STANFORD

Lecture 15

MAC Design by Weighted Sums

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Announcements & Agenda

- Announcements
  - PS7 - Final homework, due June 5.
  - Section 5.4 continued

Final Exam:

2-week homework extension to 6/6

continuous 25-hour period starting 6/7

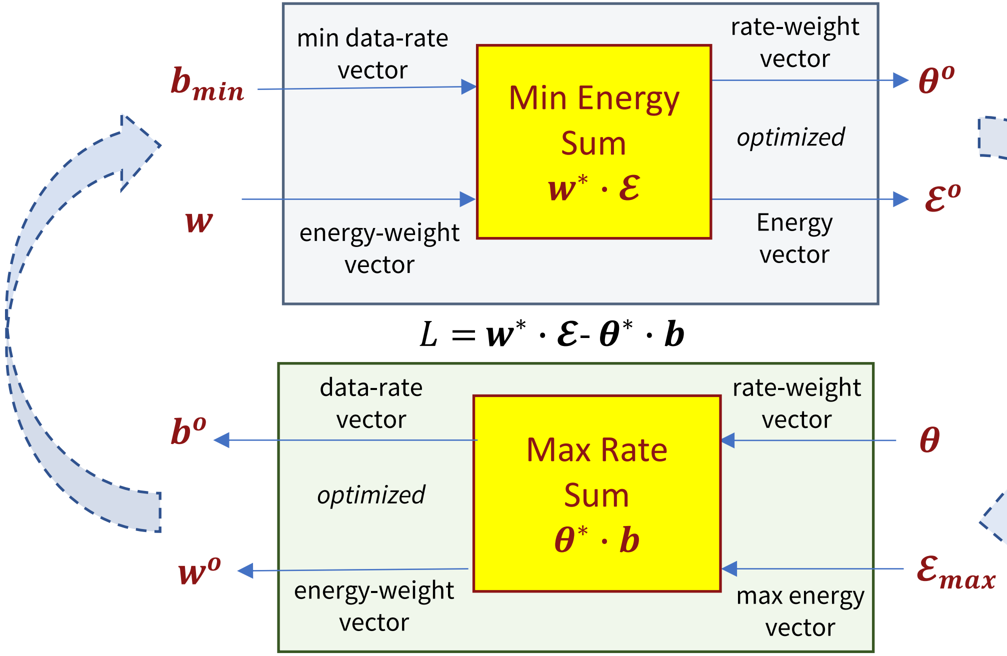
tell me preference.

- Agenda
  - Designs with Weighted Sums
  - Maximum rate sum solution (maxRMAC)
  - Minimum energy-sum solution (minPMAC)
  - MAC Design: Specify rate and energy vectors (admMAC)

Designs with weighted sums

Section 5.4.3

Multiuser Optimization



- Design for rates and/or energy assignments that are feasible $b' \in \mathcal{C}(b)$?

Tonal Lagrangian

- Minimize (over $R_{\boldsymbol{xx}}(u)$) weighted sum at any given (think temporary) $\boldsymbol{\theta}$ where $\boldsymbol{b}$ and $\boldsymbol{w}$ are the specified values:

$$L(R_{\boldsymbol{XX}}, \boldsymbol{b}, \boldsymbol{w}, \boldsymbol{\theta}) = \sum_{n=0}^{\bar{N}-1} \left\{ \underbrace{\sum_{u=1}^{U} \left[w_u \cdot \text{trace}\left\{ R_{\boldsymbol{XX}}(u,n) \right\} - \sum_{u=1}^{U} \theta_u \cdot b_{u,n} \right]}_{L_n(R_{\boldsymbol{XX}}(n), \boldsymbol{b}_n, \boldsymbol{w}, \boldsymbol{\theta})} \right\} + \theta_u \cdot b_u ,$$

With fixed $\boldsymbol{\theta} \succcurlyeq \boldsymbol{0}$ each tone can be individually minimized

- which produces then for tone n:

$$L_{min}(\boldsymbol{\theta}, n) \triangleq \min_{\{R_{\boldsymbol{XX}}(u,n)\}, b_{u,n}} L_n(R_{\boldsymbol{XX}}(n), \boldsymbol{b}_n, \boldsymbol{w}, \boldsymbol{\theta}) .$$

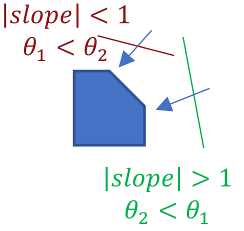
- Then, max over $\boldsymbol{\theta}$

$$L^* = \max_{\boldsymbol{\theta}} \sum_{n=0}^{\bar{N}-1} L_{min}(\boldsymbol{\theta}, n) \triangleq \max_{\boldsymbol{\theta}} L_{min}(\boldsymbol{\theta}) ,$$

- and satisfy tonal GDFE (achievable region) constraint

$$\boldsymbol{b}_n \in \left\{ \boldsymbol{b}_n \mid 0 \le \sum_{\boldsymbol{u} \subseteq \boldsymbol{U}} b_{u,n} \le \log_2 \left| \left(\sum_{u=1}^{U} \tilde{H}_{u,n} \cdot R_{\boldsymbol{XX}}(u,n) \cdot \tilde{H}^*_{u,n} \right) + I \right| \right\} = \mathcal{A}_n \left(\{ R_{\boldsymbol{XX}}(n) \}, \bar{H}_n \right) . \quad (5.2$$

The tonal achievable-region constraint



- Maximum $\sum_{u=1}^{U} \theta_u \cdot b_{u,n}$ occurs at $\mathcal{A}_n(R_{\boldsymbol{XX}}(u,n), \boldsymbol{b}_n)$ vertex (think slope -1 line and pentagon),
  - Given $R_{\boldsymbol{XX}}(u,n) \rightarrow R_{\boldsymbol{XX}}(u)$; equivalently max $\sum_{u=1}^{U} \theta_u \cdot b_u$ occurs at $\mathcal{A}\ (R_{\boldsymbol{XX}}(u), \boldsymbol{b}\ )$ vertex.
- That max-weighted-sum vertex has specific $\boldsymbol{\theta}$, with $\theta_{\pi^{-1}(U)} \geq \theta_{\pi^{-1}(U-1)} \geq \ldots \geq \theta_{\pi^{-1}(1)}$.
  - Proof see Theorem 5.4.1, and also discussion in Sec 5.4.4.1. (Same order all tones)
  - Alternative to testing all orders – optimum order is inferred from the (converged) real vector θ.
- The user data rates in $\mathcal{A}_n(R_{\boldsymbol{XX}}(u,n), \boldsymbol{b}_n)$ must satisfy the (sum of) **tonal-GDFE constraint(s):**

$$b_{u,n} = \log_2 \left\{ \frac{|R_{yy}(u,n)|}{|R_{yy}(u-1,n)|} \right\} = \log_2 \left| \sum_{i=1}^{u} \tilde{H}_{\pi^{-1}(i),n} \cdot R_{\boldsymbol{XX}}(\pi^{-1}(i),n) \cdot \tilde{H}^*_{\pi^{-1}(i),n} + I \right| - \log_2 \left| \sum_{i=1}^{u-1} \tilde{H}_{\pi^{-1}(i),n} \cdot R_{\boldsymbol{XX}}(\pi^{-1}(i),n) \cdot \tilde{H}^*_{\pi^{-1}(i),n} + I \right|$$

- For given $\boldsymbol{\theta}$, min weighted rate sum over $R_{\boldsymbol{XX}}(u,n)$ maximizes the convex sum:

$$\sum_{u=1}^{U} \theta_u \cdot b_{u,n} = \sum_{u=1}^{U} \left\{ \underbrace{\left[\theta_{\pi^{-1}(u)} - \theta_{\pi^{-1}(u+1)}\right]}_{\delta_{\pi^{-1}(u)} \leq 0} \cdot \log_2 \left| \sum_{i=u}^{U} \tilde{H}_{\pi^{-1}(i),n} \cdot R_{\boldsymbol{XX}}(\pi^{-1}(i),n) \cdot \tilde{H}^*_{\pi^{-1}(i),n} + I \right| \right\} .$$

Equal Theta

- Successive equal theta values
  - can happen, often!
  - This usually happens when there are secondary user components.

- The corresponding rate-sum difference term(s) is (are) zero.

- Only the sum rate of the corresponding users can be varied $b_{\pi^{-1}(u)} + b_{\pi^{-1}(u)+1}$ is optimized.

- The corresponding vertices for swapping the order (more generally varying when 3 or more) need to be “vertex-shared” in a proportion that causes the desired data rate to be achieved.

- Complicated program in matlab to do this, although concept is straightforward.

