Jessica Heyman
student ID 5356724
email jheyman@stanford.edu, jeshe852@student.liu.se

March, 2005

All you need to know about the D transform - at least in EE379a

Bottom line

The D transform is the z transform but with $z^{-1} = D$.

Hence,

$$X(D) = \sum_{k=0}^{\infty} x_k D^k$$

There is also a two-sided version for non-causal signals:

$$X(D) = \sum_{k=-\infty}^{\infty} x_k D^k$$

D stands for "delay", meaning that for example $2D + 4.5D^{-5}$ corresponds to the time sequence $2x_{k+1} + 4.5x_{k-5}$.

Properties

causality	region of convergence (ROC) is inside the innermost pole
stability	region of convergence includes the unit circle
minimum phase	all poles are outside the unit circle, all zeroes are on or outside the unit circle

Inverse D transform

If you need to convert a D transform back into the time domain and the transform is not a simple polynomial of the form above that you can convert by looking at it, you either:

(i) use $z^{-1} = D$ and use any of your old z transform tables. If you threw it out, burnt it or lost it, I recommend http://cnx.rice.edu/content/m10119/latest/.

(ii) use

$$\frac{r}{D-r} \leftrightarrow \begin{cases} -\left(\frac{1}{r}\right)^n u(n) & |r| > 1 \\ \left(\frac{1}{r}\right)^n u(-n-1) & |r| < 1 \end{cases}$$

It has been empirically proven that you can get through all of EE379a with just this one inverse D transform.

Link to the Fourier transform

If we write

$$D = e^{-j\omega T}$$

we get the Fourier transform on the unit circle. You might have seen (or you might not) the same formula used for the z transform,

$$z = e^{j\omega T}$$

which is the same thing, since $z^{-1} = D$.