EE 379 A

Special Help Session (MAR. 10th 2:15 ~ 3:05 Skill Aud)

Jungsub Byun

① ISI channel model

② Comparison table (performance)

③ Equalizer

④ EQ table (IIR)

⑤ FLE and BFLE table (FIR)

⑥ Error propagation

⑦ precoding

⑧ PR channel (differential encoder)

⑨ Diversity

⑩ dfecolor tutorial

(nff, nbb, Δ(delay))

dferake

① ISI channel Model

$$y(t) = \sum_k \|p\| x_k g(t-kT) + \underbrace{n_p(t) * \varphi_p^*(-t)}_{n(t)}$$

$$\Rightarrow \frac{N_0}{2} = E(|n_k|^2)$$

$$\Rightarrow \frac{N_0}{2} Q(\omega) \quad \text{PSD}$$

PSD of $n(t)$ no longer white

where

$$g(t) = \varphi_p(t) * \varphi_p^*(-t)$$

$$= \frac{p(t) * p^*(-t)}{\|p\|^2}$$

$$y_k = \underbrace{\|p\| x_k}_{\text{signal}} + \underbrace{\|p\| \sum_{m \neq k} x_m g_{k-m}}_{\text{ISI}} + \underbrace{n_k}_{\text{noise}}$$

$$Y(D) = \|p\| X(D) Q(D) + N(D) \Rightarrow \frac{N_0}{2} Q(D)$$

Nyquist criterion (a) $g_k = \delta_k$

(b) $\frac{1}{T} \sum_{n=-\infty}^{\infty} Q(\omega + \frac{2\pi n}{T}) = 1$

② Comparison table (performance)

	ZFE	MMSE-LE	MMSE-DFE	ZF-DFE	precoder	PR channel with differential-encoder	diversity
can-cellation ISI	ISI is forced to zero	Not flatten the channel near the channel notch	Cancel 'trailing' ISI	cancel ISI completely	• cancel ISI without DFE. • problem is power boost at Tx	• Simple (encoder) decoder • No feedback • No power loss at Tx	The diversity represents the use of multiple channels from a single message source to the receiver to improve performance (SNR↑)
Noise enhancement	Large	Smaller	N	Y			
	Simpler linear Equalizer	linear EQ	can lead to error propagation • Aim is to find W(D), B(D) to maximize SNR	• Aim is to find W(D), B(D) to eliminate ISI		• no EP. Even though, Slight neighbor (of constellation) increase at Rx,	

	TPC	LP
EP	completely recover	slight effect
CS	destroyed main-trains	

$\bar{P}_e$ (ZF-DFE) $\geq$ precoded PR system $\bar{P}_e = \bar{N}_e Q\left(\frac{d_{min}}{2\sigma}\right)$

$SNR_{ZFE} \leq SNR_{MMSE-LE,U}$

$SNR_{ZF-DFE} \leq SNR_{MMSE-DFE,U}$

$SNR_{MMSE-LE,U} \leq SNR_{MMSE-DFE,U}$

$SNR_{ZFE} \leq SNR_{ZFE-DFE}$

DFE can't do worse than MMSE-LE and ZFE

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③ Equalizer

$\overline{\mathcal{E}_x} = 1$, $SNR_{MFB} = 10\,dB$ $\quad SNR_{MFB} = \dfrac{\|p\|^2 \overline{\mathcal{E}_x}}{\sigma^2}$ ① $\Rightarrow \sigma^2$

Tx

② $W(D)$ $\dfrac{\frac{1}{\|p\|\,Q(D)}}{\frac{1}{\|p\|\,(Q(D)+\frac{1}{SNR_{mfb}})}}$ ZFE : MMSE-LE

③ w_0 (ZFE / MMSE-LE)

Canonical factorization of DFE

MMSE-DFE
ZF-DFE

loss factor
γ_0 (MMSE-DFE)
η_0 (ZF-DFE)

$\tilde{R} = \gamma_0 \tilde{g} \tilde{g}^*$
$Q = \eta_0 P_c P_c^*$

④ σ^2_{EQ} (equalizer noise variance)

$\sigma^2_{ZFE} = \dfrac{N_0}{2} \dfrac{w_{ZFE,0}}{\|p\|}$

$\sigma^2_{MMSE-LE} = \dfrac{N_0}{2} \dfrac{w_{MMSE-LE,0}}{\|p\|}$

$\sigma^2_{MMSE-DFE} = \dfrac{N_0}{2} \dfrac{1}{\|p\|^2 \gamma_0}$

$\sigma^2_{ZF-DFE} = \dfrac{N_0}{2} \dfrac{1}{\|p\|^2 \eta_0}$

⑤ $SNR_{R,u}$ (SNR_R)

