"Extended" ZF Equalizer

Suppose, as usual, we can factor Q(D) as Q(D) = $\eta_0 P(D)P^*(D^{-*})$ where $P(D)$ is a minimum phase, causal, and monic polynomial. Assume Q(D) has a zero on the unit circle. Lets consider the consequences of this.

a) Prove that the zeros of Q(D) are always double.

b) What can you say about the noise variance at the output of a ZF Equalizer?

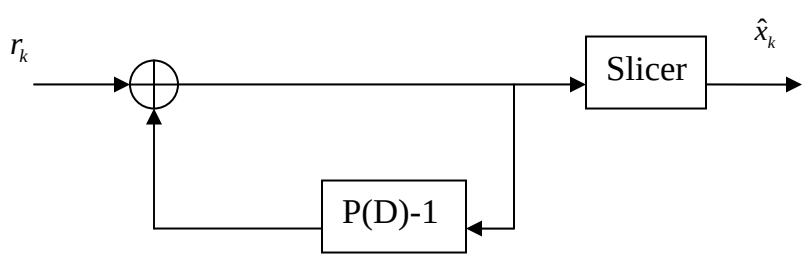
Now Suppose that before we do any equalization we pass the sampled Matched filter output $Y(D) = |p|$Q(D)X(D) + N(D) through a WMF filter to get R(D) = X(D)P(D) + N'(D). Lets examine the output of the WMF.

c) What is the PSD of the noise term N'(D)?

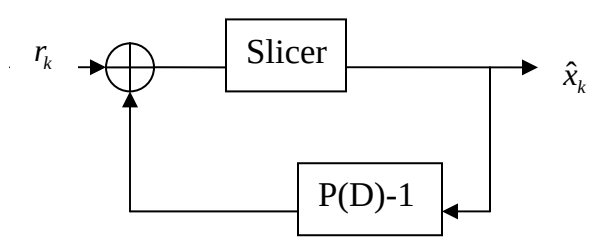
d) In general, are we losing anything by initially passing Y(D) through a WMF?

Now, to implement the ZF equalizer we pass R(D) through the filter $\frac{1}{\text{P(D)}}$.

e) Show that the filtering with $\frac{1}{\text{P(D)}}$ could be implemented as a simple feedback system with filter P(D)-1 as shown below.



Notice that the output of the above feedback loop is the input to the slicer. Suppose we now move the slicer inside the feedback loop as shown below.



Please explain

f) what is the affect of moving the slicer inside the loop on the symbol component in the absence of noise?

g) what is the affect of moving the slicer inside the loop on the noise component?

h) What is the name of this structure?

i) Can you now explain why the ZF-DFE exists and is stable (noise variance is finite) even if the ZF eqaulizer does not exist (noise variance is infinite)?

j) Can you explain (no equations) why is the performance of the ZF-DFE superior to the ZF?