Coming Attraction: The Stanford minPMAC program(s)

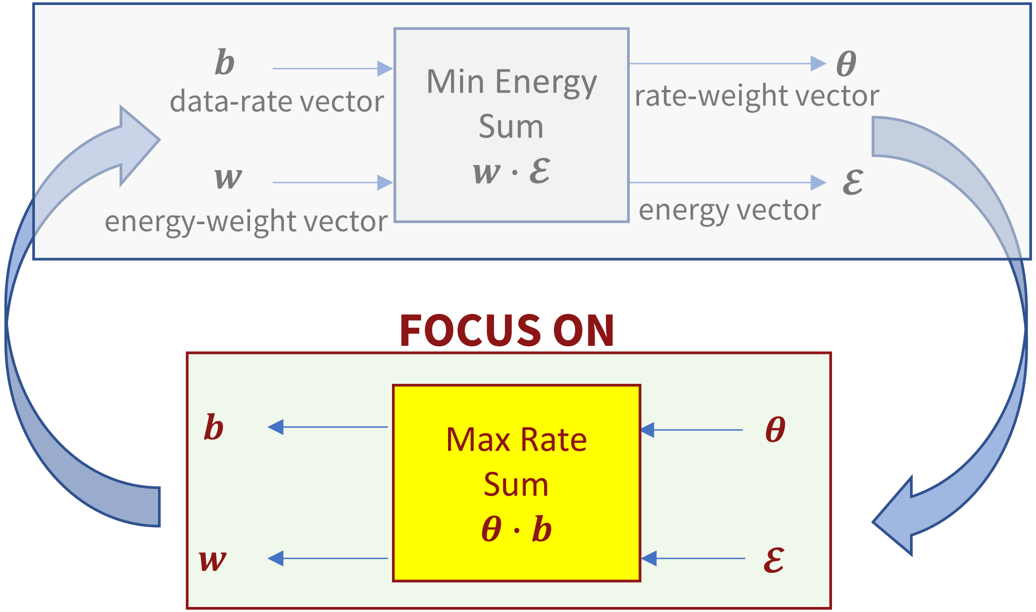
Maximum rate-sum solution (maxRMAC)

Section 5.4.4

Maximize weighted rate sum

- **Maximize weighted rate** sum b, given $\boldsymbol{\mathcal{E}}$, with
  - no rate limit and
  - given rate weights $\boldsymbol{\theta} \succcurlyeq \mathbf{0}$, (non-negative).
    - How important relatively are the users?
    - Given some energy/power budget.

$$\max_{\{R_{\boldsymbol{xx}}(u)\}} \sum_{u=1}^{U} \theta_u \cdot b_u$$
$$ST: \quad \boldsymbol{\mathcal{E}}_{\boldsymbol{x}} \preceq [\mathcal{E}_{1,max}\ \mathcal{E}_{2,max}\ ...\mathcal{E}_{U,max}]^* = \boldsymbol{\mathcal{E}}_{max}^* \succeq \mathbf{0}$$
$$\boldsymbol{\mathcal{E}} \succeq \mathbf{0} \quad .$$



maxRMAC_cvx

- function [Eun, w, bun] = maxRMAC_cvx(H, Eu, theta, cb)
  - inputs

H is the channel tensor $L_y \times U \times \bar{N}$
Eu is the given $1 \times U$ energy vector $\boldsymbol{\mathcal{E}}$
theta is the rate weight vector $\boldsymbol{\theta}$
cb =1 cplx, =2 real

  - outputs

Eun is the $U \times \bar{N}$ matrix containing $\mathcal{E}_{u,n}$
(does not handle MAC $L_x > 1$)
w is the energy-weight vector $\boldsymbol{w}$
bun is the $U \times \bar{N}$ bit matrix containing $b_{u,n}$
(Eun is related to **bun** directly, fixed theta)

There is also a non-cvx version that runs faster,
but can deviate by 1-2% from CVX accuracy.
The $\boldsymbol{w}$ weights often are scaled versions of one another.

```
> H=zeros(1,2,1);
>> H(1,1,1)=80;
H(1,2,1)=60;
>> Eu=[1 1];
>> theta=[1 1];
>> cb=2;
>> [Eun, w, bun] = maxRMAC_cvx(H, Eu, theta,cb)

Eun =
   1.0000
   1.0000
w =
   0.6394
   0.3597
bun =
   6.3220
   0.3219
>> sum(bun,2)' =   6.3220   0.3219
>> sum(bun,'all') =   6.6439
>> [Eun, w, bun] = maxRMAC_cvx(H, Eu, theta, 1);
>> Eun' =   1.0000   1.0000
>> w' =1.2789   0.7194
>> bun =   11.6443   0.6437
>> sum(bun) =   12.2880
```

If N=1 and complex bb, maxRMAC adjusts energy. Otherwise, use even N>1 if cb=2 (real bb)

Always trade dimensions for energy

maxRMACMIMO

- Runs slower, but allows variable number of transmit antennas per user: Lxu (also uses CVX),
- Also has an Rxxs output, for which trace{Rxxs(u)} is Eu.

```
[Rxxs, Eun, w, bun] = maxRMACMIMO(H, [1 1], Eu, theta, 1)

Rxxs =  2x1 cell array
   {[1.0000]}
   {[1.0000]}
Eun =
   1.0000
   1.0000
w =
   1.2789
   0.7194
bun =
  11.6443
   0.6437
sum(bun) =   12.2880
```

MIMO Examples

```
>> H2 =
    4    3    2    1
    5    6    7    8
[Rxxs, Eun, w, bun] = maxRMACMIMO(H2, [2 2], [5 6], [1 1] , 1)
Rxxs = 2x1 cell array
Eun =
   5.0000
   6.0000
W' =   0.3881   0.3275
bun =
   7.6484
   5.8337
>> Rxxs{:,:}
=   3.5983   2.2458
    2.2458   1.4017
=   1.6863   2.6971
    2.6971   4.3137
```

Both users are MIMO

```
[Rxxs, Eun, w, bun] = maxRMACMIMO(H2(:,1:3), [2 1], [5 6], [1 1] , 1)

Rxxs =  2x1 cell array
Eun =
   5.0000
   6.0000
W' =   0.3811   0.3144
bun =
   7.5867
   4.1392
>> Rxxs{:,:}
 =
   3.9149   2.0611
   2.0611   1.0851
 =
   6.0000
```

One user is MIMO

- The second antenna on user 1 helps both users' data rates.   Why?

Vector DMT Examples

```
>> Eu = [    1     1 ];
>> theta = [    1     1];
> H4
H4(:,:,1) =   80   60
H4(:,:,2) =   80   60
H4(:,:,3) =   80   60
H4(:,:,4) =   80   60
[Eun, w, bun] = maxRMAC_cvx(H4, 4*Eu', theta', 1)
Eun =
   1.0000   1.0000   1.0000   1.0000
   1.0000   1.0000   1.0000   1.0000
w' =   0.6396   0.3598
bun =
  12.6441  12.6441  12.6441  12.6441
   0.6438    0.6438    0.6438    0.6438
>>  [Eun, w, bun] = maxRMAC_cvx(H4, 2*Eu', theta', 2)
Eun =
   1.0000   1.0000
   1.0000   1.0000
w' =   0.6396   0.3598
bun =
   6.3220   6.3220
   0.3219   0.3219
>>sum(bun,'all') =  13.2879
>>[Rxx, bsum , bsum_lin] = SWF([1 1], H4(:,:,1:2), [1 1],ones(1,1,2), 2);
Rxx(:,:,1) =               Rxx(:,:,2) =
    1    0                    1    0
    0    1                    0    1
bsum =  13.2879
bsum_lin =   2.1174
```