$SNR_u(R) = SNR(R) - 1$

- ZFE, $u = \dfrac{\overline{\mathcal{E}_x}}{\sigma^2_{ZFE}}$
- MMSE-LE, $u = \dfrac{\overline{\mathcal{E}_x}}{\sigma^2_{MMSE-LE}} - 1$
- MMSE-DFE, $u = \gamma_0 SNR_{MFB} - 1$
- ZF-DFE, $u = \eta_0 SNR_{MFB}$

⑥ $P_e = Q(\sqrt{SNR_u})$: 2PAM

④

	ZFE	MMSE-LE	MMSE-DFE	ZF-DFE
EQ transfer characteristic $W(D)$	$\frac{1}{\lVert p\rVert Q(D)}$	$\frac{1}{\lVert p\rVert\left(Q(D)+\frac{1}{SNR_{MFB}}\right)}$	$\tilde{Q} = Q + \frac{1}{SNR_{MFB}} = r_0 G G^*$ ← canonical factorization ↓ feed forward: $W_{MMSE\text{-}DFE}(D) = B(D)W_{MMSE\text{-}LE} = \frac{1}{\lVert p\rVert r_0 G^*}$ feed back: $B=G$	$Q(D) = \eta_0 P_c(D) P_c^*(D^{-*})$ feed forward: $\frac{1}{\eta_0 \lVert p\rVert P_c^*(D^{-*})}$ feed back: $1-B(D) = 1-P_c(D)$
and then calculate w_k, w_0 : center tap of eq. in time domain		P. 173 table 3.1		
PSD of noise samples $\overline{R}_{ee}(D)$	$\frac{N_0/2}{\lVert p\rVert^2 Q(D)}$	$\frac{N_0/2}{\lVert p\rVert^2\left(Q(D)+\frac{1}{SNR_{MFB}}\right)}$	$\frac{N_0/2}{\lVert p\rVert^2 r_0} \cdot \frac{B(D)}{G(D)} \cdot \frac{B^*(D^{-*})}{B^*(D^{-*})}$ ($B=D$, 1, 1)	$\frac{N_0/2}{\eta_0 \lVert p\rVert^2}$
noise variance σ_e^2	$\frac{N_0}{2}\frac{w_{ZFE,0}}{\lVert p\rVert}$	$\frac{N_0}{2}\frac{w_{MMSE\text{-}LE,0}}{\lVert p\rVert}$	$\sigma_e^2 = r_{ee}(0) = \lVert f\rVert^2 \frac{N_0/2}{r_0 \lVert p\rVert^2}$ ($\lVert f\rVert^2 = 1$) Let $F(D) = \frac{B}{G} = 1$	$\frac{N_0/2}{\eta_0 \lVert p\rVert^2}$
$SNR_{R.u}$	$\frac{\overline{\mathcal{E}}_x}{\sigma^2_{ZFE}}$	$\frac{\overline{\mathcal{E}}_x}{\sigma^2_{MMSE\text{-}LE}} - 1$	$r_0 = \frac{1+\frac{1}{SNR_{MFB}}}{\lVert g\rVert^2}$ $SNR_{MMSE\text{-}DFE,u} = r_0 SNR_{MFB} - 1$	$\eta_0 = \frac{1}{\lVert P_c\rVert^2} = \frac{1}{\sum_{k=-\infty}^{\infty} \lvert P_{c,k}\rvert^2}$ $SNR_{ZFDFE} = \eta_0 SNR_{MFB}$
α (Bias) $\frac{1}{\alpha}$	No 1	Yes $1 - \frac{1}{SNR_{MMSE\text{-}LE}}$ • MMSE receivers reduce noise power at the expense of introducing a bias	Yes $\left(\frac{SNR_{mmse\text{-}dfe}}{SNR_{mmse\text{-}dfe.u}}\right)^{-1}$	No 1

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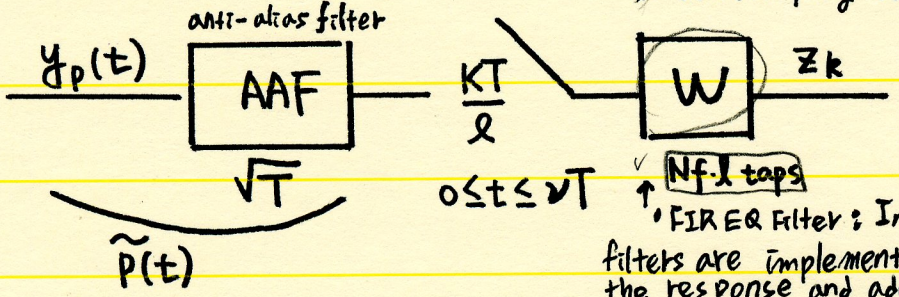
⑤ FLE at RCVR



l: oversampling factor per symbol period

$l = 1$

noise variance $\sigma^2 = \frac{N_0}{2} \cdot l$

FIR EQ Filter: In real implementation, non-causal filters are implemented by truncating and shifting the response and adding appropriate delay.

