1-Consider an optical communication system. In this system two messages are sent over the optical link. We have:

$X_0 = 1 \qquad P(X = 1) = 1/3$
$X_1 = 2 \qquad P(X = 2) = 2/3$

The received signal is Y = X + N, noise is white and Gaussian, but the variance of the noise is proportional to the amplitude of the signal i.e. $\sigma_i^2 = \alpha^2 X_i^2$ i=0,1. Find the optimum decision rules and P_e.

Solution:

$N|X_0 \sim N(1, \alpha^2)$

$N|X_1 \sim N(2, 4\alpha^2)$

$I = \max_{i=0,1}\{ P_{X|Y}(X_i \mid v) \} = \max_{i=0,1}\{ P_{(X,Y)}(X_i , v) \}$

$$P_{(X,Y)}(X_0 , v) = P(X = X_0) P_{Y|X}(v \mid X_0) = \frac{1}{3\sqrt{2\pi\alpha^2}} e^{\frac{-(v-1)^2}{2\alpha^2}}$$

$$P_{(X,Y)}(X_1 , v) = P(X = X_1) P_{Y|X}(v \mid X_1) = \frac{2}{3\sqrt{8\pi\alpha^2}} e^{\frac{-(v-2)^2}{8\alpha^2}}$$

Solving equation $P_{(X,Y)}(X_1 , v) = P_{(X,Y)}(X_0 , v)$ we can find the decision rules:

$v \in (-\infty , 0) \qquad I=1$

$v \in (0, 4/3) \qquad I=0$

$v \in (4/3, \infty) \qquad I=1$

The decision regions are shown in fig.1

$P_e = P(X = X_0) P_{e|0} + P(X = X_1) P_{e|1}$

$= 1/3 \Pr\{v > 4/3 \text{ or } v < 0 \mid X = 0 \} + 2/3 [1-\Pr\{ v > 4/3 \text{ or } v < 0 \mid X = 1 \}]$

$= 1/3\, Q(\frac{1/3}{\alpha}) + 1/3\, Q(\frac{1}{\alpha}) + 2/3\, [Q(\frac{2/3}{2\alpha})-Q(\frac{2}{2\alpha})]$

$= Q(\frac{1/3}{\alpha}) - 1/3 Q(\frac{1}{\alpha})$

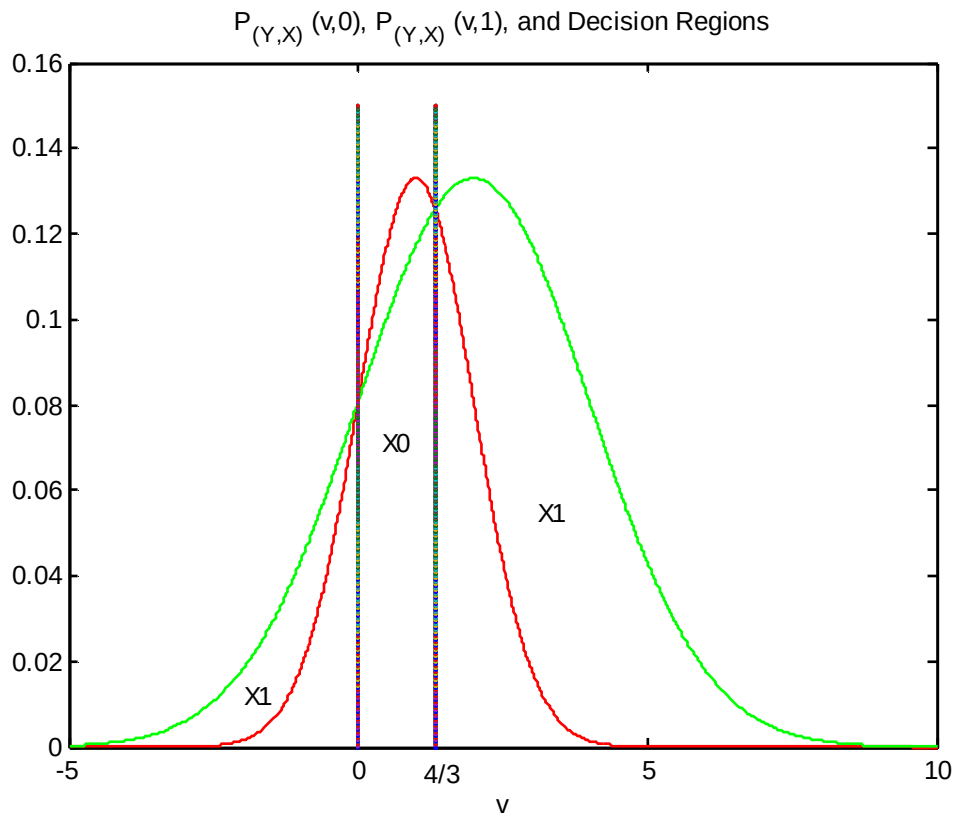

Fig.1

2-(P-3-41) *Forcing Partial Response*
Consider a H(D) = 1+.9D channel with AWGN noise variance σ^2. We would like to convert this to a 1+D channel

a. Design an equalizer that will convert the channel to a 1+D channel

b. The received signal is $y_k = x_k + .9x_{k-1} + n_k$ where n_k is the AWGN. Find the autocorrelation of the noise after going through the receiver designed in part (a). Evaluate r_0, $r_{\pm 1}$, and $r_{\pm 2}$. Is the noise white?

c. Do you think that the noise would be more or less correlated if were convert a 1+.1D channel to a 1+D channel?