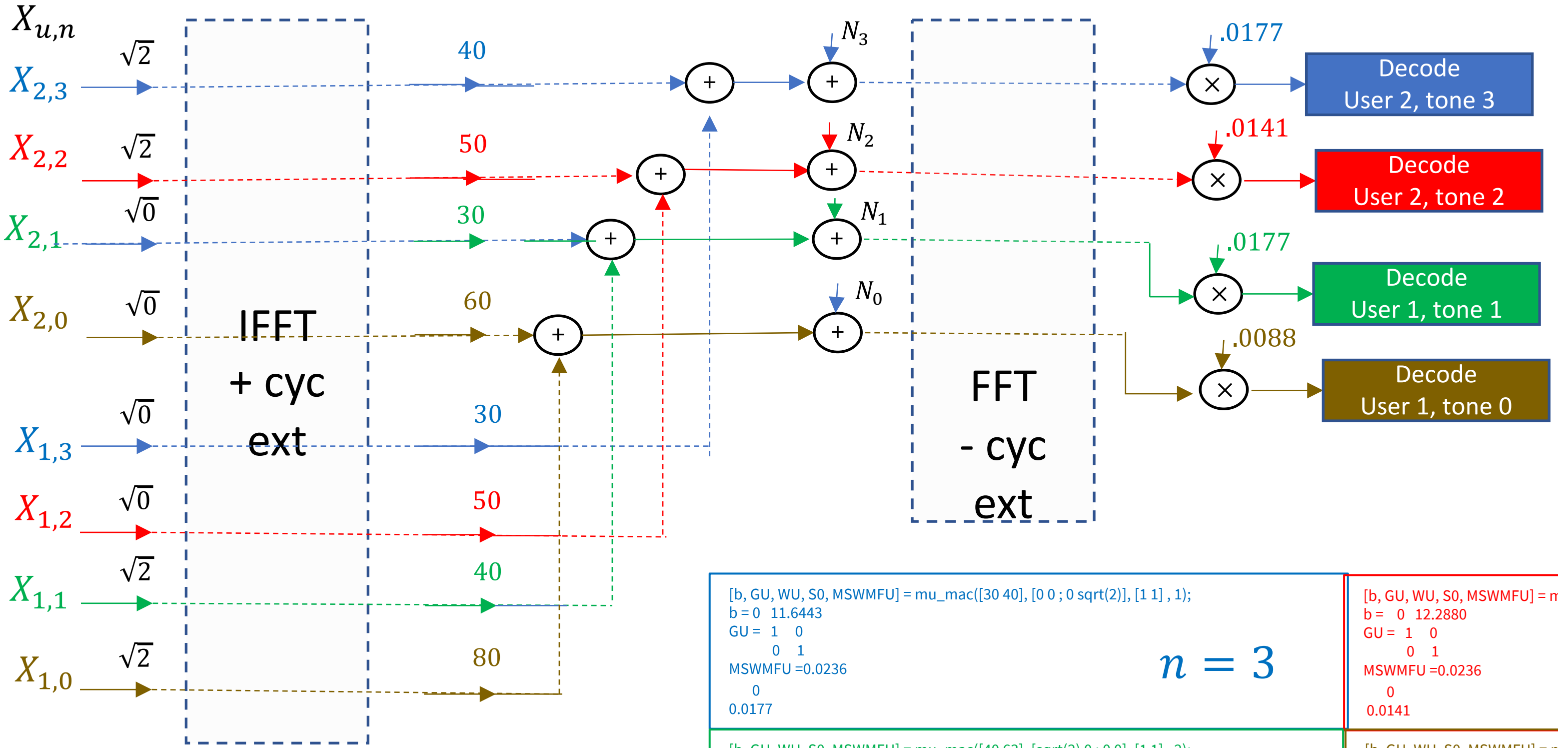
Linear substantially less– why?

```
>> H4x=zeros(1,2,4);
>> H4x(:,:,1)= [80 60];
>> H4x(:,:,2)= [40 30];
>> H4x(:,:,3)= [50 50];
>> H4x(:,:,4)= [30 40];
>> [Eun, w, bun] = maxRMAC_cvx(H4x, 4*Eu, theta,1)

Eun = 1.9984   1.9978   0.0037   0.0000
      0.0000   0.0000   1.9980   2.0020

w' =   0.5004   0.4995

bun =  13.6428  11.6427   3.3711   0.0023
        0.0000   0.0000   8.9181  11.6435

sum(bun'*theta') =  49.2206

>> [Rxx, bsum , bsum_lin] = SWF(Eu, H4x(:,:,1:4), [1 1], ones(1,1,4), 1)
Rxx(:,:,1) =   2.0001       0
                    0       0
Rxx(:,:,2) =   1.9996       0
                    0       0
Rxx(:,:,3) =   0.0003       0
                    0  2.0000
Rxx(:,:,4) =        0       0
                    0  2.0000
bsum =  49.2206
bsum_lin =  48.4228
```

Not Hermitian symmetric

Linear is close – why?

System diagram for previous example



No feedback between users on these FDM – tonal GDFEs

```
[b, GU, WU, S0, MSWMFU] = mu_mac([30 40], [0 0 ; 0 sqrt(2)], [1 1] , 1);
b = 0  11.6443
GU =  1    0
         0    1
MSWMFU =0.0236
      0
0.0177
```

$n = 3$

```
[b, GU, WU, S0, MSWMFU] = mu_mac([50 50], [sqrt(2) 0 ; 0 0], [1 1] , 2);
b =   0  12.2880
GU =  1    0
         0    1
MSWMFU =0.0236
      0
 0.0141
```

$n = 2$

```
[b, GU, WU, S0, MSWMFU] = mu_mac([40 63], [sqrt(2) 0 ; 0 0], [1 1] , 2);
b =   11.6443        0
GU =  1    0
         0    1
MSWMFU =0.0236
   0.0177
        0
```

$n = 1$

```
[b, GU, WU, S0, MSWMFU] = mu_mac([80 60], [sqrt(2) 0 ; 0 0], [1 1] , 2);
b =    6.8220        0
GU =  1    0
         0    1
MSWMFU =
   0.0088
        0
```

$n = 0$

- FDM solution simplifies to linear receiver as optimum.

maxRESMAC – only energy-sum constraint

- function [Eun, w, bun] = maxR**ES**MAC(H, Etotal, theta, cb)
  - inputs

> H is the channel tensor $L_y \times U \times \bar{N}$
> Etotal is the total energy sum $\mathcal{E}$
> theta is the rate weight vector $\boldsymbol{\theta}$
> cb =1 cplx, =2 real

  - outputs

> Eun is the $U \times \bar{N}$ matrix containing $\mathcal{E}_{u,n}$
> (does not handle MAC $L_x > 1$)
> w is the order/rate vector $\boldsymbol{w}$
> bun is the $U \times \bar{N}$ bit matrix containing $b_{u,n}$

```
[Eun, w, bun] = maxRESMAC(H4x, 8, [ 1 1]',1)

Eun =
   2.0000   1.9996  -0.0000   0.0000
   0.0000   0.0000   2.0007   1.9997
w =   0.4998   0.4998
bun =
  13.6439  11.6440  -0.0000   0.0005
   0.0000   0.0000  12.2885  11.6436
>> sum(bun,'all') =   49.2206
>> sum(Eun,'all') =   8.0000
>> sum(bun') = 25.2884   23.9322

>> [Rxx, bsum , bsum_lin] = SWF(Eu, H4x(:,:,1:4), [1 1], ones(1,1,4), 1);

Rxx(:,:,1) =              Rxx(:,:,3) =
   2.0001   0              0.0003      0
      0     0              0          2.0000
Rxx(:,:,2) =              Rxx(:,:,4) =
   1.9996   0              0      0
      0     0              0   2.0000
bsum =   49.2206
bsum_lin =   48.4228
```

- Also [Rxxs, Eun, w, bun] = maxRESMAC**MIMO**(H, Lxu, Etotal, theta, cb)

Minimum energy-sum solution (minPMAC)

Section 5.4.4.5

minimize weighted energy sum

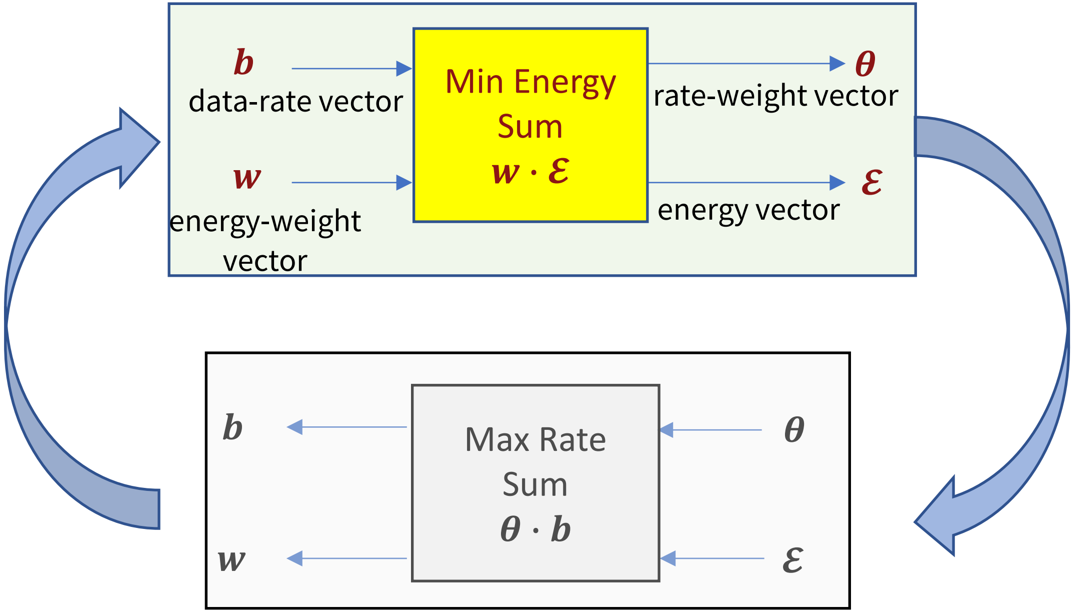
- **Minimize weighted energy** sum $\mathcal{E}$, given $\boldsymbol{b}$, with
  - no energy limit and
  - given energy weights $\boldsymbol{w} \succcurlyeq \boldsymbol{0}$, (non-negative).
    - How important relatively are the users?
    - Given some fixed desired user rates.

$$\min_{\{R_{\boldsymbol{xx}}(u)\}} \quad \sum_{u=1}^{U} w_u \cdot \underbrace{\text{trace}\left\{R_{\boldsymbol{xx}}(u)\right\}}_{\mathcal{E}_u}$$

$$ST: \quad \boldsymbol{b} \succeq [b_{1,min}\ b_{2,min}\ \ldots b_{U,min}]^* = \boldsymbol{b}^*_{min} \succeq \boldsymbol{0}$$

$$\boldsymbol{\mathcal{E}} \succeq \boldsymbol{0} \quad .$$

FOCUS ON



minPMAC

- [Eun, theta, bun, FEAS_FLAG, bu_a, info] = minPMAC(H, bu, w, cb)
  - inputs

    H is the channel tensor $L_y \times U \times \bar{N}$ **Freq Domain**
    bu is the given data-rate $\boldsymbol{b}$
    w is the energy weight vector $\boldsymbol{w}$
    cb =1 cplx; cb =2 real