$\underline{Y}_k = P\underline{X}_k + \underline{N}_k$: Matrix Channel Model

(denotes vector)

$z_k = \underline{W}\underline{Y}_k$: equalizer output (feedforward)

· Matrix dimension

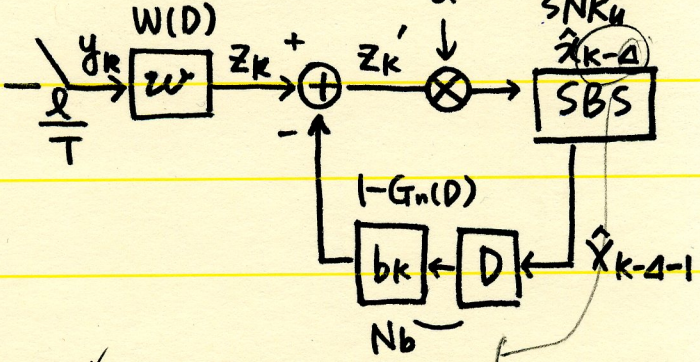
$\underline{X}_k$: $(N_f + \nu) \times 1$

P_k : $(N_f \cdot l) \times (N_f + \nu)$

$\underline{Y}_k$: $N_f \cdot l \times 1$

$\underline{W}$: $1 \times N_f \cdot l$

FLE MMSE-DFE



Δ (delta) = delay of system is approximately the sum of the channel and equalizer delays in symbol period.

In linear equalizer $\Delta = \frac{\nu + N_f}{2}$

In the DFE, $\tilde{\underline{Y}}_k = \tilde{P}\underline{X}_k + \tilde{N}_k$ (Matrix channel Model)

'~' tilda denotes an augmented vector for the feedback path.

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FLE	ZFE	MMSE-LE	MMSE-DFE	ZF-DFE
1 R_{xY}	$\bar{\mathcal{E}}_x 1_\Delta^* P^*$ $1_\Delta = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ $N_f+\nu$ $(\Delta+1)$th position	$\bar{\mathcal{E}}_x 1_\Delta^* P^*$	$*$: conjugate transpose ✓ $R_{x\tilde{Y}} = \bar{\mathcal{E}}_x 1_\Delta^* \tilde{P}^*$	set $\frac{N_0}{2}=0$ by letting the $SNR_{mfb} \to \infty$, in the FIR MMSE DFE $R_{x\tilde{Y}} = \bar{\mathcal{E}}_x 1_\Delta^* \tilde{P}^*$
Cross-Correlation Matrices				
2 Autocorrelation Matrices R_{YY}	$\bar{\mathcal{E}}_x PP^*$ ✗ $SNR_{mfb} \to \infty$ neglect σ^2	$\bar{\mathcal{E}}_x PP^* + \sigma^2 I$ $(\sigma^2 = \frac{N_0}{2}\ell)$	$R_{\tilde{Y}\tilde{Y}} = \bar{\mathcal{E}}_x \tilde{P}\tilde{P}^* + R_{\tilde{n}\tilde{n}}$ $I: N_f \times N_f$ $= \ell \frac{N_0}{2}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$	$R_{\tilde{Y}\tilde{Y}} = \bar{\mathcal{E}}_x \tilde{P}\tilde{P}^*$
EQ filter tap w 3	$R_{xY} R_{YY}^{-1}$	$R_{xY} \cdot R_{YY}^{-1}$	$\tilde{w} = [w \,\vdots\, -b] = R_{x\tilde{Y}} R_{\tilde{Y}\tilde{Y}}^{-1}$ '~' Tilda denotes an augmented vector because of feedback path for the DFE	$\tilde{w} = [w \; -b] = R_{x\tilde{Y}} R_{\tilde{Y}\tilde{Y}}^{-1}$
4 σ^2	$\sigma^2 = \frac{N_0}{2}\ell$ (ISI) $\bar{\mathcal{E}}_x - R_{xY} W^* + \|w\|^2 \sigma^2$ (noise enhancement) Cannot neglect σ^2	$\bar{\mathcal{E}}_x - R_{xY} W^*$	$\bar{\mathcal{E}}_x - R_{x\tilde{Y}} W^*$ where $R_{xY} = R_{Yx}^*$	residual ISI $\bar{\mathcal{E}}_x - \tilde{w} R_{x\tilde{Y}}^* + \|w\|^2 \sigma^2$ noise $\sigma^2 = \frac{N_0}{2}\ell$
5 SNR_u	$\frac{\bar{\mathcal{E}}_x}{\sigma^2_{ZFE}} - 1$	$\frac{\bar{\mathcal{E}}_x}{\sigma^2_{MMSE-LE}} - 1$	$\frac{\bar{\mathcal{E}}_x}{\sigma^2_{MMSE-DFE}} - 1$	$\frac{\bar{\mathcal{E}}_x}{\sigma^2_{ZF-DFE}} - 1$

