## Homework Help - Problem Set 2

[PS2.1 32SQ QAM help (1.14)] Odd $b$ constellations improve the granularity of constellation information carriage, but somewhat increase complexity. Often wireless systems avoid odd bits/QAM-symbol. Wireline systems that often design to higher accuracy will use them. This problem tries to take advantage of the next largest even- $b$ 's constellation by examining selection of every other point.
a. Think of the two checker-boards that arise (black squares and red squares perhaps). Is there any difference in black and red, or are they symmetric and therefore should have the same properties ( $b$ and $\mathcal{E}_{x}$ ? This symmetry observation simplifies finding the energy, which you should be able to do quickly if you know the energy of the 64SQ, which you can get from the formula in class and notes, since you kn ow $d=2$.
b. What happens to the minimum distance squared when every other point is used? It should enlarge - by how much? This should be obvious.
c. The nearest neighbor computation $\left(N_{e}\right)$ averages the number of nearest neighbors. How many interior points have 4 nearest neighbors? Obviously, there are no points with more than 4 . What about 3? Are there any with only two common decision boundaries? Recall that $N_{e}$ counts a boundary even if it is not at minimum distance. Assume all the boundaries are at minimum distance, and then find the minimum distance.
d. 32 CR eliminates some corner points better than 32 SQ or 64 SQ , so you should expect it to be slightly better. To compare 32 CR and 32 SQ , first set the energies equal, and find the difference in minimum distances? You should realize the noise variance $\sigma^{2}$ does not change, which is why it is not specified here. This is purely a packing/shaping issue of 32 points. You should see an improvement, but it will take 3-digit accuracy to see it, so it is small.
e. Think about the two closest easy constellations surrounding 32 points you saw the larger one earlier in this problem. What about the smaller one if we again take every other point from 32SQ. While the average bits/symbol is the same, the overall average energy/symbol increases if $P_{e}$ remains the same (that is keep the same $d_{\min }$ in all). We'll see in Chapter 4 that the equal energy distribution on all symbols is optimum for this AWGN channel - variable energy is suboptimum. This part is just to show you (and you have to compute the different energy that is
not given in this helper to do it) that time-varying constellations ON AN AWGN are a bad idea. So for instance, if you had a wireless channel that someone tells you will vary, SOMETIMES when it is moving, but is otherwise a constant AWGN when not moving, so called "time-varying modulation" would not be a good idea when the channel is already good - it increases energy or reduces data rate.
[PS2.2 Shaping Gain (1.19) help] This problem focuses attention on the boundary of constellations and the consequent shaping gain. First you should look up the shaping gain formulae from Section 1.3. Then understand the use of energy and volume in both. For two dimensions, volume simplifies to area. So we try to compare packing points into a triangle, a circle, and a hexagon with this problem. The common reference for all is a unit-area square, so also find its energy - known as its "second moment." Your intuition should already be telling you which two are better than the square and which is worse. Now you just have to compute these shaping gains/losses. The problem provides formulas online for an equilateral triangle's, circle's, and hexagon's second moment and area. Thus, you don't need to do the double integral. A uniform distribution implies simply dividing by the area so no new integrals need be performed. There is some minor algebra to get the quantities needed, and then the parameter $a$ or $r$ needs to be selected so the area is the same for any two regions being compared. Use rectangular as the reference.
[PS2.3 Basic QAM Design (1.22) help] This program attempts to familiarize yourself with a simple design. The idea is to start to associated required $S N R \mathrm{~s}$ or Q-function input arguments with data rates and $P_{e}$ levels.
a. Find the formulas for the Q -function inputs with SQ are CR constellations (You can use your results from PS2.1 if you like also here as they may be convenient). You may reverse these formulas. Once you have the arguments, it is then possible to back-out the $M$ value and find the corresponding data rate. You may round if error is less than $2 \%$.
b. The data rate is the bits/symbol times the symbol rate, which is given.
c. Now we get a tough situation, we cannot really keep the same data rate. This problem is to make you aware of the granularity of bit rates. It might be nice to have some kind of fractional transmission (it's coming in different ways later with coding and also again in Chapter 4's multicarrier.)
d. The data rate will drop in this problem because of the more stringent error requirement.
