Homework Help - Problem Set 6

[Stanford EE379 Channel Model]

a. The solution is the convolution of the (normalized) transmit filter (or basis function), $\varphi(t)$, and the channel impulse response $h_c(t)$.

b. Use the the Parseval's like relationships or simply note the integral of the basis function has unit value. $\varphi_h(t)$ normalizes the pulse response to have unit norm.

c. The solution convolves the normalized pulse response with itself (conjugated) and reversed in time to create $q(t)$, which is a channel autocorrelation with $q(0) = 1$. Your answer should have (conjugate) symmetry about $t = 0$.

d. This last part allows you to see the ISI in discrete time.

e. The peak distortion finds the maximum 8 PAM value for $d = 1$ (not $d = 2$ so divide the usual values for PAM by 2), times $\|h\|$ (note not $\|h\|^2$) and the summed absolute values of the discrete time ISI values.

f. This part simply computes the MS distortion so you can compare. You need a square root on MS distortion to make a sensible comparison with peak distortion.

g. The AWGN power-spectral density is a variable σ^2, so it needs to be added to MS distortion before the square-root of the sum is take with MS analysis. Otherwise, it is plugging into the PAM formula, or you may find it easier to just use $d/2$ times the inverse square root of this sum.

[Bias and SNR]

a. The receiver is unbiased if $\mathbb{E}[\alpha \cdot y_k / x_k] = x_k$. Split the convolution of $\|h\| \cdot \sum_m x_m \cdot q_{k-m}$ into two parts for when $m = k$ and $m \neq k$. The bias should now be clear because the average of any value of x_{k-m} for $m \neq 0$ is zero. You may now insert values from Problem 3.3 (PS5.1). This value

b. Recall x_k and n_k are independent. Basically, you need to square $\alpha \cdot y_k - x_k$ and take the mean. Simplify and you should see the $\mathcal{D}_{MS}$ in your expressions as you do. You can plug in values then, and divide the result into $\mathcal{E}_x$ to get the unbiased SNR for the value of α you found in Part a. No one said the value in Part a provides a good SNR yet, which is the subject of the next part.

c. Differentiate with respect to α the answer for Part b and set to zero. This is a maximum for a single scalar α and the new SNR should be higher than the answer in Part b. You should see the familiar $SNR = SNR_u + 1$ arising if you do this correctly, albeit in a slightly different way than lecture or text.

d. Specifically, look at $\mathbb{E}[\alpha \cdot y_k / x_k]$ and show it is not equal to x_k.

e. This is straightforward and you should know by heart the SNR_{MMFB} calculation formula.

f. There is a strange result on ISI-free channels that the biased SNR exceeds the SNR_{MFB}, which simply reinforces that SNR_u is the one to use. The actually SNR_{MMSE} is often close if $SNR_u = SNR_{MMSE} - 1$ is large and the 1 can be ignored, and the latter can never exceed the matched-filter bound. There will be a new SNR later that actually can be achieved, but you don't need to know that here. It can be less than SNR_{MFB} but always no less than SNR_u and often considerably higher.

[**Noise Enhancement**] This problem explores the linear equalizers' difference as the ISI becomes more severe (larger $|a|$)

a. This is straightforward algebra on the ZFE and MMSE-LE formulas in lecture and text. Don't forget congugates if the channel might be complex baseband.

b. Using the results of the previous part to plot and you should see that the ZFE gets really bad for severe ISI while not far off for mild ISI, from the always better MMSE-LE.

c. This is algebra and quadratic equation, but tries to cause thought about the channel zeros size relative to SNR.

d. The equalizer basically cancels the channel's roots with high SNR and that should be dvident in your results.

e. The center tap should always be the largest here, and you are finding it's value as a function of $\|h\|$, a, and SNR. You may want to go back to time domain after a partial-fraction expansion of the equalizer.

f. This part has you evaluating how the MMSE and ZF approaches deviate as ISI becomes more severe.

[**ISI Quantificaton**]

a. This channel is studied in class and in the text, so much of the work is done already. For $\mathcal{D}_p$, find the q_k and note it has only 3 values. Only 2 contribute to peak distortion along wiht $\|h\|$ and $|x|_{max}$, which you can determine from constellation and from energy.

b. Here, the peak distortion includes the worst-case ISI from the two contributing terms and evaluates P_e (uncoded). It's almost a coin flip, so illustrates how bad peak distortion is as a measure on a simple channel.

c. The mean-square distortion is much less and your P_e will be under 0.1 now, still not good but 9 out of 10 bits getting through ok.

d. Neither system is very good yet, but the MS distortion is better.

Eventually, as you progress in this class, you will be able to make this channel play very well with no rate loss, even though it appears hopeless at this point.

[**Peak Distortion**] This is an easy question, so avoid complicating it. It basically shows the peak-distortion measure is not so bad as the ISI reduces from the 0.9 value to 0.5