	ZFE	MMSE-LE	MMSE-DFE / ZF-DFE	
P	$y_k = x_k - 0.5x_{k-1} + n_k$ ✓ a discrete-time IIR channel with response given by $\Delta = 0$ $N_f = 3$ $\nu = 1$ $(P(D) = 1 - 0.5D)$ $P = \begin{array}{c} \\ N_f \end{array}\begin{array}{c} \overbrace{\quad}^{\nu+1} \quad \overbrace{\quad}^{N_f-1} \\ \begin{bmatrix} 1 & -0.5 & 0 & 0 \\ 0 & 1 & -0.5 & 0 \\ 0 & 0 & 1 & -0.5 \end{bmatrix} \end{array}$	$y_k = x_k - 0.5x_{k-1} + n_k$ $\Delta = 0$ $N_f = 2$ $N_b = 1$ $P = \begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -0.5 \end{bmatrix}$ N_f $\nu + N_f$ $\tilde{P} = \begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -0.5 \\ 0 & 1 & 0 \end{bmatrix}$ J_Δ^* $(\Delta+2)$th position	
R_{xY}	$R_{xY} = \bar{\varepsilon}_x [\overbrace{1\ 0\ 0\ 0}^{N_f+\nu}]P^*$ 1_Δ^* $(\Delta+1)$th position	MMSE-DFE, ZF-DFE $R_{x\tilde{Y}} = [\bar{\varepsilon}_x [1\ 0\ 0] P^* \underset{n_b}{0}]$ 1_Δ^*	$\tilde{w} = [w \vdots -b]$ '~' Tilda – augumented vector because of feedback path for DFE	
R_{YY}	$\bar{\varepsilon}_x PP^*$	$\bar{\varepsilon}_x PP^* + \sigma^2 I$ $\sigma^2 = \ell \frac{N_0}{2}$	MMSE-DFE $R_{\tilde{Y}\tilde{Y}} = \begin{bmatrix} \bar{\varepsilon}_x PP^* + \sigma^2 I & \bar{\varepsilon}_x P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ J_\Delta^* & \\ \bar{\varepsilon}_x [0\ 1\ 0] P^* & \bar{\varepsilon}_x I \end{bmatrix}$ $N_f \times N_f$ J_Δ $(\Delta+2)$th position $= (\bar{\varepsilon}_x \tilde{P}\tilde{P}^* + R_{\tilde{n}\tilde{n}})$	by letting the SNR $\to \infty$ in the FIR MMSE-DFE ZF-DFE $\sigma^2 = 0$ $R_{\tilde{Y}\tilde{Y}} = \begin{bmatrix} \bar{\varepsilon}_x PP^* & \bar{\varepsilon}_x PJ_\Delta \\ \bar{\varepsilon}_x J_\Delta^* P^* & \bar{\varepsilon}_x I \end{bmatrix}$ $= \bar{\varepsilon}_x \tilde{P}\tilde{P}^*$	
σ^2_{eq}	$\bar{\varepsilon}_x - R_{xY}w^* + \lVert w \rVert^2 \sigma^2$ ✓	$\bar{\varepsilon}_x - R_{xY}w^*$	$\sigma^2 = \bar{\varepsilon}_x - R_{x\tilde{Y}}w^*$ (MMSE-DFE) $\sigma^2 = \bar{\varepsilon}_x - R_{x\tilde{Y}}w^* + \lVert w \rVert^2 \sigma^2$ (ZF-DFE)	$R_{\tilde{n}\tilde{n}} = \begin{bmatrix} .125 & 0 & 0 \\ 0 & .125 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{N_0}{2}\ell \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ I
	✓ • Matrix channel Model $(\underline{Y}_k = P\underline{X}_k + \underline{N}_k)$ where P is Toeplitz matrix	$\begin{bmatrix} y_k \\ y_{k-1} \\ x_{k-1} \end{bmatrix} = \begin{bmatrix} 1 & -.5 & 0 \\ 0 & 1 & -.5 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \end{bmatrix} + \begin{bmatrix} n_k \\ n_{k-1} \\ 0 \end{bmatrix}$ $\tilde{Y}$ $\tilde{P}$ X_k $\tilde{n}_k$ $x_{k-1} = X_{k-\Delta-1}$ (feedback!) $(\tilde{\underline{Y}}_k = \tilde{P}\underline{X}_k + \tilde{\underline{N}}_k)$	In FLE, ($\bar{\varepsilon}_x$, $\frac{N_0}{2}$ given) $P \to R_{xY} \to R_{YY} \to w \to \sigma^2_{(eq)}$ $\to SNR_u \to P_e$	

