Canonical Factorization Tutorial

I found myself slightly confused when we first using canonical factorization and I spent a good while trying to figure out what exactly the examples were doing. I figured, that much frustration could have been avoided, had there been a quick tutorial included in the appendix on canonical factorization.

This tutorial walks through the steps on how to use Matlab to determine the canonical factorization for polynomial. The numbers used in this tutorial follow those from example 3.6.2.

First, determine what the Q(D) + 1/SNR_{MFB} will be for the polynomial that you are trying to factor. In this case,

$$Q(D) + 1/SNR_{MFB} = -.\frac{25jD^{-2} - 0.625*(1+j)D^{-1} + 1.5625*(1+.1) + 0.625*(1+j)D + .25jD^{2}}{1.5625}$$

Now, we need to take the roots of this polynomial using the roots() command in Matlab. Form the vector for Q, ignoring the constant in the denominator (it won't matter). Then, take the roots of this polynomial using roots(Q). The results are shown below.

```
Q =

    0 + 0.2500i  -0.6250 - 0.6250i   1.7188          -0.6250 + 0.6250i       0 + 0.2500i

roots(Q)

ans =

  -0.2759 - 2.1718i
   2.1718 + 0.2759i
   0.7443 - 0.7443i
  -0.1402 + 0.1402i
```

Select all the roots with ||r|| < 1, these will be used to form the canonical factorization. For this example, the 3rd and 4th roots will be used

$$r1 = 0.7443 - 0.7443i = .4510*e^{j.0612}$$
$$r2 = -0.1402 + 0.1402i = .4510*\ e\text{-}^{j1.632}$$

Then,

$$G(D) = (1-r1)*(1-r2) = (1 - .4510*e^{j.0612})*(1 - .4510*\ e\text{-}^{j1.632})$$
$$= 1-.4226(1+j)D + .2034jD^{2}$$

Now, we use this to find $?_0 = \frac{1 + 1/\ SNR_{MFB}}{||G(D)||^2} = \frac{1 + .1}{1 + (.4226)^2(1+j)(1-j) + (.2034)^2(j)(-j)}$.

$?_0 = 0.7866.$

Finally, we get the canonical factorization

$Q(D) + 1/SNR_{MFB}$

$= ?_0 * G(D) * G^{*}(D^{-*})$
$= .7866*(1 - .4510*e^{j.0612})*(1 - .4510* e-^{j1.632}) (1 - .4510*e-^{j.0612})*(1 - .4510* e^{j1.632})$