19.9448   17.0320   10.0000    2.9680    0.0552    2.9680

En = 2.3345    2.3758    2.3843    2.3758    2.3345    2.0976         0    2.0976

bn = 2.3028    2.6869    2.8008    2.6869    2.3028    1.4266         0    1.4266

Nstar = 7

b = 1.7370

SNRdmt = 10.0484

>> sum(En) = 16.0000      ;      >> sum(bn) = 15.6332
```

Margin Adaptive DMT

- function [gn,en,bn,Nstar,b_bar_check,margin]=DMTma(H,NoisePSD,Ex_bar,b_bar,N,gap)

OUTPUTS

gn	en	bn	Nstar	b_bar_check	margin
channel SNRs	energy distribution	bit distribution	# of Used dim	target bbar	Margin dB

INPUTS

H	NoisePSD	Ex_bar	b_bar	N	gap
sampled channel response	AWGN Variance/ Dimension Or cpx sample	energy/ Dimension Or cpx sample	target bbar	FFT size	Γ dB

- >> [gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,8,0)
- gn = 19.9448 17.0320 10.0000 2.9680 0.0552 2.9680 10.0000 17.0320
- en = 0.6043 0.5958 0.5545 0.3175 0 0.3175 0.5545 0.5958
- bn = 1.8532 1.7393 1.3552 0.4790 0 0.4790 1.3552 1.7393
- Nstar = 7
- b_bar_check = 1
- margin = 3.5410

It works real or complex, but (again) be areful.

Continuing

- >> [gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,8,8.8)
- gn = 19.9448 17.0320 10.0000 2.9680 0.0552 2.9680 10.0000 17.0320
- en = 4.5844 4.5193 4.2061 2.4088 0 2.4088 4.2061 4.5193
- bn = 1.8532 1.7393 1.3552 0.4790 0 0.4790 1.3552 1.7393
- Nstar = 7
- b_bar_check = 1.0000
- margin = -5.2590

Negative margin – can’t do it!

[gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,16,0);

>> margin = 4.1445

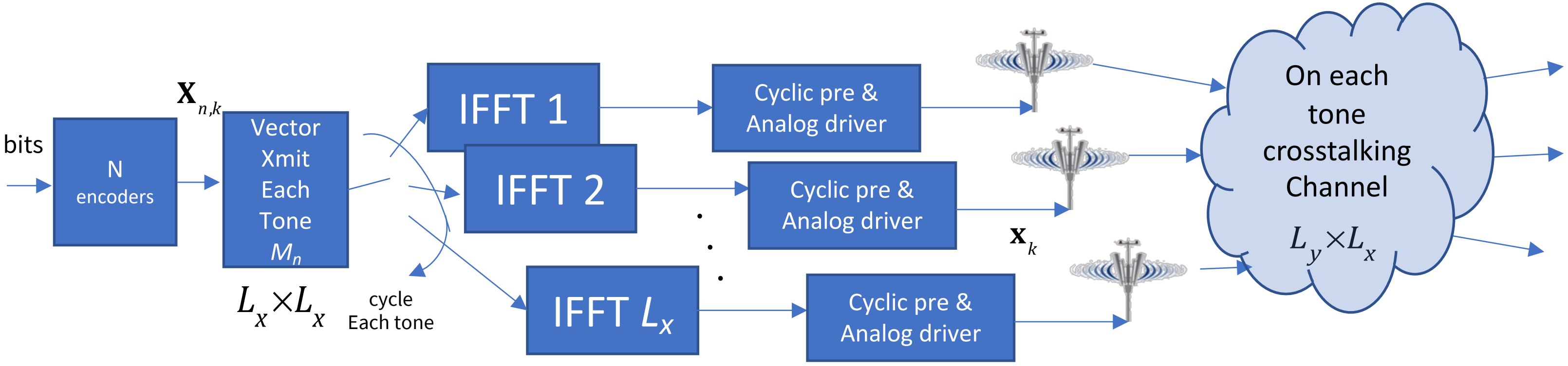
[gn,en,bn,Nstar,b_bar_check,margin]=DMTma([.9 1],.181,1,1,1024,0);

>> margin = 4.7267

Vector DMT/OFDM

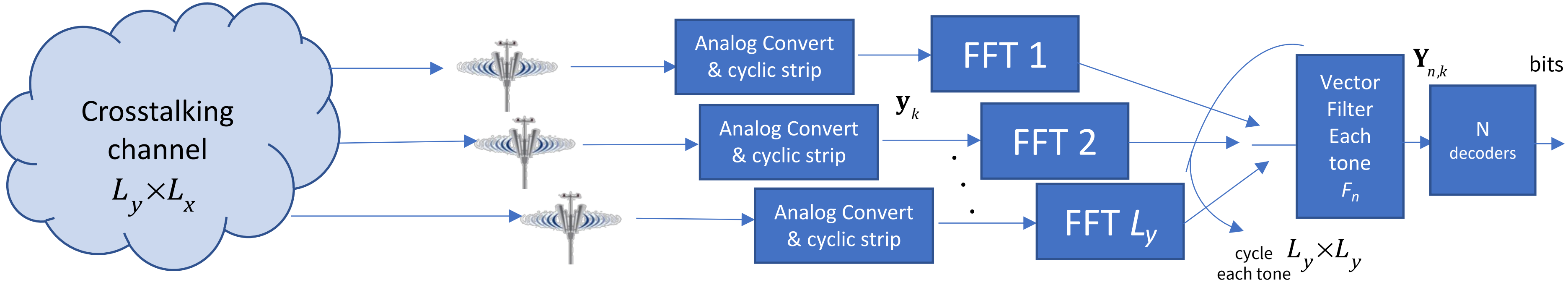
Section 4.7

Vector DMT/OFDM Transmitter



$$NL_x^2 + L_x N \log_2(N) << (L_x N)^2$$

Vector DMT/OFDM Receiver



$$\tilde{H}_n = F_n \cdot \Lambda_n \cdot M_n^*$$

$$\boxed{NL_y^2 + L_y N \log_2(N) << (L_y N)^2}$$

- Just much larger number of dimensions, each a scalar AWGN, $L = min(L_x, L_y)$
- $L \cdot N$ dimensions
- Can water-fill over them all (if total energy constraint, which is common)

End Lecture 2