  - outputs

    Eun is the $U \times \bar{N}$ matrix containing $\mathcal{E}_{u,n}$
    (does not handle MAC $L_x > 1$)
    theta is the order/rate vector $\boldsymbol{\theta}$
    bun is the $U \times \bar{N}$ bit matrix containing $b_{u,n}$
    FEAS_FLAG = 1 (one order), 2 vertex sharing
    bu_a is user rate vector
    Info is table of information (see examples)

```
>> H=zeros(1,2,1)  % dimensioning a tensor
H =  0    0
>> H(1,1,1)=80;
>> H(1,2,1)=60
H=  80   60
>> b = [3
3];
>> w = [ 1
1];
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H, b, w, 2);
Eun =
   0.6300
   0.0175
theta =
   1.2800
   1.2956
bun =
   3.0000
   3.0000
>> info.order{:}' =    1    2
>> 0.5*log2(1+(6400*Eun(1))/(1+3600*Eun(2))) = 3.0000
>> 0.5*log2(1+(3600*Eun(2))) = 3.0000
```

Decode user 1 last (best position or top)

- Called subroutines (6)
  - minPtone, Lag_dual_f, eval_f, hessian, fm_waterfill_gn , start_Ellipse
  - Now at web site

Section 5.4.4.5

Increase N to 4 tones

```
>> H=zeros(1,2,4);
>> H(1,1,:)=80 ;
>> H(1,2,:)=60 ;
[Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H, 4*b', w', 2)
Eun =
   0.6300   0.6300   0.6300   0.6300
   0.0175   0.0175   0.0175   0.0175
theta =
   1.2800
   1.2956
bun =
   3.0000   3.0000   3.0000   3.0000
   3.0000   3.0000   3.0000   3.0000
FEAS_FLAG =    1
bu_a' =  12.0001   12.0000
info.order{:} =    1    2
```

Real bb, so ignore upper-half tones that repeat lower half.

4 real dimensions

```
[Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H, 5*b', w', 2)
Eun =
  5.0917   5.0917   5.0917   5.0917
  0.0500   0.0500   0.0500   0.0500
theta =
 10.2400
 10.2840
bun =
  3.7500   3.7500   3.7500   3.7500
  3.7500   3.7500   3.7500   3.7500
FEAS_FLAG =    1
bu_a' =  15.0000  15.0000
info.order{:} =     1     2
```

Result is correct, but energy too high, perhaps.

- Unequal gains on tones

```
>> H4x(:,:,1) = [ 80 60];
>> H4x(:,:,2) = [ 40 30];
>> H4x(:,:,3) = [ 50 50];
>> H4x(:,:,3) = [ 30 40];
```

No conjugate symmetry, Must be BB complex

```
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H4x  8*b', w', 1)
Eun =
  1.6187   1.6184   0.0008   0.0000
  0.0002   0.0000   1.6180   1.6186
theta =
  3.2380
  3.2384
bun =
 12.6610  11.3385   0.0007   0.0000
  0.6780   0.0005  11.9822  11.3391

FEAS_FLAG =   1
bu_a' =  24.0000    23.9998
>> info.order{:}'  % =    1    2
```

8 real dimensions FDM showing again

Still decode user 1 last (best position or top) all tones, both examples, slightly not "FDM"

Equal Thetas

```
[Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC([80 80], b', w', 2)
Eun =
   0.3200
   0.3198
theta =
   1.2800
   1.2800
bun =
   0.4999
   5.5001
FEAS_FLAG =    2
bu_a =
   3.0000
   3.0000
info =  2x6 table
 bu_v              Eun      bun      theta    order      frac     clusterID
0.49964   5.5004  {1 × 2}  {1 × 2}  {1 × 2}  {1 × 2}  0.49996      1
 5.5000  0.49964  {1 × 2}  {1 × 2}  {1 × 2}  {1 × 2}  0.50004      1
>> info.bu_v'*info.frac  =
   3.0000
   3.0000
>> info.order{:} =
    1     2
    2     1
```

Not the desired b, but sum is same.

Order switches

```
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC([80 80 80], [5 4 3], [1 1 1], 2)
Eun = [873.8222    873.8105    873.8076]
Theta' =  1.0e+03 *  [5.2429    5.2429    5.2429 ]
bun =
   0.5000
   0.2925
  11.2075
FEAS_FLAG =    2
bu_a' =  5.0000    4.0000    3.0000
 3 × 7 table

            bu_v               Eun        bun        theta      order      frac     clusterID
 11.208   0.5        0.29248  {1 × 3}  {1 × 3}  {1 × 3}  {1 × 3}  0.33511     1
0.29248 11.208      0.5       {1 × 3}  {1 × 3}  {1 × 3}  {1 × 3}  0.42492     1
0.5       0.29248  11.208     {1 × 3}  {1 × 3}  {1 × 3}  {1 × 3}  0.23998     1

>> (info.bu_v'*info.frac)' % =   5.0000    4.0000   3.0000
>> info.order{:}  %=
    3    2    1
    1    3    2
    2    1    3
```

Split vertices 50/50 in decoder share
Transmitter with $\Gamma = 0$ dB can send
3 bits for all symbols, both users – Why?

Split vertices 17/50, 21/50, 12/50 in decoder share

Or with really good code can do a "coded-GDFE" (like coded OFDM) via the separation theorem, but which user is decoded first? (I think "any")

Single-tone channel with U > L

- Forces a secondary user component (equal theta for >= 2 user components)

```
Htemp = [ 80   40   30
          30  -50  -15 ];
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(Htemp, [7 5 6], [1 1 1], 1);
Eun' % =    0.0560   0.0891   0.0967
theta'% = 0.1968   0.1968   0.2892

>> bu_a' =        7.0000   5.0000   6.0000
MAC User ID           3        2        1          (not same as Matlab's index)
Matlab ID             1        2        3
FEAS_FLAG =    2
bu_a' =            7.0000   5.0000   6.0000
info =  2x6 table
       bu_v               Eun          bun        theta        order        frac    clusterID

  5.8731   6.1269     6  {1×3 dble}  {1×3 dble}  {1×3 dble}  {1×3 dble}  0.58514    1
  8.5895   3.4105     6  {1×3 dble}  {1×3 dble}  {1×3 dble}  {1×3 dble}  0.41485    1

>>> info.bu_v'*info.frac = 7.0000   5.0000   6.0000

>> info.order{:} =
   1   2   3
   2   1   3
```

The text in Section 5.4.4.4 continues This example to 4 tones

Also shows at lower data rates, the vertex sharing goes away with N>1, because effectively "tone-sharing"

58/42 split for users 2 and 1, which move to top and cancel user 3
User 3 treats both users 2 and 1 as noise

Multiple Clusters

```
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC([80 60 80 60], [3 4 5 2], [1 1 1 1], 2)

Eun' =      1.0e+04 *   0.0001   0.0001   2.0973   2.0970
theta' =    1.0e+04 *   8.3886   8.3887   8.3886   8.3887
FEAS_FLAG =   2
>> bu_a' =        3.0000   4.0000   5.0000   2.0000
MAC User ID          2        3        4        1        (not same as Matlab's index)
Matlab ID            1        4        3        2
```

(Cluster 2 is at top – and cancels Cluster 1; Cluster 1 treats Cluster 2 as noise)

info = 4 × 6 table

bu_v				Eun	bun	theta	order	frac	clusterID
7.5017	0.49796	0.49834	5.502	{1x4 dble}	{1x4 dble}	{1x4 dble}	{1x4}	0.69995	1
7.5017	5.4983	0.49384	.50169	{1x4 dble}	{1x4 dble}	{1x4 dble}	{1x4}	0.30005	1
7.5017	0.49796	0.49384	5.502{	(1x4 dble}	{1x4 dble}	{1x4 dble}	{1x4}	0.35714	2
0.50165	0.49796	7.4984	5.502	{1x4 dble}	{1x4 dble}	{1x4 dble}	{1x4}	0.64286	2

```
>> info.frac(3:4)'*info.bu_v(3:4,[1,3]) =   3   5
>> info.frac(1:2)'*info.bu_v(1:2,[2,4]) =   4   2
>> info.order{:} =
   3   1   2   4
   3   1   4   2
   3   1   2   4
   1   3   2   4
```

Program with # clusters > 1
Reverses order within clusters
(ok for 1 cluster, no reversal)

