Solution

a)

If Q(D) has a zero on the unit circle then this means it has a factor of the form $(1-e^{ja}D)$.
In our factorization of Q(D) we can take this factor to be part of P(D). Therefore, in $P^*(D^{-*})$ there must be a factor of the form $(1-e^{-ja}D^{-1})$. It is clear both zeros occur at the same location e^{-ja}.

b)

Since P(D) has a zero on the unit circle this means that $\frac{1}{P(D)}$ has a pole on the unit circle.
Hence, the corresponding time resposone p_k does not decay to zero. So the noise variance is

$$\sigma^2 = \frac{N_0}{2|p|^2}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \frac{1}{P(e^{-j\omega T})P^*(e^{-j\omega T})}\,d\omega = \frac{N_0}{2|p|^2}\sum_{i=1}^{n}|p_k|^2 = \infty$$

c)

The PSD of N(D) is $\frac{N_0}{2}$Q(D). After passing through the WMF the PSD becomes

$$\begin{aligned}
R_{N'}(D) &= \frac{N_0}{2}Q(D)\left[\frac{1}{\eta_0|p|P(D)}\frac{1}{\eta_0|p|P^*(D^{-*})}\right] \\
&= \frac{N_0}{2}Q(D)\left[\frac{1}{|p|^2\eta_0^{\,2}P(D)P^*(D^{-*})}\right] \\
&= \frac{N_0}{2}Q(D)\left[\frac{1}{|p|^2\eta_0 Q(D)}\right] \\
&= \frac{N_0}{2|p|^2\eta_0}
\end{aligned}$$

Hence, the output noise is white.

d)

No, we don't lose anything. The transformation is generally invertible so no information has been lost.

e)

Call the input to the Slicer S(D). Then we have that

$$S(D) = R(D) + S(D)(P(D)-1)$$
$$S(D)\left[1-(P(D)-1)\right] = R(D)$$

Hence, $S(D) = \frac{R(D)}{P(D)}$

f) In the absence of noise the slicer will have no affect.

g) The slicer removes any noise that would otherwise propagate through the feedback loop.

h) This is just a ZF-DFE.

i) Moving the slicer makes the system stable since the output is bounded. So even if P(D) has a zero on the unit circle the output is always stable since the slicer output (signal + noise) is always bounded.

j) The ZF-DFE removes the noise fed to the filter 1-P(D), making sure that the noise will not be fed back in the input to the slicer. This could only improve the performance.