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ABSTRACT

We define a family of high-rate binary LDPC codes for ADSL. We present simulation results that show the performance that can be achieved by these codes on a binary additive white Gaussian noise (AWGN) channel. LDPC codes exhibit performance very close to the capacity limit with moderate decoding complexity and are thus well suited for channel coding applications in ADSL systems.

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1. Introduction

At the last SG15/Q4 rapporteur meeting, we presented an introduction to low-density parity-check (LDPC) codes, described their decoding by a message-passing algorithm also known as the sum-product algorithm, and discussed performance and implementation aspects [1]. The simulation results of [1],[2] confirmed that LDPC codes and turbo codes offer similar coding gains for multilevel transmission. LDPC codes achieve asymptotically an excellent performance without exhibiting "error floors" and admit a wide range of trade-offs between performance and decoding complexity. For these reasons, they represent an alternative to turbo codes for DSL transmission.

In this contribution, we describe the LDPC codes that are being proposed for use in ADSL systems. The LDPC codes employed in [1] were obtained via a random construction [3]. Here, we define a family of high-rate binary LDPC codes that are obtained by a deterministic construction. We present simulation results that show the performance that can be achieved by these codes on a binary additive white Gaussian noise (AWGN) channel.

In a companion contribution [4], we define multilevel encoding based on binary LDPC codes and address the set of requirements listed in [5] and [6] for ADSL transmission.

2. Construction of LDPC codes

High-rate LDPC codes appear to have certain advantages over convolutional or turbo-codes in communications and storage applications [7]. LDPC codes, when designed properly, can outperform turbo-codes. For example very long LDPC codes which are within 0.0045 dB of the Shannon limit have been constructed in [8]. Furthermore, LDPC codes appear not to suffer from error floors at bit-error rates of 10^{-8}. The sparseness of the parity-check matrices of LDPC codes results in decoding algorithms that are competitive in terms of complexity compared to serially or parallel concatenated turbo codes. Finally, no interleaver is needed between the LDPC encoder and the channel because interleaving can be implicitly incorporated into the LDPC code.

A binary LDPC code is a linear block code [9], [3] described by a sparse parity-check matrix H, i.e., H has a low density of ones. The class of LDPC matrices must satisfy the following regularity constraint: each column contains a small fixed number j of ones and each row contains a small fixed number k of ones. Equivalently, an LDPC matrix with M rows and N columns can be described by a bipartite graph with two kinds of nodes: there are N symbol nodes, which correspond to each bit in the code, and M check nodes, which correspond to the parity checks represented by the rows of the matrix. The connectivity of the graph is such that the parity-check matrix H is the incidence matrix of the bipartite graph. The regularity constraint implies that each symbol node is connected to j check nodes and each node is connected to k symbol nodes.

Codes defined by graphs can be decoded by the sum-product algorithm (SPA). However, the SPA performs well and essentially achieves the performance of a maximum-likelihood decoder only for graphs without short cycles and, therefore, one requires that the graph has no 4-cycles. This 4-cycle-free condition translates into the condition that the corresponding parity-check matrix has no two rows that have overlapping ones in more than one position. High-rate 4-cycle free LDPC matrices only exist if the rate satisfies a combinatorial bound, viz., for a given number M of rows, the block length N is upper bounded by

$$N \le \frac{M(M-1)}{j(j-1)} .$$

Binary high-rate LDPC codes without 4-cycles can be constructed by randomly generating columns, which contain exactly j ones. The parity-check matrix H is built up iteratively by adding a new column if this column does not form a 4-cycle with the previously generated columns of H. In general, this method does not provide matrices with a fixed row weight.

An alternative method was proposed by Gallager [9] where the M%N matrix H is built form j sub-matrices of size (M/j)%N, which are themselves LDPC matrices with column weight one and row weight k. In particular, M must be a multiple of j.

These random constructions provide LDPC codes with reasonable distance properties. An alternative to these random constructions are deterministic constructions, which are easy to obtain and lead to codes that can be simply defined via a small number of parameters.

One approach to the deterministic construction of LDPC codes is based on "array codes." Array codes are two dimensional codes that have been proposed for detecting and correcting burst errors [10]. The definition of array codes has an algebraic structure analogous to Reed-Solomon codes with operations defined on rings as opposed to Galois fields. When array codes are viewed as binary codes, their parity-check matrices exhibit sparseness which can be exploited for decoding them as LDPC codes using the SPA or low-complexity derivatives thereof. Therefore, array codes provide the framework for defining a family of LDPC codes that lend themselves to deterministic constructions.