Cluster 2 in GDFE receiver has

User 4 with $\langle b_4 \rangle$=

$$7.5 \times .643 + 0.5 \times .357 = 3$$

User 2 with $\langle b_2 \rangle$=

$$0.5 \times .643 + 7.5 \times .357 = 5$$

Users 3 and 1 both cancelled

Cluster 1 in GDFE receiver has

User 3 with $\langle b_3 \rangle$=

$$5.5 \times .30 + 0.5 \times .70 = 4$$

User 1 with $\langle b_1 \rangle$=

$$0.5 \times .30 + 0.5 \times .70 = 2$$

Treats users 1 and 2 as noise

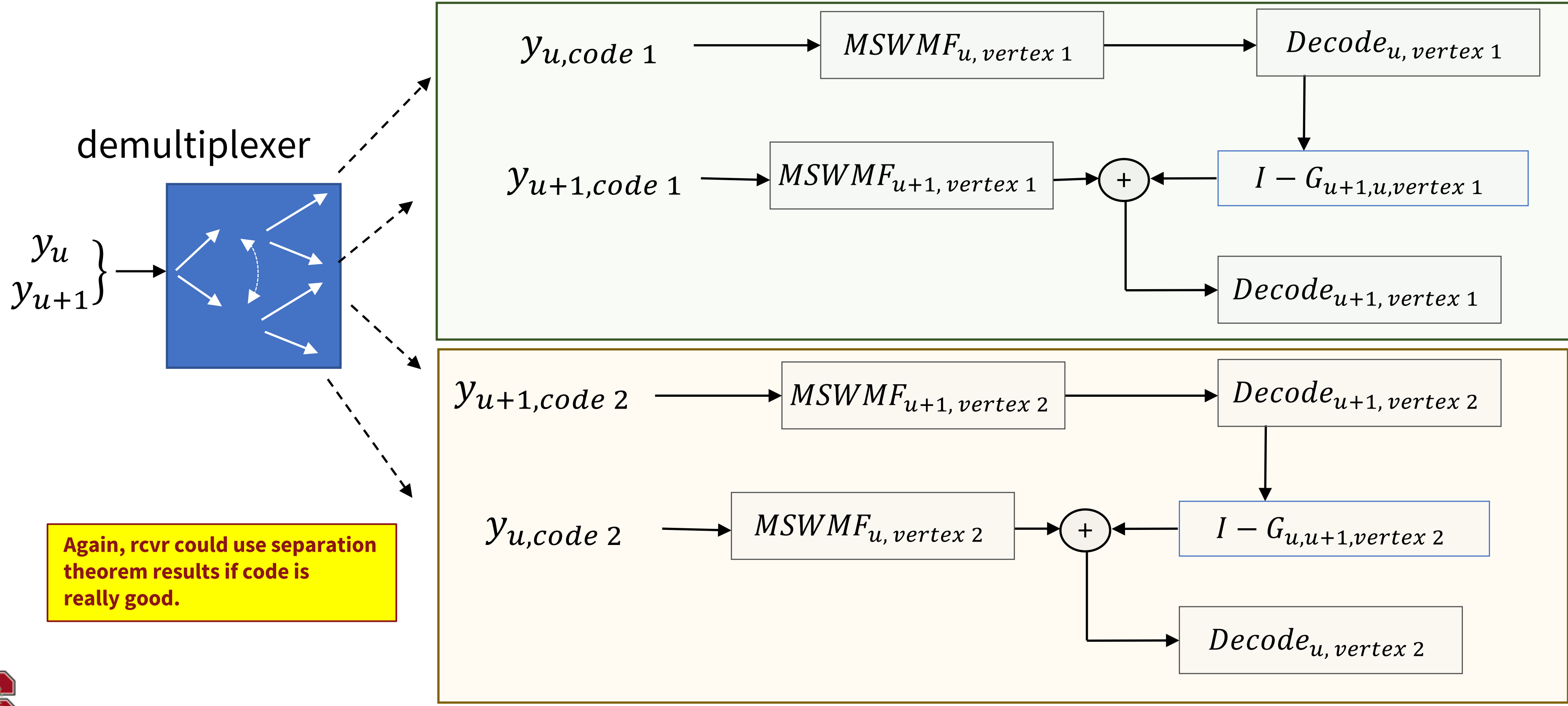
Encoder occurrence frequencies?

cluster 2 \ cluster 1	0.7	0.3
0.36	.192	.108
0.64	.448	.252

Each user has 4 encoders/decoders with roughly 4, 9, 5, and 2 turns from each 20–symbol packet transmitted in larger frame.

illustrate cluster receiver

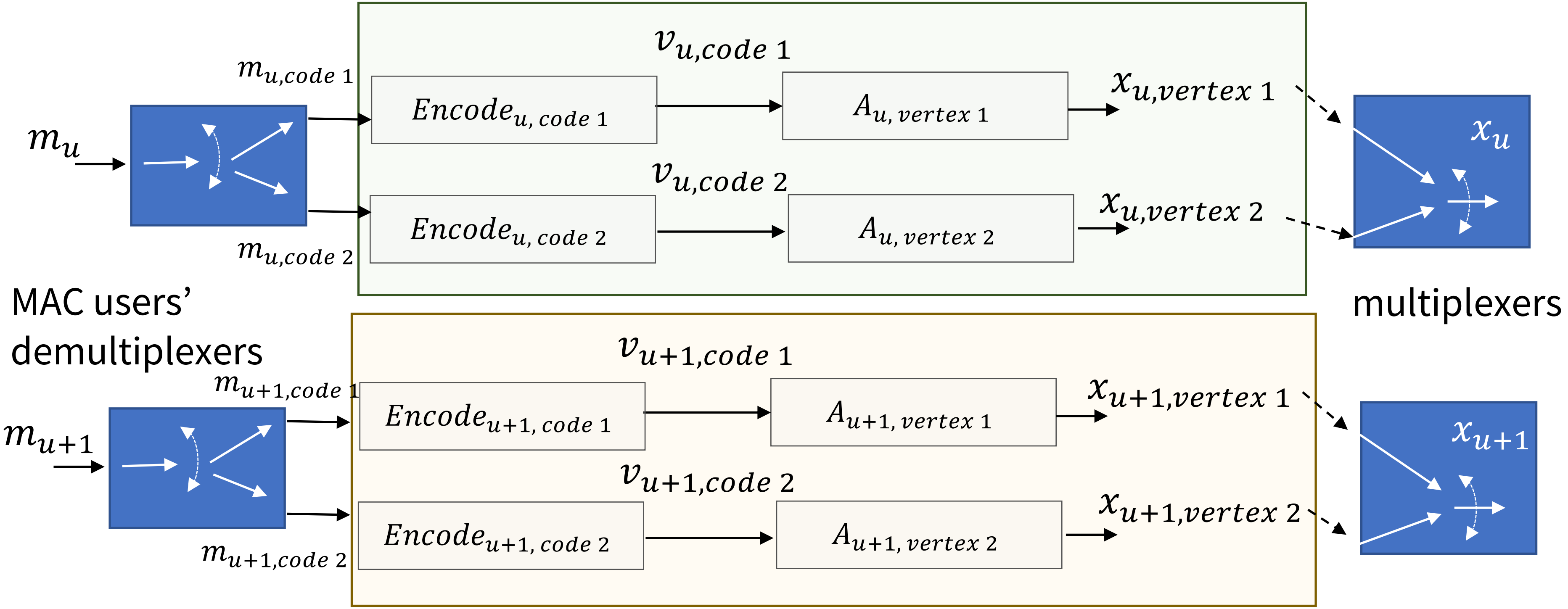
- Two (U) GDFE receivers for frac(1) …. frac(U) of the received symbols



Again, rcvr could use separation theorem results if code is really good.

illustrate cluster encoder modulator

- Two (U) encoders/modulator transceivers for frac(1) …. frac(U) of the received symbols

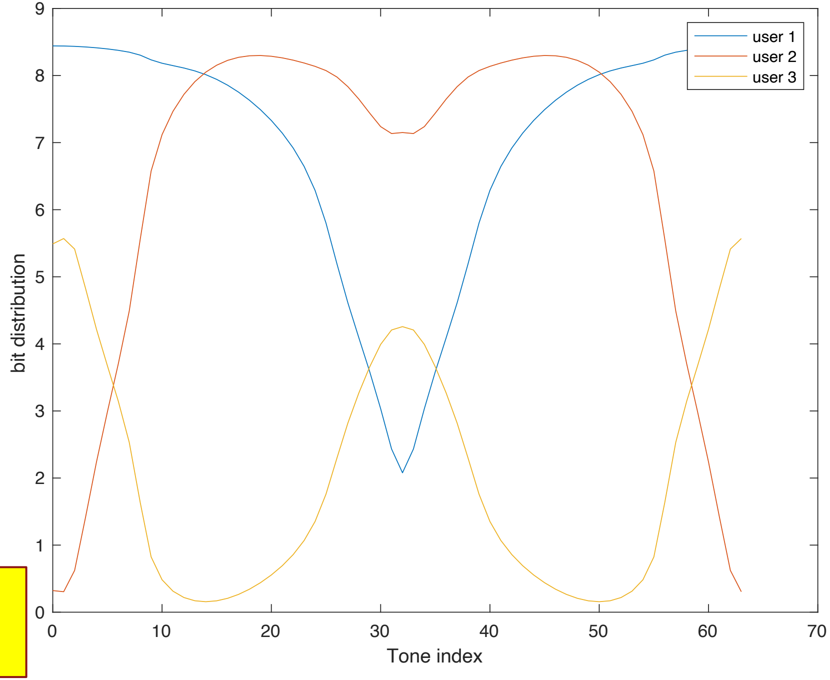


Multicluster imposes packet-level synchronization so that each user encoder at any symbol time corresponds to only encoder in each and every other user's set.