[PS2.4 Hex Constellation (1.34) help] This program illustrates a little better packing of points, but also the complications it creates. The honeycomb packing is tighter, so more points can be squeezed into the constellation.
a. We actually have a time-varying modulator now, even though a previous problem (if worked correctly, hint, hint) showed time-varying modulators are not so wise on the AWGN. But this one is only slight variation. Count
the always-on points (shaded) - how many more symbol values would you need to get an integer number of bits? Is that happening? (The rotation of the extra point here basically zeros the constellation's average value. That thought might comfort some simpliflcations you make to proceed.
b. The data rate is the bits/symbol times the symbol rate, which is given.
c. This is not that hard in that most of them are already in the given figure. Just, what do the outer regions look like? For this and other constellations, these need not all look the same.
d. trivial.
e. The energy calculation is where it is comforting to know the average value is zero. Each of the points (some of them are the same) has an energy, so add them times their probabilities.
f. Each point also has a number of nearest neighbors, now exceeding what you were used to earlier. However, same basic process works.
g. The last two questions set this up. Remember that $\operatorname{SNR}=\overline{\mathcal{E}}_{x} / \sigma^{2}$ - i.e., note the bar.
h. Find a QAM CR constellation formula in text and put in the right number of levels to compare. Honeycomb should look better in terms of packing, but not necessarily nearest neighbors. This is a classic tradeoff between good packing and neighbors in coding.
i. Be creative - it's simple, but you'll need to earn this point.
[Baseband Equivalents (1.35] help] This problem largely tests your understanding of the various quantifies already defined, but now in the context of a baseband channel, associated with inphase/quadrature data transmission (QAM). Largely, the intent is for you to gain familiarity with design levels that are reasonably and how to work smoothly with quanties like dBm or $\mathrm{dBm} / \mathrm{Hz}$.
a. These are the QAM basis functions, where you need to understand the carrier frequency $f_{c}$ can be anything in the general approach - that is, you don't need to know it for most of the basic design. In practice of course, a range is specified and this problem also has such. Of course the carrier frequency has to be at least greater than the symbol rate. Use 50 MHz if you like.
b. This part is easy, but you need to think dB . If you know the power is 0 dbm ( 1 milliwatt), what is $\mathcal{E}_{x}$, the ENERGY/symbol? It is power x symbol period. Do this is dB , so simple add. $0-10 \log 10$ of symbol rate. The noise psd is the energy per symbol if one-sided PSD, but the given PSD is two-sided at $-103 \mathrm{dBm} / \mathrm{Hz}$. Thus this is relative to the energy/dimension, which is 3 dB less than the energy/symbol for quadrature modulation. Do this right, and you get a really nice round number that signifies 4 orders of magnitude.
c. You could do detailed analysis, but this is set up for immediate solution with the gap approximation. You need to know the gap for $10^{-7}$ (its in Lecture 3), and then use the gap approximation. You should also get a really nice round number here.
d. At this point, we're looking for the easy answer. You know how many bits per symbol, and this is nice even integer, so what constellation is that?
e. Now, you just need to draw the full modulator corresponding to your basis functions, mapping from messages ( $b$ or $\bar{b}$ at a time) into integers for QAM constellation selected, then show the baseband basis function, and the quadrature conversion to carrier frequency.
f. Continue now with the demodulator. You can use Hilbert Transform version, create baseband, and then show matched filters and sampler.
g. This is where we first introduce you to understanding power/bandwidth trade off in real terms. What happens to the number of bits per symbol if you double the symbol rate? Is this a better design? How far can you go on this and continue to increase data rate at same $P_{e}$. You are seeing the fallacy of the AWGN model, which has infinite bandwidth, at least up to the half the bandwidth allowed in this problem.
h. We pull off the constraints. Take this all the way down to QPSK and see what data rate this unrestricted AWGN can support given the limits of this problem and "uncoded" QAM.
[2-tap channel (1.38) help] This problem familiarizes you somewhat with a channel that changes the signal, thus perhaps preventing orthogonality of two signals but also possibly increasing gain effectively through the second (multi-) path.
a. This first part is easily drawn and it is just levels for the 4 different time slots.
b. This part allows you to effectively take the difference of the two signals and square, which is pretty simple integrate (just adding and multiplying constant levels). Your distance should be proportional to $A^{2} T$ / There is a before filtering distance, and an after filtering distance. This channel has gain, so a larger output distance is possible with multipath. In this case we also allow positive gain on the two paths, which could be caused by a transmit or intermediate amplification that occurs, which differs in gain on the two paths.
c. The $P_{e}$ for unfiltered with gain $A$ will be not as good as the multipath channel because the extra path not only exists, but even has a larger gain than the other (delayed) path. Multipath inteference is not necessarily bad - this is the basic characteristic exploited by MIMO systems. We'll see much more on that later.