✓ The vector of data symbols in the feedback path

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⑥ Error Propagation in DFE

- DFE structures use previous SBS decisions $\{\hat{x}_i\}_{i=k-1}^{\infty}$ to decide $\hat{x}_k$

— In our analysis we assumed that the previous SBS decisions are correct

— If a decision is incorrect, it may affect the next decision and lead to a burst of errors (i.e. error propagation)

— can degrade DFE performance a lot at high P_e's like 10^{-3}, actually <at $P_e = 10^{-6}$ of DFE, — EP is not issue.>

— current communication systems are 'coded' systems where the DFE is expected to operate at such high P_e's

* Probability P that we get error free at time K given error at time $(K-\alpha)$ $\quad P = \frac{1}{M}$

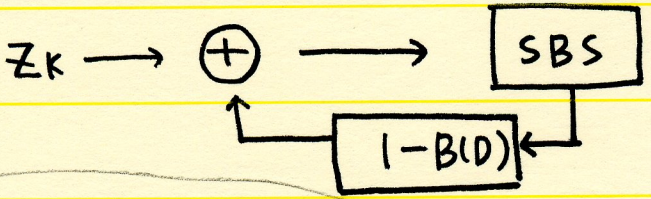
① $E\left(\frac{\text{number of error propagations}}{\#\text{ of no EP}}\right) = \sum_{K=1}^{\infty} K \cdot (P) \cdot (1-P)^{K-1} = M$ (geometric distribution) ② $P_e = \tilde{N}_e Q\left(\frac{d_{min}}{2\sigma}\right)$ ↑

② $\tilde{N}_e = E\left(\frac{\text{number of error propagations}}{\#\text{ of no EP}}\right) \cdot (\text{number of nearest neighbors})$ ↑

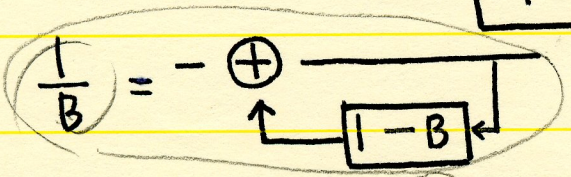
⑦ Precoding (Tomlinson)

$$Z(D) = B(D)X(D) + E(D)$$: feedforward equalizer output

Receiver DFE



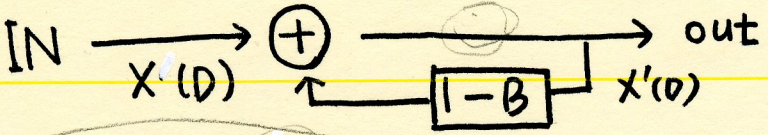
* The feedback filter removes the trailing ISI using past decision



⇐ nonlinear element

precoding : move $\frac{1}{B}$ to transmitter

Transmitter : pre-cancels trailing ISI

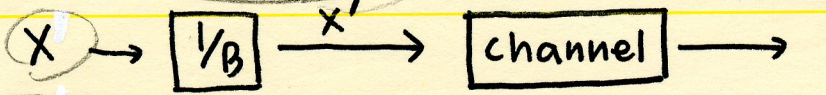


$$\frac{OUT}{IN} = \frac{1}{B}$$

$$X'(D) = \frac{X(D)}{B(D)}$$

: need more transmit power.

✓ problem is power boost.