64-tone example

```
h64=cat(3,[1 0 .8 ; 0 1 1],[.9 -.3 0 ; .5 -1 -1],[0 .2 0 ; .4 -.63 0],[0 0 0 ; 0 .648 0])*10;
>> H64 = fft(h64, 64, 3);
[Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H64, [445 412 132]', [1 1 1]',1);
Elapsed time is 215.045054 seconds.
>> theta =
    2.6917
    2.6917
    2.2404
FEAS_FLAG =    2
bu_a =
  445.0000
  412.0000
  132.0000

      %  bu_v              Eun        bun        theta    order   frac    clusterID

   445.27   411.72    132  {1x3x64 }  {1x3x64 }  {1x3}   {1x3}   0.99    1
   416.71   440.29    132  {1x3x64 }  {1x3x64}   {1x3}   {1x3}    0.01   1
>> info.order{:} =
    3    2    1
    3    1    2
 >>sum(Eun') = 65.9552   60.2757   40.4454
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H64, [445 412 132]', [3 1
.75]',1);
>> sum(Eun')  =  61.0398  58.6166  49.6402
>> bu_a =  445.0000  412.0000  132.0001
```

- Every tone has a cluster of 2 users (3 & 2)
  - Codes for 2 and 3 occur 1/100 and 99/100 symbols
- User 1 is removed via GDFE cancellation
  - Has only 1 code (same all symbols)



The GDFE's energize all 3 users, but some "FDM"

Corresponding MAC Design (N = 8), HWH7

```
N=8;
U=3;
Ly=2;
cb=1;
Lxu=[1 1 1];
bsum=zeros(1,N);
H8 = fft(h, N, 3);
[Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H8, [54 51 16]', [1 1 1]',1)

GU=zeros(U,U,N);
WU=zeros(U,U,N);
S0=zeros(U,U,N);
Bu=zeros(U,N);
MSWMFU=zeros(U,Ly,N);
AU=zeros(3,3,N);
 for n=1:N
  AU(:,:,n)=sqrtm(diag(Eun(:,n)));
 end
 for n=1:N
  [Bu(:,n), GU(:,:,n) , WU(:,:,n),S0(:,:,n), MSWMFU(:,:,n)] = ...
   mu_mac(H(:,:,n), AU(:,:,n), Lxu, cb);
 end
bvec=sum(Bu');
Bsum(index) = sum(bvec);

>> FEAS_FLAG =    1
 >> bvec =   54.7544   51.8108   16.1050
>> bu_a =   54.0000   51.0000   16.0000
```

Design commands, Divide by 8

Did not take 8/9 of energy because that is outside the minPMAC process

But Eun x 9/8 is actual energy

8 Feedback Sections

```
>> GU
GU(:,:,1) =   1.0e+04 *
   0.0001 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0001 + 0.0000i  -1.7799 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0001 + 0.0000i
GU(:,:,2) =
   1.0000 + 0.0000i  -0.2004 + 0.0132i   0.1823 + 0.2030i
   0.0000 + 0.0000i   1.0000 + 0.0000i   1.3125 - 0.1839i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
GU(:,:,3) =
   1.0000 + 0.0000i  -0.8690 + 0.1232i  -0.0108 + 0.0889i
   0.0000 + 0.0000i   1.0000 + 0.0000i   0.1887 + 0.0287i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
GU(:,:,4) =
   1.0000 + 0.0000i  -1.7737 + 0.5843i  -0.4677 + 0.3006i
   0.0000 + 0.0000i   1.0000 + 0.0000i   0.8075 + 0.5128i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
GU(:,:,5) =   1.0e+03 *
   0.0010 + 0.0000i  -1.7618 + 0.0000i  -9.6891 + 0.0000i
   0.0000 + 0.0000i   0.0010 + 0.0000i   0.0024 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0010 + 0.0000i
GU(:,:,6) =
   1.0000 + 0.0000i  -1.7737 - 0.5843i  -0.4677 - 0.3006i
   0.0000 + 0.0000i   1.0000 + 0.0000i   0.8075 - 0.5128i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
GU(:,:,7) =
   1.0000 + 0.0000i  -0.8690 - 0.1232i  -0.0108 - 0.0889i
   0.0000 + 0.0000i   1.0000 + 0.0000i   0.1887 - 0.0287i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
GU(:,:,8) =
   1.0000 + 0.0000i  -0.2004 - 0.0132i   0.1823 - 0.2030i
   0.0000 + 0.0000i   1.0000 + 0.0000i   1.3125 + 0.1839i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
```

8 Feedforward Sections

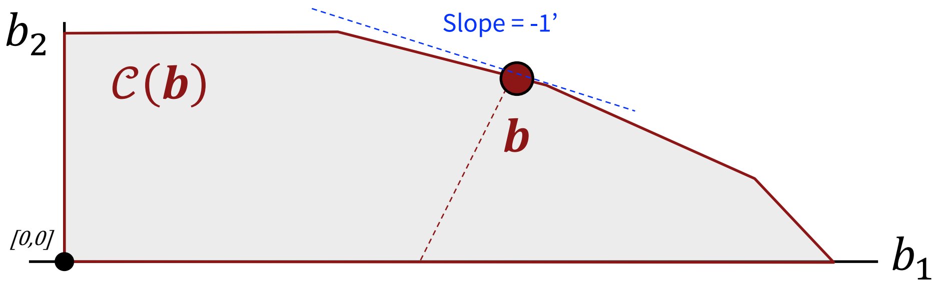
```
>> MSWMFU
MSWMFU(:,:,1) =   1.0e+03 *
   0.0000 + 0.0000i   0.0000 + 0.0000i
  -2.2006 + 0.0000i   4.6158 + 0.0000i
   0.0001 + 0.0000i  -0.0003 + 0.0000i
MSWMFU(:,:,2) =
   0.0397 + 0.0155i   0.0086 + 0.0183i
   0.0584 + 0.0483i  -0.0191 - 0.1594i
   0.0481 + 0.0390i   0.0017 - 0.1107i
MSWMFU(:,:,3) =
   0.0406 + 0.0365i  -0.0162 + 0.0203i
   0.0135 + 0.0147i   0.0294 - 0.0304i
   0.2279 + 0.0713i   0.0673 - 0.1243i
MSWMFU(:,:,4) =
   0.0666 + 0.1166i  -0.0648 - 0.0085i
   0.0123 + 0.0189i   0.0479 + 0.0042i
   0.0819 + 0.0380i   0.0158 - 0.0243i
MSWMFU(:,:,5) =   1.0e+02 *
   7.7990 + 0.0000i  -7.7990 + 0.0000i
   0.0006 + 0.0000i   0.0009 + 0.0000i
  -0.0013 + 0.0000i   0.0010 + 0.0000i
MSWMFU(:,:,6) =
   0.0666 - 0.1166i  -0.0648 + 0.0085i
   0.0123 - 0.0189i   0.0479 - 0.0042i
   0.0819 - 0.0380i   0.0158 + 0.0243i
MSWMFU(:,:,7) =
   0.0406 - 0.0365i  -0.0162 - 0.0203i
   0.0135 - 0.0147i   0.0294 + 0.0304i
   0.2279 - 0.0713i   0.0673 + 0.1243i
MSWMFU(:,:,8) =
   0.0397 - 0.0155i   0.0086 - 0.0183i
   0.0584 - 0.0483i  -0.0191 + 0.1594i
   0.0481 - 0.0390i   0.0017 + 0.1107i
```

- So, this is design, see problems 7.1-4

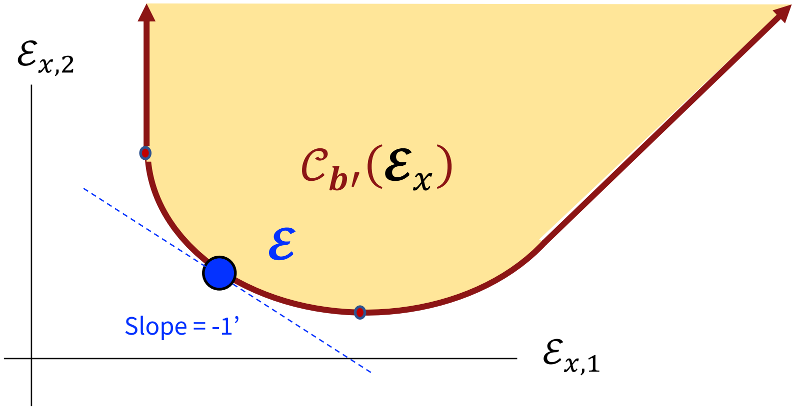
admMAC and admissibility

Section 5.4.5

Capacity region(s)



- $\mathcal{C}(\boldsymbol{b})$ contains all possible weighted **rate** sums $\sum_{u=1}^{U} \theta_u \cdot b_u$ that meet **energy-vector** constraint $\boldsymbol{\mathcal{E}} \preccurlyeq \boldsymbol{\mathcal{E}}_x$.
- The max-b-sum point is "highest" $\boldsymbol{b}$ in $\mathcal{C}(\boldsymbol{b})$.
- **If $\boldsymbol{\mathcal{E}} \preccurlyeq \boldsymbol{\mathcal{E}}_x$, then $\boldsymbol{b}$ is admissible.**
- If not, find minimum energy-sum to achieve $\boldsymbol{b}$.

