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Abstract

This contribution proposes to use Multi-Tone Turbo Trellis Coded Modulation (MTTCM) technique as an option for the forward error correction in ADSL modems. The contribution describes the encoder structure and the encoding rules for all constellation sizes.

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I. Encoder structure of Turbo code

Turbo codes [1] have an impressive near-Shannon limit error correcting performance. The superior performance of Turbo codes over convolutional codes has raised a lot of interest in the research community to find new applications for Turbo codes such as deep space communications [2-3], bandwidth efficient Turbo trellis coded modulation (TTCM) [4-7], CDMA mobile radio systems [8] and PCS [9] applications. In this paper, we present a new application for Turbo codes in systems that utilize multi-tone modulation techniques. In particular, the technique proposed in this contribution is suitable for Asymmetric digital subscriber line (ADSL) modems with discrete multi-tone (DMT) technology.

Figure 1 illustrates the structure of the Turbo encoder. The systematic data, d_k, and two parity bits from the output of the recursive systematic convolutional (RSC) encoders, y_k^1 and y_k^2, are the outputs of the Turbo encoder. The two RSC encoders are in a parallel structure similar to [1]. The length of memory for each RSC encoder is v and consequently, the total number of states for each decoder is $M = 2^v$. The received systematic data and parity bits from the i^{th} encoder are represented as d_k^r and $y_k^{r_i}$ respectively.

For the specific application of the Turbo code to ADSL, we propose block lengths of N =2176,1088,544, and 272. These block lengths are chosen so that they are an integer multiple of the number of DMT frames (68) in one ADSL super-frame. Moreover, if the Reed-Solomon (RS) encoder block length is chosen to be a multiple of this block size (N), the Turbo block length also becomes an integer multiple of the RS encoder block size. Thus the Turbo block boundaries can be made to coincide with the super-frame and RS block boundaries.

An N-bit interleaver or permuter separates the two RSC encoders. Thus, the input to the second encoder is the interleaved version of the input information data. If we transmit the data as shown in figure 1, the encoder is rate $\frac{1}{3}$. There is an option of puncturing the parity bits to achieve different coding rates. The role of the interleaver in the performance of the Turbo code is important especially when the Turbo block size (N) is small. The significance of the interleaver design is discussed later. In this approach, we recommend an additional interleaver to be used for the systematic input data path. Each RSC encoder has the generator matrix $[1, \frac{g_{FF}(D)}{g_{FB}(D)}]$ where $g_{FF}(D)$ and $g_{FB}(D)$ are the feed-forward and feedback polynomials respectively. The feedback polynomial is usually selected a primitive polynomial.

Figure 2 demonstrates the RSC encoder recommended for Turbo codes with feedback and feed-forward generator polynomials equal to 15_{oct} and 17_{oct} respectively.

Figure 3 shows the combination of Turbo encoder and multi-tone modulation for a system utilizing DMT technology. The Turbo encoder receives the information bits as input and it associates one

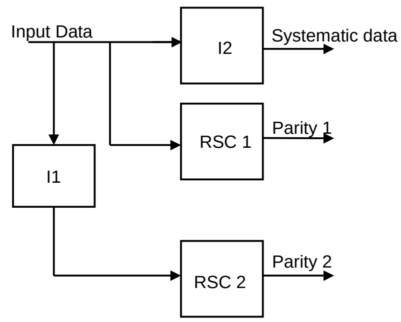

Fig. 1. Turbo encoder with additional interleaver.

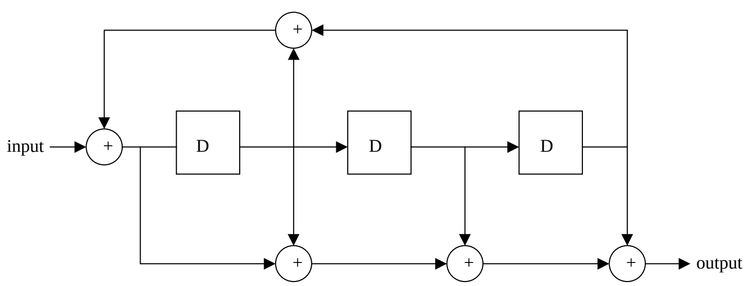

Fig. 2. RSC encoder for Turbo codes with generator matrix $[1, \frac{17_{oct}}{15_{oct}}]$.