$\overline{\mathcal{E}_x}$ ↑ increase

→ solution : Modulo arithmetic at transmitter

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Modulo M-PAM

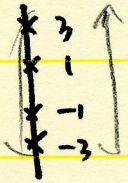
(straightforward moving of the filter $\frac{1}{B(D)}$ to the transmitter could result in significant transmit power increase. To prevent most of this power increase, modulo arithmetic is employed to bound the value of x_k'.

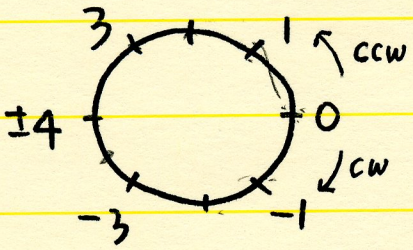
$$T_M(x) \triangleq x - Md \left\lfloor \frac{x + M\frac{d}{2}}{Md} \right\rfloor$$

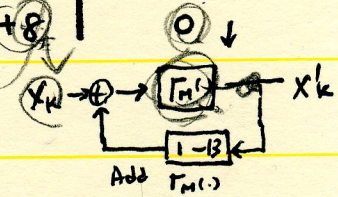
$X'(D) = \frac{X(D)}{B(D)}$

x	$T_M(x)$
ccw +3	3
cw −3	−3
+4.5	−3.5
−6	+2
+0.5	+0.5
+8	0

$M=4$ 4-PAM

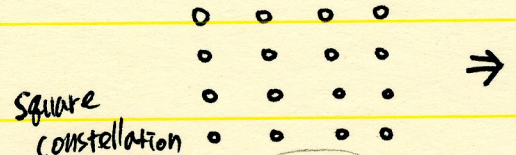




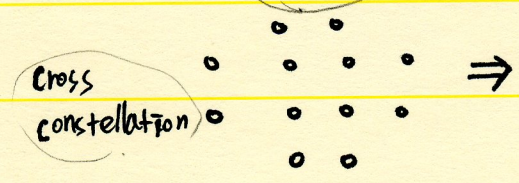


Input power increase $\begin{cases} \frac{M^2}{M^2-1} & \text{(PAM)} \\ \frac{M}{M-1} & \text{(SQ-QAM)} \end{cases}$

M	loss (SNR)
2	1.2dB
4	0.3dB
⋮	⋮
∞	0 dB



$X' \Rightarrow X'(D) = X'(D)B(D)$



loss more

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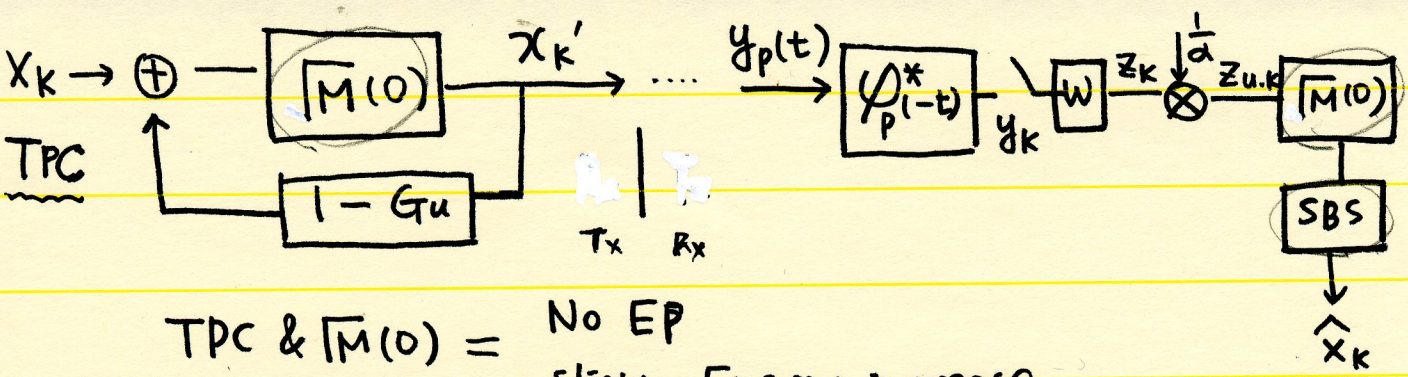

TPC & $\overline{M}(0)$ = No EP
slight Energy increase

- TPC has some power increase even with the modulo operator.
- TPC completely avoids any loss due to EP.