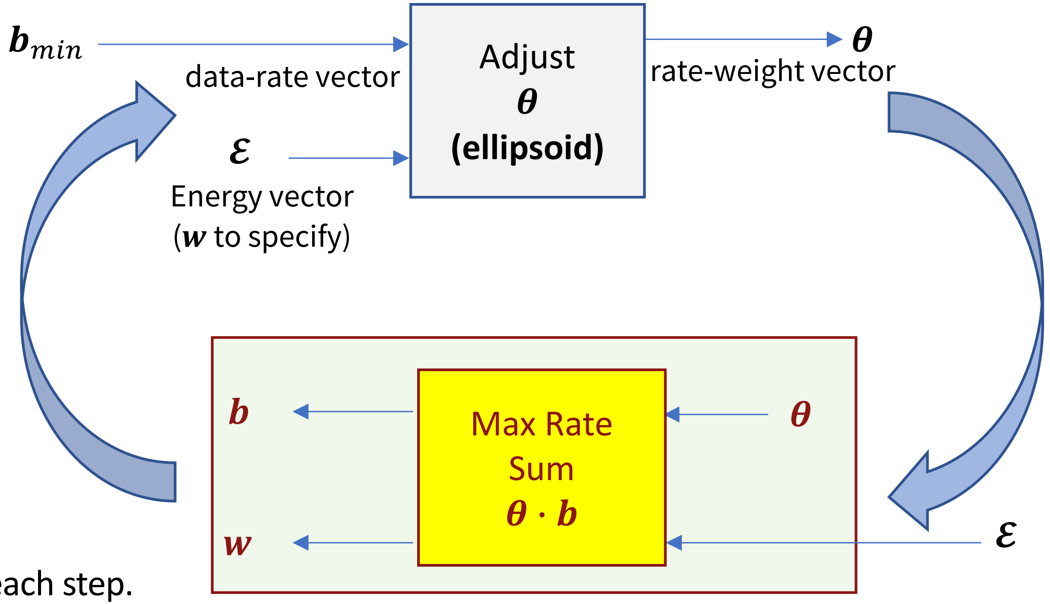


- $\mathcal{C}_{\boldsymbol{b}'}(\boldsymbol{\mathcal{E}}_x)$ contains all possible weighted **energy** sums $\sum_{u=1}^{U} w_u \cdot \mathcal{E}_u$ that meet **rate-vector** constraint rate vector $\succcurlyeq \boldsymbol{b}'$.
- The min-E-sum point is "lowest" $\boldsymbol{\mathcal{E}}$ in $\mathcal{C}_{\boldsymbol{b}'}(\boldsymbol{\mathcal{E}})$.
- **If $\boldsymbol{b}' \succcurlyeq \boldsymbol{b}$, then $\boldsymbol{\mathcal{E}}$ is admissible.**

Admissible: meet both $\boldsymbol{b}$ and $\boldsymbol{\mathcal{E}}$ constraints

Using admMAC to find solution

- Specify the $\boldsymbol{b}_{min}$ and $\boldsymbol{\mathcal{E}}$.
- Find the weights $\boldsymbol{w}$ and $\boldsymbol{\theta}$.
- When $\boldsymbol{b}_{min} \nsucceq \boldsymbol{b}$, change $\boldsymbol{\theta}$.
- If there is a solution in $\mathcal{C}(\boldsymbol{b})$.
- This process finds it.
    - Both are convex methods that improve at each step.



$$\min_{\{R_{\boldsymbol{XX}}^{(u,n)}\}} \quad 0$$

$$ST: \quad 0 \preceq \boldsymbol{b}_{min} \preceq \sum_n [b_{1,n}\ b_{2,n}\ ...b_{U,n}]$$

$$0 \preceq \sum_n \text{trace}\left\{R_{\boldsymbol{XX}}(u,n)\right\} \preceq [\mathcal{E}_1\ \mathcal{E}_2\ ...\ \mathcal{E}_U]^* = \boldsymbol{\mathcal{E}}_{\boldsymbol{x}}$$

$$\boldsymbol{b}_n \in \mathcal{A}_n(\bar{H}_n, \{R_{\boldsymbol{XX}}(u,n)\}_{u=1,...,U} \quad .$$

admMAC program

```
function [FEAS_FLAG, bu_a, info] = admMAC(H, Lxu, bu, Eu, cb)

 admMAC determines whether the target rate vector bu is
 feasible for (noise-whitened) channel H and energy/symbol Eu via rate region

  Inputs:
    - H: Ly-by-Lx-by-N channel matrix. H(:,:,n) denotes the channel for
      the n-th tone.
    - Lxu: number of transmit antennas of each user. It can be either a
      scalar or a length-U vector. If it is a scalar, every user has
      Lxu transmit antennas; otherwise user u has Lxu(u) transmit
      antennas.
    - bu: target rate of each user, length-U vector.
    - Eu: Energy/symbol on each user, length-U vector.
    - cb=1 cplex bb, cb=2 for real baseband
  Outputs:
    - FEAS_FLAG: indicator of achievability. FEAS_FLAG=0 if the target
      is not achievable; FEAS_FLAG=1 if the target is achievable by a
      single ordering; FEAS_FLAG=2 if the target is achievable by
      time-sharing
    - bu_a: U-by-1 vector showing achieved sum rate of each user.
    - info: various length output depending on FEAS_FLAG
      --if FEAS_FLAG=0: empty
      --if FEAS_FLAG=1: 1-by-5 cell array containing
        {Rxxs, Eun, bun, theta, w} corresponds to the single vertex
      --if FEAS_FLAG=2: v-by-6 cell array, with each row representing
        a time-shared vertex {Rxxs, Eun, bun, theta, w, frac}
```

```
info's row entries in detail (one row for each vertex shared
    - Rxxs: U-by-N cell array containing Rxx(u,n)'s if Lxu is a
      length-U vector; or Lxu-by-Lxu-by-U-by-N tensor if Lxu is a
      scalar. If the rate target is infeasible, output 0.
    - Eun:  U-by-N matrix showing users' transmit energy on each tone.
      If infeasible, output 0.
    - bun: U-by-N matrix showing users' rate on each tone. If
      infeasible, output 0.
    - theta: U-by-1 Lagrangian multiplier w.r.t. target rates
    - w: U-by-1 Lagrangian multiplier w.r.t. energy constraints
    - order: U-by-1 user order
    - frac: fraction of dimensions for each vertex in time share (FF =2
      ONLY)
 subroutines called (directly)
  maxRMAC_cvx
  maxRMACMIMO
```

- Both energy and rate are inputs for MAC.
- FEAS_FLAG = 0 if not admissible.
- Info is similar to minPMAC.

Back to simple channel, pick pt on boundary

H = [80 60];
>> bu_min = [1.72;4.92];
>> Eu = [1;1];
Lxu=[1 1];

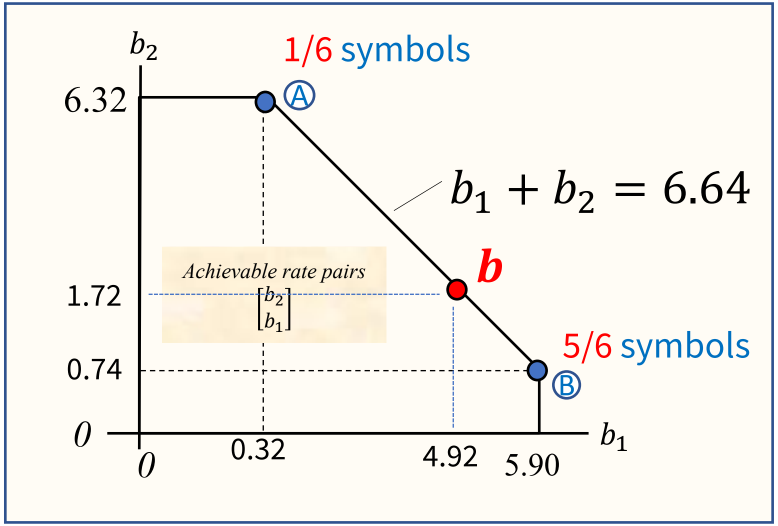
```
[FEAS_FLAG, bu_a, Rxxs, Eun, theta, w,
info] = admMAC(H, Lxu, bu_min, Eu, cb);
```

flag = 2
bu_a =
1.7210
4.9229
info = 2x8 table

bu_v		Rxxs	Eun	bun	theta	order	frac	cID
6.322	.32189	{2x1}	{1x2 }	{1x2}	{1x2}	{1x2}	0.17621	1
.73684	5.9071	{2x1}	{1x2}	{1x2}	{1x2}	{1x2}	0.82379	1

>>Rxxs % 2 × 1 cell array same both vertices
{[1]}
{[1.0000]}>> info.Eun{:,:}
> Eun = % same both vertices
1.0000
1.0000
>> theta =
0.9272
0.917
>> w =
0.5969
0.3302

