## Homework Help - Problem Set 7

**[DFE is even better (3.7)]** This problem explores generalizing the .9 in text/class examples to a general variable $a$. When $|a| << 1$, there is little ISI and as $|a| \to 1$, ISI worsens. ($|a| > 1$ actually starts to lessen ISI again, but that is not part of this problem.)

a. Use the results of Prob 3.6, which is PS6.3, and solutions are at the web site as of March 2. Check your/solutions' correct $Q(D) + 1/SNR_{MFB}$ and extract the $\gamma_0$, $r_1$ and $r_2$. Use $r_1 \cdot r_2^* = 1$.

b. The causal factor has the form $1 + r \cdot D$ where minimum phase choice causes a choice such that $|r| < 1$. So look at the two roots and use the one corresponds to an $|r| < 1$. Then $B(D)$ follows easily, and $W(D)$ uses the other $r$ choice in its denominator with the anti-causal factor.

c. The loss factor is a moderately complex function of $a$ and $b$, along with $\|h\|^2$, $\bar{\mathcal{E}}_x$, and $\sigma^2$. It sets up the ability to plot the function for given signal and noise energies. Recall that the parameter $b$ was defined as $b = \|h\|^2 \cdot (1+1/SNR_{MFB})$. As $|a|$ increases the loss should also increase. The largest $\gamma$ for this channel would be limited to that of the ZF-DFE, which is easily shown to be 3 dB, so your plot should not have losses greater than that number anywhere. It's also much better than the losses with the LEs.

**[Noise-Predictive DFE (3.8)]** The noise predictive DFE is equivalent to the DFE in performance, but finds an alternative implementation that leverages a linear equalizer's output, and then attempts to use previous decisions to predict the noise in the current sample from previous noise estimates. This particular equivalent DFE form might be useful with iterative GRAND decoding schemes that guess noise values. It also creates a performance-boost option to a system that nominally uses only a linear equalizer. Unfortunately, it usually requires longer filters and higher complexity, for the same performance. Thus, while the reader may encounter some with strong "noise-predictive" is better beliefs, this author has not found any situations where the usual DFE would not be simpler. Nonetheless, we include it as a homework problem to basically show the equivalence and find the filters as it helps the student understand the MMSE basics from another perspective.

a. One way to approach this problem is to recognize that the usual MMSE-DFE's FF filter is the cascade of a MMSE-LE and $B(D)$ (or optimally

$G(D)$). Try to see that by reorganizing the NP-DFE's filters/transfers a little.

b. This follows directly from the Part a hint above.

c. Again, what happens to the MMSE-DFE if we sub-optimally set $B(D) = 1$? (that is no feedback means what trivially?)

d. Returning to the original diagram, trace how the system is actually trying to predict the noise or error sequence, whence the name.

e. All MMSE systems are biased, no matter how we rearrange the filters; and the removal prior to the SBS decision is the same also.

[**Equalizer for Two-Band Channel with Notch (3.19)**] This is a simple version of a channel with an inband notch, which helps the student understand what an equalizer does in such situations.

a. Plot the channel magnitude versus frequency. You should get a notch right in the middle of the band,.

b. A notch is bad news for a linear equalizer, which essentially attempts to divide by zero at the notch frequency and enhances greatly the noise. Thus DFEs are clearly the desired solution here.

c. The ZFE-DFE for a channel that is already causal and monic is pretty easy. Use it's formula to quickly find the corresponding transmit energy required using the appropriate PAM constellation for this (consequently) real-baseband channel.

d. Now that we have the energy determined in last part, let's see what the improvement for MMSE is over ZFE with the DFE. You will need to use the matlab roots command for a polynomial that has 6 zero coefficients with its 9 values that are nonzero at only the end points and at time zero.

[**Finite-Delay Tree Search (3.23)**] This problem investigates finite-delay tree search, which as method for generating soft information as well as reducing error-propagation effects. It enhances the DFE structure to have a list of potential feedback inputs instead of just one. FTDS is used productively in various situations, especially so-called "turbo-equalization" systems where the equalizer is viewed as a constraint for iterative decoding (that you learned earlier in this class in Chapter 7, even though this problem is in Chapter 3 and precedes iteratively decoding in Chapter 7. The channel goes back to $1 + a \cdot D$ that by now you will be familiar.

a. Careful, $|a|$ is easy, but it is now complex with nonzero imaginary part.

b. This is straightforward use of MFB SNR formula.

c. Basically you need to take care with complex arithmetic, but this otherwise looks like what you already know well.

d. You are welcome to use Prob 3.6/3.7 results, which can make this easy.

e. Again, you can use the earlier problem's formulas here. Just the eventual numbers and performance will be different.

f. Now finally to the tree-search part. Since FTDS attempts to do a little more "sequence detection" than SBS, it necessarily must be at least as good and probably better. You should be able to lower bound its SNR by the ZF-DFE SNR.

g. Don't be afraid if you get maximum performance (subject to the always correct $\hat{x}_{k-2}$) presumption.

h. with $\nu > 1$, don't expect the MFB to be attained even with a most-delayed last-tap correctly decided. Think in terms of how much energy in total versus how much in the FDTS part that uses only 2 symbols. It's kind of like the ZF-DFE formula, just a little more elaborate for two taps.

**[Ifinite-Length EQ]** This problem addresses an infinite-length discrete-time channel. The $D$-transform-based formulas all still apply, but they can be pole-zero equalizers/filters.

a. The $\|h\|^2$ now involves an infinite discrete sum, but all follows. Your FF ZFE filter may consequently look pretty simple and FIR.

b. The MMSE-LE will no longer be purely FIR.

c. Similarly the ZF-DFE FF will look pretty simple, although this ignores the fact that the matched filter was implemented prior to the sampling device and may not be trivial. The FB section has infinite length.

d. The MMSE-DFE is pole-zero in both FF and FB sections, but does have a better performance as expected. Thus, the extra effort is paying for the MMSE, even though it may look simple to "just cancel the channel pole."