	TPC	LP
EP	completely recover (No EP)	'slight' effect (small EP)
CS (constellation shape)	destroyed (increase Tx pwr)	maintains (preserve Tx pwr)

- The Laroia precorder largely preserves the shape of the transmitted constellation

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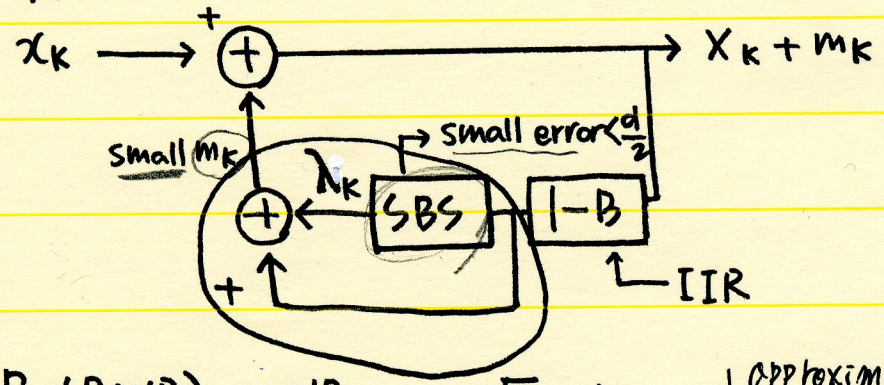
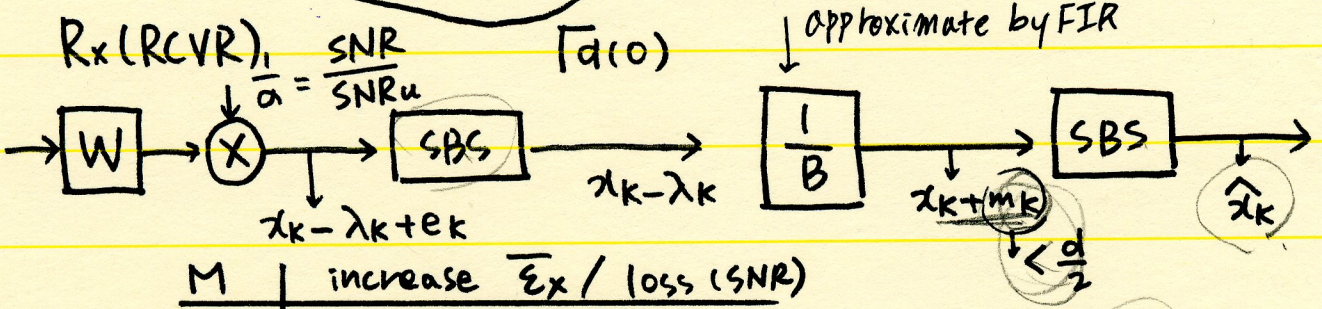
Laroria (flexible) precoder : Any constellation

LP

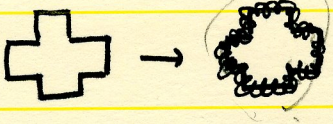
Tx



Rx (RCVR), $\overline{\alpha} = \frac{SNR}{SNRu}$ $\Gamma a(0)$



M	increase $\overline{\mathcal{E}_x}$ / loss (SNR)
2	1.2 dB loss
4	⋮
∞	0 dB



- less power increase
- shape maintain

X_{mit} $\overline{\mathcal{E}_x} \rightarrow \overline{\mathcal{E}_x} + \frac{d^2}{12} \approx \overline{\mathcal{E}_x}$ for large constellation

$\overline{\mathcal{E}_m}$

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⑧ PR channel (partial response)