>> info.bun{:,:} =
6.3220 0.3219 % top vertex
0.7368 5.9071 % bottom vertex
>> info.order{:} =
2 1
1 2
]
>> info.bu_v'*info.frac =
1.7210
4.9229 (vertex-sharing checks)




```
[FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] =
admMAC(H, Lxu, bu_min+[1 ; 1], Eu, cb);
```

flag = 0

```
[FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] =
admMAC(H, Lxu, bu_min+[1 ; 1], Eu, cb);
```

flag = 0

Confirms point is on boundary

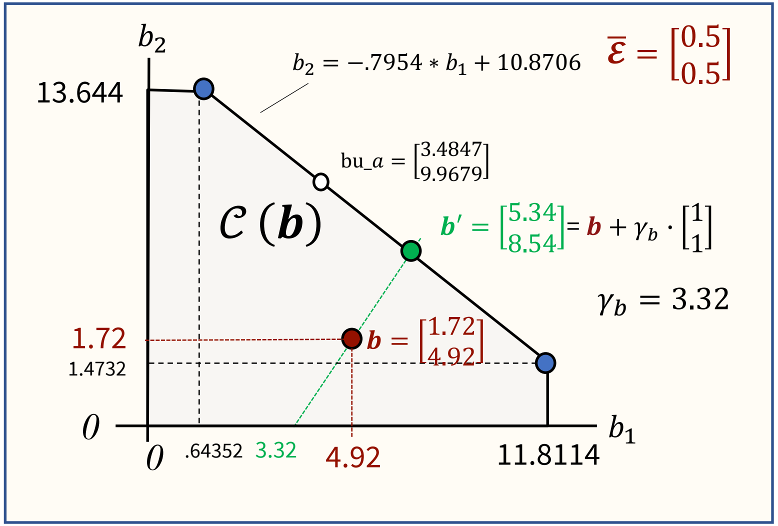
- FEAS_FLAG = 2, vertex sharing
  - One vertex has equal thetas is enough

Complex channel, same rate pt – NOT on boundary

```
>> [FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] = admMAC(H, Lxu, bu_min, 2*Eu, 1);
flag =    2
>> bu_a =
   3.4847
   9.9679
>> info.bu_v =            info.frac
  1.4732   11.814          .83472
  13.644   .64352          .16528
>> Eun =
   2.0000
   1.9989
```

- FEAS_FLAG = 2, vertex sharing

```
>>info.order{:} =
   1   2
   2   1
>> info.bu_v =
   1.4732  11.8142
  13.6440   0.6435
>> slope=(info.bu_v(1,1)-info.bu_v(1,2))/…
(info.bu_v(2,1)-info.bu_v(2,2)) =   -0.7954

>> int=info.bu_v(1,1)-slope*info.bu_v(1,2) =
  10.8706
>> rsum=sum(bu_a)=   13.4526
>> gammab=(rsum-bu_min(2)-bu_min(1))/2 =
        3.4063
```



γ_b = 3.4 or roughly 10.2 dB (complex so x 3 dB)

```
[FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] = admMAC(H, Lxu, bu_min +4*[1; 1], 2*Eu, 1)
flag =    0
[FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] = admMAC(H, Lxu, bu_min +3.4*[1; 1], 2*Eu, 1)
flag =    0
[FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] = admMAC(H, Lxu, bu_min +3.3*[1; 1], 2*Eu, 1)
flag =    2
bu_a' =    5.2878    8.6585
sum(bu_a) =   13.9464
```

```
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H, bu_min, [1 1], 1)
Eun =
   0.0109    0.0081
theta =
   0.0312    0.0385
bun =
   1.7200    4.9200
FEAS_FLAG =    1
bu_a =
   1.7200
   4.9200
```

FEAS_FLAG = 1, single vertex
Lower sum energy
About 20 dB, so

Equal energy weight used here; others likely also will work.

Preferred Design

admMAC

- There can be multi-solution vertex-sharing when in the interior of $\mathcal{C}(\boldsymbol{b})$.
  - Slope (or hyperplane normal vector) is not necessarily = -1 $(\cdot \boldsymbol{1})$.
- minPMAC can use any $\boldsymbol{w}$**,** including all equal (so energy sum), but does not guarantee a point in $\mathcal{C}_{\boldsymbol{b}}(\boldsymbol{\mathcal{E}})$**.**
- minPMAC may be preferred design method as it tends to produce larger margin.
  - Unless any user's energy is too large.
  - Then use admMAC , judiciously with the energies found – limiting any user energies that exceed allowed amounts.
- If admMAC does not work, use it's $\boldsymbol{w}$ in minPMAC, try again the cycle.
- If the first admMAC run does produces FEAS_FLAG = 0 , at least one user data rate is too high.
- Could do similar cycle with maxRMAC, and use admMAC's $\boldsymbol{\theta}$ as admMAC input for the generated $\boldsymbol{b}$.

Two users, high pass and low pass

- A past 2-user MAC with memory (user 2 at 1+.9D and user 1 at 1-D)

```
>> H=zeros(1,2,8);
>> H(1,1,:)=fft([1 .9],8);
>> H(1,2,:)=fft([1 -1],8);
>> H=(1/sqrt(.181))*H;
>> b=[1 ; 1];
>> energy=[8 ; 8];
```

```
[FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] = admMAC(H,[1 1], 18*b,
(8/9)*energy,1)
FEAS_FLAG =     2
bu_a' = 19.6385          19.6385
info =  2x7 table
       bu_v            frac
_______________    _______
 21.698    17.579    0.5814
16.778    22.499    0.4186
>> buntop=info.bun{1,:,:}; bunbot=info.bun{2,:,:};
>> reshape(buntop,2,8) =
 5.2089    4.9796    3.2601    0.0047    0.0000    0.0047    3.2601    4.9796
      0    0.0001    1.0629    5.0737    5.3058    5.0737    1.0629    0.0001
>> reshape(bunbot,2,8) =
5.2089    4.9768    0.8077    0.0001    0.0000    0.0001    0.8077    4.9768
      0    0.0028    3.5152    5.0783    5.3059    5.0783    3.5152    0.0028
>> Eun =
    1.8043    1.7937    0.8580    0.0011    0.0006    0.0011    0.8580    1.7937
    0.0005    0.0006    0.9443    1.7381    1.7447    1.7381    0.9443    0.0006
>> sum(Eun,2)' =    7.1105    7.1111
```

```
[Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H,18*b, [1 1]',1)
Eun =
    1.3437    1.3352    0.3090    0.0000    0.0000    0.0000    0.3090    1.3352
    0.0000    0.0000    0.9849    1.3020    1.3098    1.3020    0.9849    0.0000
theta =    2.7878
           2.7100
bun =
    4.7970    4.5693    2.0322    0.0000    0.0000    0.0000    2.0322    4.5693
         0    0.0000    1.8721    4.6758    4.9043    4.6758    1.8721    0.0000
FEAS_FLAG =     1
bu_a =   18.0000
         18.0000
>> sum(info.Eun{:,:},2)' = 4.6321     5.8835
>> 64/9 =     7.1111
>> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H,18*b+[1 ;1], [1
1]',1)
sum(info.Eun{:,:},2)' =     5.4922     7.1035
```

Margin is 3dB
For b=[18 18]'

64-tone Example (3 users)

```
>> [FEAS_FLAG, bu_a, Rxxs, Eun, theta, w, info] = admMAC(H64, [1 1 1], [410 390 210]', 64^2/67*[1 1 1], 1)
Elapsed time is 476.254513 seconds.    flag =    2
bu_achieved =  410.3922  390.3730  210.2009
info =  3×8 table
           bu_v                     Rxxs      Eun             bun          theta      order       frac     clusterID
   481.11   386.38   143.47   {1×64}   {1×3x64}   {1×3×64}   {1×3}   {1×3}    0.72759       1
   385.85   207.15   417.96   {1×64}   {1×3x64}   {1×3×64}   {1×3}   {1×3}   0.021875       1
   207.15   417.96   385.85   {1×64}   {1×3x64}   {1×3×64}   {1×3}   {1×3}    0.25054       1
>> info.bu_v'*info.frac =  410.3922  390.3730  210.2009
>> sum(cell2mat(info.Eun),3) =
  61.1343   61.1343   61.1343
  61.1343   61.1343   61.1343
  61.1343   61.1343   61.1343
```

```
[Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H,[410 390 210]', [1 1 1], 1);
theta =   2.8865    2.8865    2.8739
FEAS_FLAG = 2
bu_a =  410.0000  390.0000  210.0002
theta =   2.9830    2.9830    2.9830
info =  2x6 table
           bu_v                Eun                  bun                  theta             frac          clusterID
   410.24    389.76    210  {1×3×64 double}  {1×3×64 double}  {1×3 double}   0.98818       1
   390.02    409.98    210  {1×3×64 double}  {1×3×64 double}  {1×3 double}   0.011823      1
>> Eun=info.Eun{1,:,:};  >> sum(Eun(1,:,:),3) =   59.5706   56.2791   66.2707
>> >> [Eun, theta, bun, FEAS_FLAG, bu_a, info]=minPMAC(H,[410 390 210]', [1.03 1 1.08], 1) bu_a' =  410
390  210
info =  2x7 table
          bu_v                    Eun             bun          theta      order      frac      clusterID
 410    394.28    205.72    {1×3×64}   {1×3×64}   {1×3}   {1×3}   0.98208       1
 410    155.42    444.58    {1×3×64}   {1×3×64}   {1×3}   {1×3}   0.017919      1

>> sum(Eun(1,:,:),3) = 60.9146   60.3152   61.0854  😀
```

- Design might ignore one of the vertices – do ¾ share, other 2.

- Solution is close to admMAC solution,
- and maybe just 1 vertex.

MAC Capacity Region

- Use admMAC in U-loop that gradually increases b_u's (for all $u = 1, \ldots, U$) until admMAC returns negative value and record the last rate pair (the last before that violation happens).

- This is the exterior of $\mathcal{C}_{MAC}(\boldsymbol{b})$.

STANFORD

End Lecture 15