We will define LDPC codes by three parameters: a prime number p and two integers k and j such that k,j [p. Let H be the jp % kp matrix defined as:

$$H = \begin{bmatrix} I & I & I & \cdots & I \\ I & \alpha & \alpha^2 & \cdots & \alpha^{k-1} \\ I & \alpha^2 & \alpha^4 & \cdots & \alpha^{2(k-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & \alpha^{j-1} & \alpha^{2(j-1)} & \cdots & \alpha^{(j-1)(k-1)} \end{bmatrix},$$

where I is the p % p identity matrix and α is a p % p permutation matrix representing a single left or right cyclic shift. For example:

$$\alpha = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad \alpha = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The above parity-check matrix gives rise to a sparse matrix that can be decoded using the SPA algorithm. Furthermore, this parity-check matrix is 4-cycle free by construction. In the case where k = p, we have array codes as defined in [10]. The case k < p corresponds to truncated array codes. Furthermore, the parameters j and k provide by construction the column and row weight of the LDPC H matrix, respectively. It is important to emphasize the similarity of the H matrix to the parity-check matrix of Reed-Solomon codes.

In Table 1, we give LDPC codes based on the array construction mentioned above that are appropriate for ADSL transmission.

	p	**j**	**k**	**(N,K)**	**Rate K/N**
Code 1	23	3	12	(276,209)	0.7572
Code 2	23	3	23	(529,462)	0.8733
Code 3	37	3	37	(1369,1260)	0.9204
Code 4	47	4	47	(2209,2024)	0.9163
Code 5	67	5	67	(4489,4158)	0.9263
Code 6	89	6	89	(7921,7392)	0.9332

Table 1: Definition of LDPC code parameters.

3. Performance on a binary AWGN channel

The performance of LDPC codes obtained via the construction described in the previous section was studied by simulation assuming an AWGN channel and binary transmission and compared to the performance of comparable length codes by MacKay obtained via a random construction. Iterative decoding with the sum-product algorithm was employed. The results are shown in Fig. 1 to 3 in terms of bit-error rate (BER), block-error rate (BLER) and corresponding channel capacities (CapB and C_Bck, respectively). The figures also indicate the BER for uncoded binary transmission.

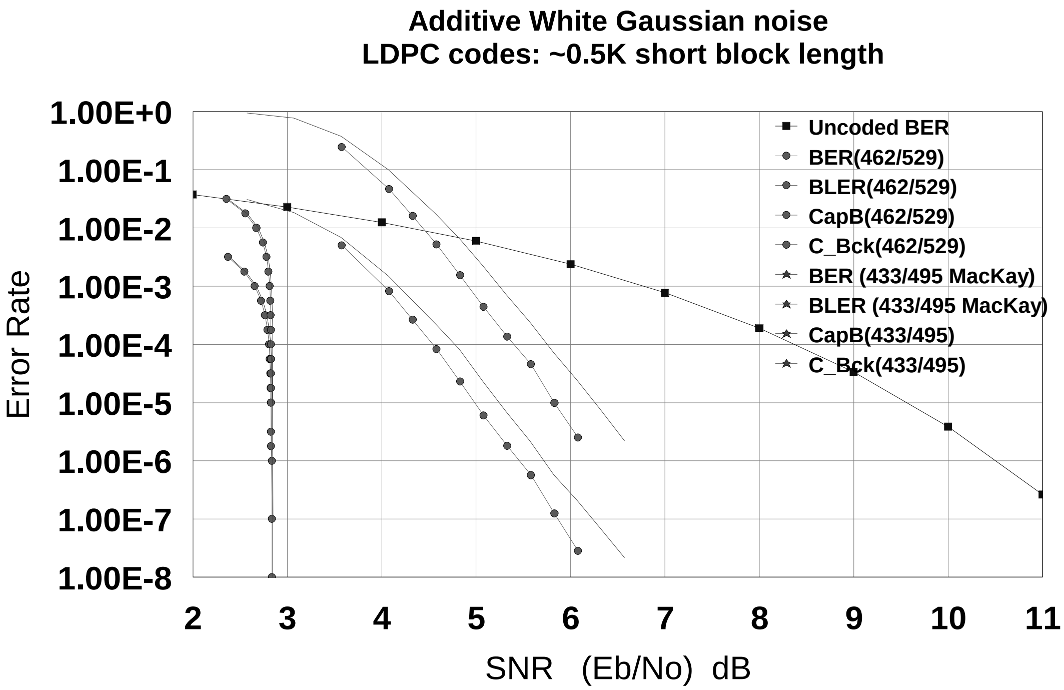

Fig. 1: Performance of LDPC codes for binary transmission over an AWGN channel: short block length.

