Homework Help - Problem Set 4

[**Multiuser Channel Types**] **(2.21)** Multiple-Access Channels (MACs) have only 1 receiver location, and all transmitters are located in different "locations." The word location has somewhat broad interpretation and could simply mean that any MAC users' transmitter cannot use the other users' input messages in any way; although loading strategies (like how much energy, bits/dimension, etc) can be shared and a common (subsymbol) clock effectively exists. MAC receivers may use/process all channel outputs. Broadcast Channels (BCs) instead reverse the situation and any receiver cannot access the channel output of other receivers. BC's have one transmitter location where all input messages can be shared. Interference Channels (ICs) have both restrictions. A single-user channel allows transmitter access to all messages and receiver access to all channel outputs.

Problem 2.21 attempts to illustrate specific examples of basic multiuser types. This should be simple.

a. The OLT is the common side where signals can be shared.

b. How many users are their really in each direction?

c. Nothing is colocated.

[**MU Detector Multiuser Margin**] **(2.22)** Problem 2.22 attempts to illustrate specific examples of basic multiuser types. This should be simple.

a. Check the two vertices - do they have the same rate sum?

b. This is essentially single-user caclulation.

c. C' is on the graph. This is basic geometry, but also tells the point at which multiuser margin can be computed. remember γ_b is the same for both users.

d. This should be easy, but recall the channel is complex baseband.

e. Use the well-known uncoded gap for this given P_e. Does that exceed your answer for the previous question?

[**Mutual-Information Vector**] **(2.23)** The minimum information vector is this course's specific way of addressing the general multi-user capacity region. IMPORTANT - It is a function of both the joint probability distribution $p_{\boldsymbol{x}\,\boldsymbol{y}}$ and the channel order $\boldsymbol{\pi}$. Thus, there can be a lot of $\mathcal{I}+min$'s that correspond to different order choices and different channels/input distributions. This course simplifies capacity-region construction (which some researchers state remains unknown, although the instructor sees no fault in the constructed capacity regions and indeed finds them as this course progresses, so until proven otherwise, it is the capacity region - in general). The constructed regions provide excellent performers for designs.

The calculation of each mutual information for each receiver and each order, and again each probability distribution, could be tedious in general. So, Lecture 7 and one homework problem introduce the idea of tables to list the orders and associated mutual information quantities, given previous users presumed decoding in the specified order. Once the table is known, the search simply finds for each user the minimum rate over all places where it must be decoded (because it is the user of interest or is given prior to the user of interest's decoding). This minimum is limiting for the given order matrix and probabilities. While perhaps overly conservative, it allows the construction of achievable regions by searching over all orders (and combinations of orders in the usual convex way, which correspond to using more than 1 order for some fraction of available time). That convex-hull process "stretches" out the achievable region to all the possible orders, given the joint distribution. The final step then cycles the $\mathcal{I}_{min}$ cacluation and consequent achievable region generation $\mathcal{A}$ through all the allowed input distributions, taking possible convex-hull combinations of these. Any of the data rates is certainly reliably achievable; any point outside this region inevitably must have a least one user running too fast than can be decoded at least at one receiver that needs it.

The reader is further referred to the example in L7 with the appropriate tables and minimization process.

a. This should be clear from the Table size.

b. The prior users are those that are decoded before the user associated with the receiver.

c. The minimum information vector requires search for lowest rate that must be decoded for this given order and input distribution for each and every user.

[**TDD Multiuser Channel**] (2.24) Time-Division Multiplexing (TDM) finds heavy use in practice for many multi-user systems, not just the MAC. Indeed, it is a form of dimension sharing, or convex-hull, of different solutions for a multiuser channel. However, the assignment of each user to its own unique dimension set is not necessarily optimum in general. Often, simultaneous energization of the same dimension with multiple users "does no harm" in that one of the users can be successfully decoded without penalty to the other. Nonetheless, the homework looks at some simple aspects of TDM in general.

Clearly, TDM shares the energy constraint over the users by its very nature, although the capacity of any user is limited by its time of use (and the rate sum would simply be the sum of users' rates with each multiplied by its fraction of occurrence). This can reduce the number of MAC capacity-region "faces" and vertices since the chain-rule points effectively go away. The homework 2.25 (PS4.3) finds those reduced numbers.

Since dimensions are a resource and considered equivalent in this course, and practically in communication, similar statements could apply to frequency division modulation (FDM) and space-division multiplexing (SDM).

a. What is the energy sum of the users on any use?
b. This is also somewhat trivial for TDD.
c. Hint - at least one of the faces is a triangle.
d. This follows easily from the fact that only one user at a time can be energized.
e. Time sharing of the vertices is implied here.
f. This as small as it gets in 3 dimensions.
g. The more general number of faces is in the text.
h. Dimensions in frequency or time on an ISI-free channel are essentially just mathematical indexing.

Vector Gaussian MAC help (2.25) Problem 2.25 is similar to many exercises in the notes, but in particular has a secondary user that may be energized. The problem simply reinforces the MAC receiver design from notes and lectures, so look to those examples, but try to do it yourself step by step to gain insight to each step as you do it.

Problem 2.25 is meant to challenge intuition a bit, and probably the most difficult on this assignment. Fortunately, the needed integral is provided. Note that at sampling rate 1, the *sinc* functions given are zero at all sampling instants except at sample time 0. This makes the discrete-time Fourier Transform essentially just a repeat of the continuous-time brick-wall Fourier Transform of *sinc*. The corresponding frequency responses are easily then determined in matlab. For instance, the sequence [11] has transform $1 + e^{\jmath\omega}$. Each user water-filling to the same level in its own (separately occupied) band is certainly SWF if a common water-fill level can be found for all.

a. For the matrix AWGN, this arises from the size of the matrix H.
b. The given factorization makes the square root easy. Find the noise-whitened channel.
c. Is the channel rank less than the number of users? If so, it is degraded.
d. The MAC rate sum follows a simply MMSe-like formula that is $\frac{1}{2} \cdot \log_2 \frac{|R_{\boldsymbol{yy}}|}{|R_{\boldsymbol{nn}}|}$.
e. You find 3 two-user pentagons for this, which should develop your skill in getting MAC regions.
f. Somewhat tedious, but again improving facility with rate regions.
g. Use the fact that the channel is degraded to simplify this part.
h. Again, the degraded channel simplifies the amount of calculation.