interleaved systematic data bit and two parity bits for each information bit. The parity bits are partially selected and the remaining are punctured based on the information in the channel assignment controller (CAC). During the initialization of the modem, the CAC collects information about the noise spectrum and channel characteristics of each channel. Based on these measurements in the particular loop and the power mask and budget constraints, the CAC utilizes an algorithm such as water pouring algorithm and its associated bit-loading algorithm to assign a M-QAM modulation to each bin (channel). This initialization process determines the size of the constellation, M, for each channel. The CAC determines the appropriate combination of interleaved systematic data and punctured parity bits to map these bits into a complex value (I_k, Q_k). The assignment of bits to M-QAM symbols employ Gray code mapping. The CAC controls the mapping of bit sequences into M-QAM symbols by employing a rule that the mapping must assign the parity bits to the most protected bits and then assign the information bits to the remaining bits in the M-QAM constellation. The most protected bit is the one that has less probability of incorrect detection compared to the other bits. This can be defined in a symbol or in each dimension within a symbol. In Gray code mapping, the most protected bit in each dimension usually corresponds to the most significant bit. We recommend Gray code mapping for our proposal.

The second rule in assignment of the parity bits by CAC is to puncture equal number of parity bits from all the RSC encoders.

As explained earlier, each Turbo block contains N information bits. Also the total number of bits that can be transmitted during one frame in ADSL depends on the characteristics of the particular twisted pair wire that is carrying the signal. In general, there is no relationship between Turbo block size, N, and the number of bits loaded in each ADSL frame, N_f. N_f depends on the channel characteristics, i.e., the channel attenuation function and the noise and interference level in that particular line. CAC will ensure that the boundaries of the Turbo block are detected in the transmitter and the receiver and decodes each Turbo block accordingly.

The CAC also controls the flow of data in the receiver. In the receiver, we can compute the conditional probability of each transmitted bit directly from the M-QAM constellation. The details of the decoder will be discussed later.

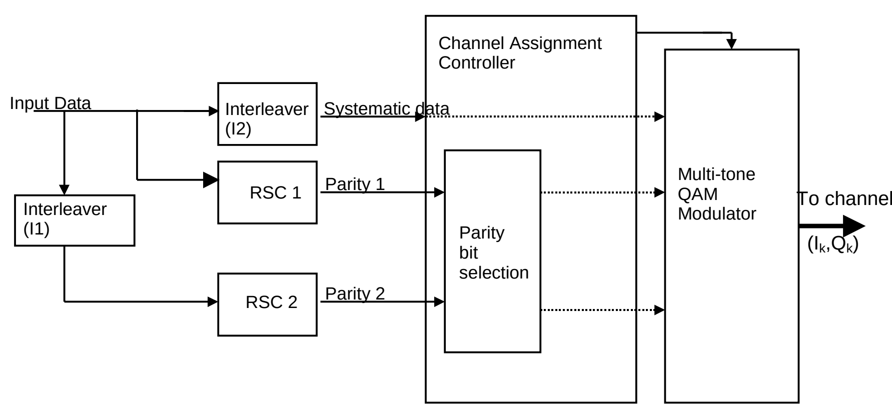

Fig. 3. Multi-Tone Turbo Trellis Coded Modulation (MTTCM) Encoder

A. Modulation for 4QAM and 1 bit/s/Hz spectral efficiency

In a 4-QAM constellation, we assign one information bit and one parity bit to each frequency bin. Since there are always two parity bits for one information bit, we puncture one parity bit from each encoder at each time interval. The code rate is $\frac{1}{2}$. Table I demonstrates the mapping and the puncturing pattern.

Figure 4 compares the performance of Turbo code to Wei code and uncoded modulation over the bit error rate (BER) range of $10^{-7} - 10^{-1}$. As it is clear from this figure, at the BER equal to 10^{-7},

information data d_k	d_1	d_2	d_3	d_4
parity y_k^1	y_1^1	-	y_3^1	-
parity y_k^2	-	y_2^2	-	y_4^2
4QAM symbol	(d_1, y_1^1)	(d_2, y_2^2)	(d_3, y_3^1)	(d_4, y_4^2)

TABLE I

MAPPING FOR 4QAM CONSTELLATION WITH RATE $\frac{1}{2}$

Turbo code has a 4.2 dB coding gain over Wei code and more than 8 dB coding gain over uncoded modulation. The decoding scheme used for the simulation of these results is the Simplified-Log-MAP algorithm and the interleaver design is 2-step S-random interleaver. Both of these techniques are described later in other contributions. Another point of this proposal in contrast to other Turbo Code designs, is that the recommended design does not exhibit an error floor at bit error rates (BER) equal to 10^{-7} for these interleaver sizes.

