## Homework Help - Problem Set 1

Constellation Help This problem attempts to acclimate the student to inner products' interchangeability in continuous and discrete time with modulation, thus representing the transmitted signal as a vector or constellation.

Part a reinforces that cos and sin of same frequency over intervals that are at least any multiple of periods, here 1 period, are orthogonal.
Parts b, c, and d encourage the student, based on Part a's orthogonality finding, to represent the waveforms as vectors or signal points in a constellation. For instance, note that $\boldsymbol{x}_{0}=\left[\begin{array}{l}1 \\ 1\end{array}\right]$.

Inner Products Help This problem could use Appendiz A.1.7's Gram-Schmidt procedure that generally finds a basis-function representation of a set of functions. However, using the hinted cosine-sum formula, an obvious set of functions should be evident given PS1.
After finding the data symbols, the inner products are most easily done in the discrete domain.
multiple basis sets Help The main point of this problem is that a given constellation may correspond to multiple basis-function sets.

Irrelevancy Help This problem examines irrelevancy, but also helps build some insight.
Your intuition in looking at $y_{3}$ given $y_{1}$ should tell you all that is left is independent noise $n_{2}$, so how could that be relevant? This will follow from the equations if you write them carefully.
However, when both $y_{1}$ and $y_{2}$ are given, both are extra looks at the signal $x$ with different unrelated noises. Thus, they both should be helpful. Again, this will follow from equations. Is it possible to generate $x$ with no noise from all $3 y$ values? What does that tell you?

Part b is similar to Part a's last question in that two independently noisy looks at the input is better than one, even if the noise is not Gaussian. Part c starts by writing the MAP condition for deciding one of the values relevant to the other. Define $p=p_{\boldsymbol{x}}(1)$ to simplify notation, and note that $1-p$ is the other message choice's probability. Simplifying this expression leads to a sum $( \pm 1)$ of 4 differences' absolute values relative to $p$. Use this relation and investigate
various $\left[y_{1} y_{2}\right]$ combinations; these correspond to the boundary lines for the regions shown in the problem statement.

On Part d, the decision region figure makes it obvious that this rule can work. To get the error probability, the convolution of two exponentials is an exponential, or you may want to review a Laplace transform for an exponential, and then multiply them for convolution. A partial-fraction expansion of the result should inverse transform back to the desired distribution, which can then be integrated to determine $P_{e}$.

For Part e, there are actually 9 regions that might need consideration, but this proceeds similar to Part c, just with a nonuniform input affecting the diagram.

Invariance Help This problem tests understanding of translational and rotational invariance.

For part a - Does the constellation look like a translation and/or rotation of an easier-analyzed constellation?
Part b, given Part a, should proceed easily by just enumerating the different decision regions for the 9 -point constellation, following the general SQ QAM approach.
Part c - simply insert the numerical values and comment.
Part d - answer this for both the original constellation and for the rotated and translated constellation. Does translation affect energy used? Does rotation affect energy used?