Solution

a. The system is shown in Fig.2.

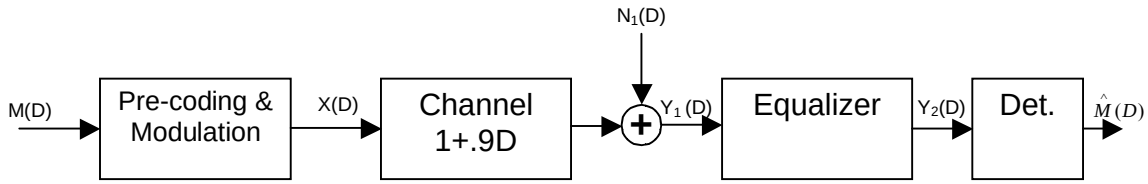

Fig.2.System for equalization by converting to P.R. and pre-coding

$Y_1(D) = X(D)(1+.9D) + N_1(D)$ $\qquad S_{n1}(D) = \dfrac{N_0}{2}$

We would like to have:

$Y_2(D) = (1+D)X(D) + N_2(D)$

So we can set: $H_{eq}(D) = \dfrac{1+D}{1+.9D}$, so we have $S_{n2}(D) = \dfrac{N_0}{2} H_{eq}(D)H^*_{eq}(D^{-*})$

b. $S_{n2}(D) = \dfrac{N_0}{2} H_{eq}(D)H^*_{eq}(D^{-*}) = 1.111 + \dfrac{.0527D}{1+.9D} - \dfrac{.0585}{1+.9D^{-1}}$

$= 1.111 + .0527D[\,1 - .9D + (-.9)^2D^2 + (-.9)^3D^3 + \ldots..] - .0585\,[1 - .9D^{-1} + (-.9)^2D^{-2} + (-.9)^3D^{-3} + \ldots]$

$= 1.05 + .0527D + .0527D^{-1} - .0474D^2 - .0474D^{-2} + \ldots\ldots.$

$R_{n2}(0) = 1.05$, $R_{n2}(1) = .0527$, $R_{n2}(-1) = .0527$, $R_{n2}(2) = .0474$, $R_{n2}(-2) = .0474$

Clearly, noise is not white after passing through equalizer, so S.B.S. detecting is not optimum.

c. As we can see in Fig.3, the equalizer for 1+.1D is far less similar to an all-pass filter which would result in having white noise at the output of the equalizer. Therefore, we expect to have more correlated noise for 1+.1D. Also, the variance of the noise is larger.

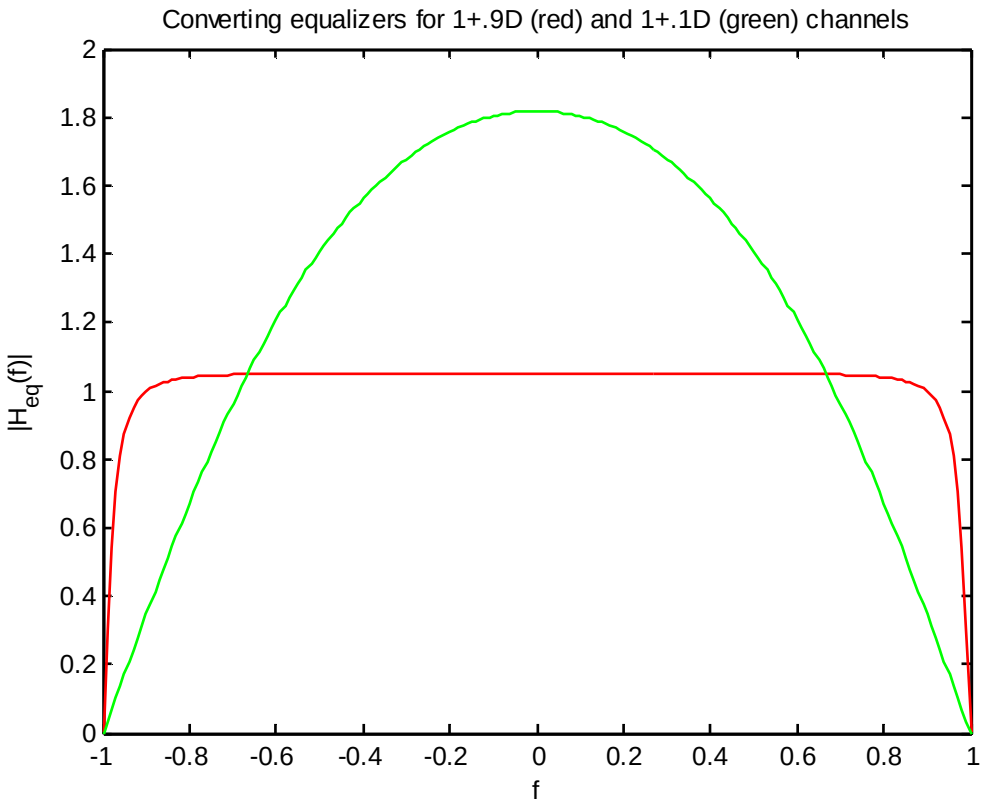
Converting equalizers for 1+.9D (red) and 1+.1D (green) channels
|H_eq(f)|
f
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Fig.3