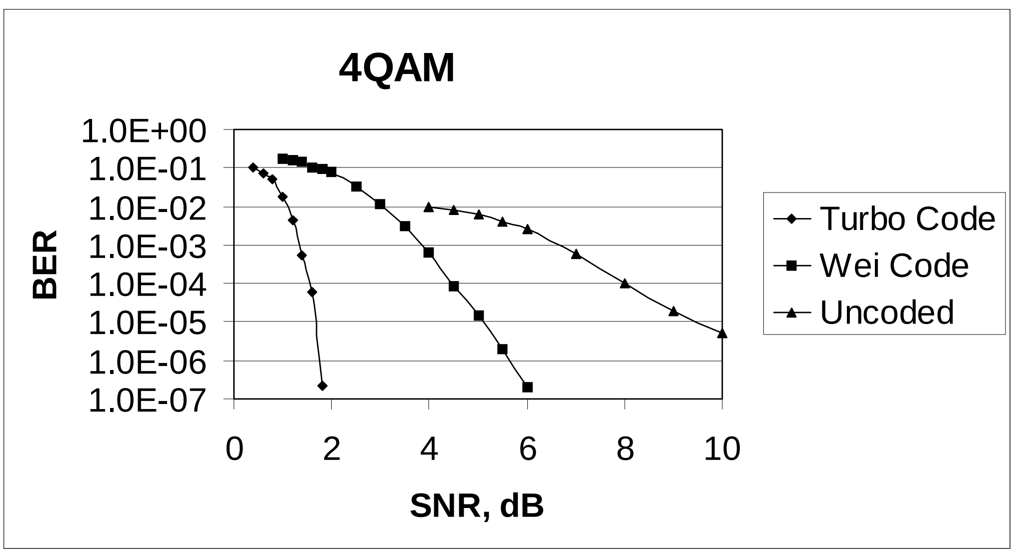

Fig. 4. Comparison between Turbo, Wei code and uncoded modulation for 4QAM constellation

Next figure compares the performance of Turbo code for 4QAM constellation with different interleavers. As we can see from this figure, 2-step S-random interleaver improves the performance of the Turbo code even compared with S-random interleaver and does not exhibit any error floor at very low BER. This is essential in utilizing any Turbo code approach. For this example, interleaver size of 1088 bits is used.

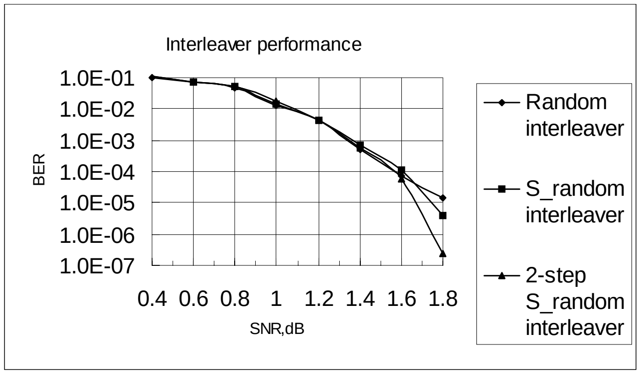

Fig. 5. Comparison between 2-step S-random interleaver and S-random and random interleavers for 4QAM constellation

B. Modulation for 16QAM and 2 bit/s/Hz spectral efficiency

In a 16QAM constellation and 2 bit/s/Hz spectral efficiency, we assign one information bit and one parity bit from each of two encoders to construct the complex symbol. For each symbol, we puncture one parity bit from each encoder and so the code rate for this code is $\frac{1}{2}$. Table II demonstrates the mapping and the puncturing pattern for this code.

information data d_k	d_1	d_2	d_3	d_4
parity y_k^1	y_1^1	-	y_3^1	-
parity y_k^2	-	y_2^2	-	y_4^2
16QAM symbol	(d_1, y_1^1, d_2, y_2^2)		(d_3, y_3^1, d_4, y_4^2)	

TABLE II

Mapping for 16QAM constellation with rate $\frac{1}{2}$

C. Modulation for 16QAM and 3 bit/s/Hz spectral efficiency

In a 16QAM constellation and 3 bit/s/Hz spectral efficiency, we assign three information bits and one parity bit from one of the encoders to construct the symbol. For each symbol, we puncture a total of five parity bits from the encoders and the code rate for this code is $\frac{3}{4}$. Table III demonstrates the mapping and the puncturing pattern for this code.