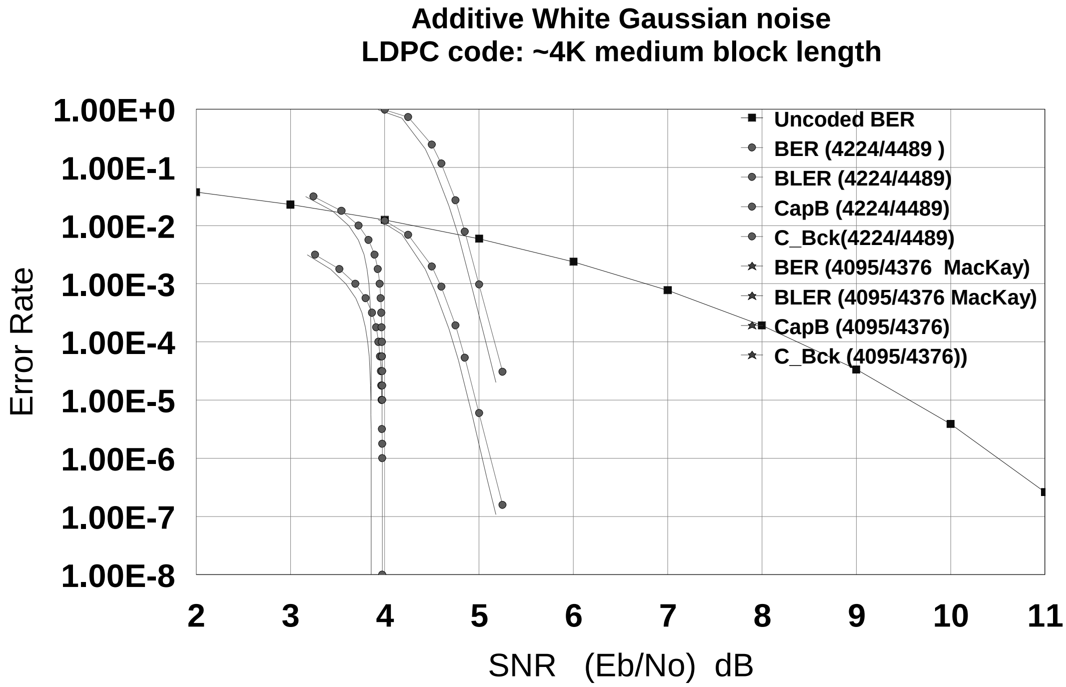

Fig. 2: Performance of LDPC codes for binary transmission over an AWGN channel: medium block length.

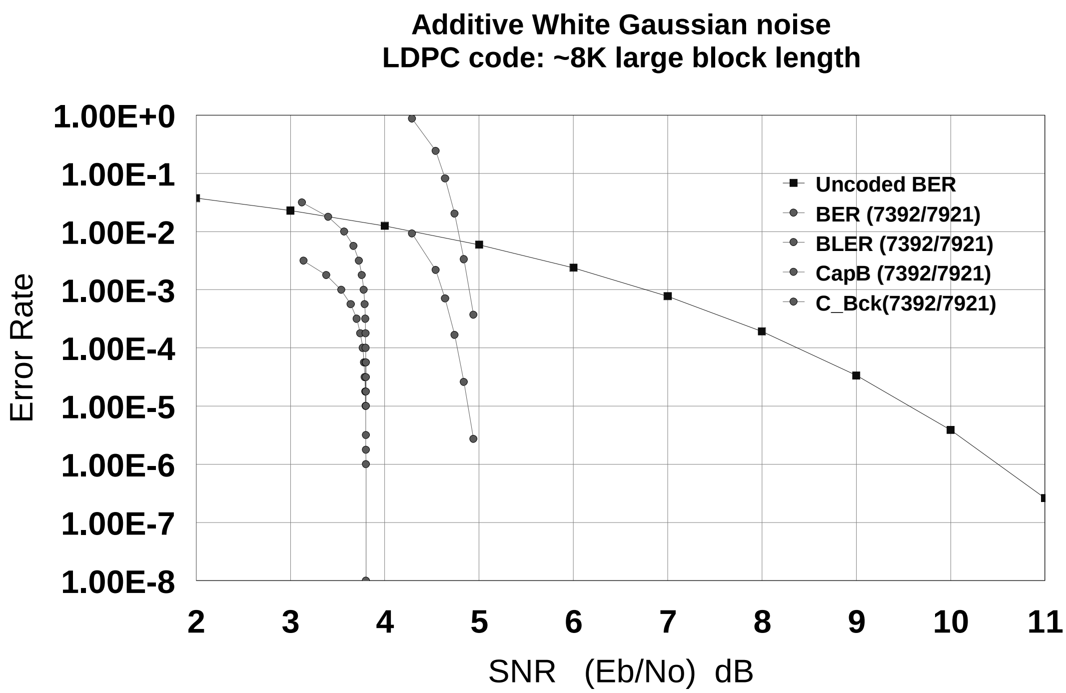

Fig. 3: Performance of LDPC codes for binary transmission over an AWGN channel: long block length.

4. Summary

We have presented a family of LDPC codes to be used for ADSL transmission. Simulation results for binary transmission over an AWGN channel show excellent performance. Performance for multilevel transmission is addressed in a companion contribution [4].

We therefore propose to include LDPC coding as an advanced coding technique in G.dmt.bis and G.lite.bis.

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