D. Modulation for 64QAM and 4 bit/s/Hz spectral efficiency

In a 64QAM constellation and 4 bit/s/Hz spectral efficiency, we assign four information bits and two parity bits to construct the symbol. For each symbol, we puncture a total of six parity bits from the

information data d_k	d_1	d_2	d_3	d_4	d_5	d_6
parity y_k^1	y_1^1	-	-	-	-	-
parity y_k^2	-	-	-	y_4^2	-	-
16QAM symbol	(d_1, d_2, d_3, y_1^1)			(d_4, d_5, d_6, y_4^2)		

TABLE III

Mapping for 16QAM constellation with rate $\frac{3}{4}$

encoders and the code rate for this code is $\frac{2}{3}$. Table IV demonstrates the mapping and the puncturing pattern for this code.

information data d_k	d_1	d_2	d_3	d_4
parity y_k^1	y_1^1	-	-	-
parity y_k^2	-	-	y_3^2	-
64QAM symbol	$(d_1, d_2, y_1^1, d_3, d_4, y_3^2)$			

TABLE IV

Mapping for 64QAM constellation with rate $\frac{2}{3}$

E. Modulation for 256QAM and 5 bit/s/Hz spectral efficiency

In a 256QAM constellation and 5 bit/s/Hz spectral efficiency, we assign five information bits and three parity bits from the encoders to construct the symbol. For each symbol, we puncture a total of seven parity bits from the encoders and the code rate for this code is $\frac{5}{8}$. Table V demonstrates the mapping and the puncturing pattern for this code.

information data d_k	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
parity y_k^1	y_1^1	-	-	-	y_5^1	-	-	y_8^1	-	-
parity y_k^2	-	-	y_3^2	-	-	y_6^2	-	-	-	y_{10}^2
256QAM symbol	$(d_1, d_2, y_1^1, d_3, d_4, y_3^2, d_5, y_5^1)$					$(d_6, d_7, y_6^2, d_8, d_9, y_8^1, d_{10}, y_{10}^2)$				

TABLE V

Mapping for 256QAM constellation with rate $\frac{5}{8}$

F. Modulation for 256QAM and 6 bit/s/Hz spectral efficiency

In a 256QAM constellation and 6 bit/s/Hz spectral efficiency, we assign six information bits and two parity bits from the encoders to construct the symbol. For each symbol, we puncture a total of ten parity bits from the encoders and the code rate for this code is $\frac{3}{4}$. Table VI demonstrates the mapping and the puncturing pattern for this code.

information data d_k	d_1	d_2	d_3	d_4	d_5	d_6
parity y_k^1	y_1^1	-	-	-	-	-
parity y_k^2	-	-	-	y_4^2	-	-
256QAM symbol	$(d_1, d_2, d_3, y_1^1, d_4, d_5, d_6, y_4^2)$					

TABLE VI

Mapping for 256QAM constellation with rate $\frac{3}{4}$

G. Modulation for 1024QAM and 8 bit/s/Hz spectral efficiency

In a 1024QAM constellation and 8 bit/s/Hz spectral efficiency, we assign eight information bits and two parity bits from the encoders to construct the symbol. For each symbol, we puncture a total of fourteen parity bits from the encoders and the code rate for this code is $\frac{4}{5}$. Table VII demonstrates the mapping and the puncturing pattern for this code.

information data d_k	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
parity y_k^1	y_1^1	-	-	-	-	-	-	-
parity y_k^2	-	-	-	-	y_5^2	-	-	-
1024QAM symbol	$(d_1, d_2, d_3, d_4, y_1^1, d_5, d_6, d_7, d_8, y_5^2)$							

TABLE VII

Mapping for 1024QAM constellation with rate $\frac{4}{5}$

H. Modulation for 4096QAM and 10 bit/s/Hz spectral efficiency

In a 4096QAM constellation and 10 bit/s/Hz spectral efficiency, we assign ten information bits and two parity bits from the encoders to construct the symbol. For each symbol, we puncture a total of eighteen parity bits from the encoders and the code rate for this code is $\frac{5}{6}$. Table VIII demonstrates the mapping and the puncturing pattern for this code.

information data d_k	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
parity y_k^1	y_1^1	-	-	-	-	-	-	-	-	-
parity y_k^2	-	-	-	-	-	y_6^2	-	-	-	-
4096QAM symbol	$(d_1, d_2, d_3, d_4, d_5, y_1^1, d_6, d_7, d_8, d_9, d_{10}, y_6^2)$									

TABLE VIII

Mapping for 4096QAM constellation with rate $\frac{5}{6}$

II. Summary

This contribution should be presented under the activity in G.gen.

We propose to add Multi-Tone Turbo Trellis Coded Modulation as an option for FEC in G.lite.bis and G.dmt.bis.

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