$$H(D) = (1+D)^l \ (1-D)^n$$

$(1+D)^l$: low pass decay, $(1-D)^n$: high pass decay

low pass

Input
1 1 1 1 1 1
1 -1 1 -1 1 -1

$H(D) = 1+D$: Duobinary

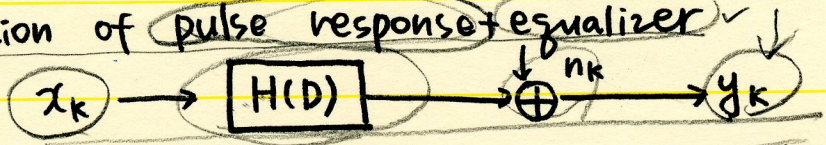
$1-D$: alternate mark inversion

high pass

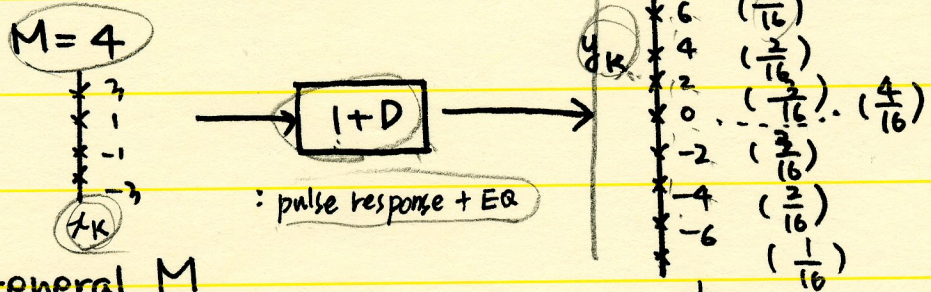
$(1+D)(1-D) = 1-D^2$ Modified duobinary

$\frac{1}{2}$

* Combination of pulse response + equalizer ✓



$H(D) = 1 + h_1 D + \cdots\cdots + h_\nu D^\nu$ without noise



General M

$-2(M-1) \quad \cdots \quad 0 \quad \cdots \quad 2(M-1)$

probability $\left(\frac{1}{M^2}\right)\left(\frac{2}{M^2}\right)\cdots\left(\frac{1}{M}\right)\cdots\left(\frac{2}{M^2}\right)\left(\frac{1}{M^2}\right)$

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$H(D) = 1+D$ PR channel

The messages at time k $M = 2$ $\begin{matrix}+1\\0\end{matrix}$

Tx differential encoder $\bar{m}_k = m_k \ominus \bar{m}_{k-1}$

$x_k = [2\bar{m}_k - (M-1)]\frac{d}{2}$ $\begin{matrix}+1\\-1\end{matrix}$ (encoder output $\bar{m}_k$ are converted into the channel input symbols $s_{(x_k)}$ according to

Rx decoder $(= \bar{m}_k \oplus \bar{m}_{k-1}) = \left(\frac{y_k}{d} + (M-1)\right)_M = m_k$ $\begin{matrix}1\\0\end{matrix}$

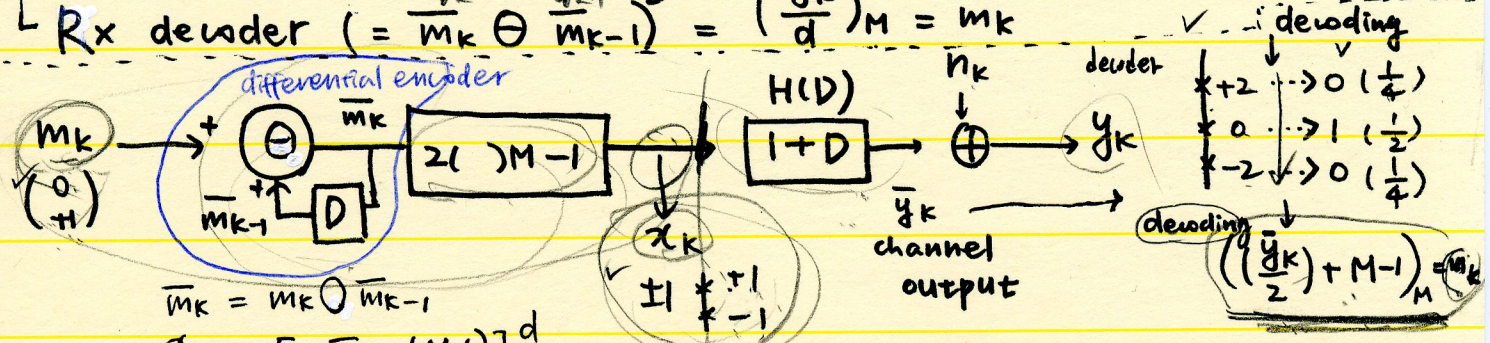
$y_k \quad y_{k-1}$

$H(D) = 1-D$

Tx differential encoder $\bar{m}_k = m_k \oplus \bar{m}_{k-1}$

$x_k = [2\bar{m}_k - (M-1)]\frac{d}{2}$

Rx decoder $(= \bar{m}_k \ominus \bar{m}_{k-1}) = \left(\frac{y_k}{d}\right)_M = m_k$



$\bar{m}_k = m_k \ominus \bar{m}_{k-1}$

$x_k = [2\bar{m}_k - (M-1)]\frac{d}{2}$

($\bar{m}_k = 0 \longrightarrow x_k = -1$

$\bar{m}_k = 1 \longrightarrow x_k = +1$

y_k	m_k
2	0
0	1
-2	0

$N_e \triangleq \sum_{i=1}^{M-1} N_i P_x(i) = 2\left(1 - \frac{1}{M^2}\right)$

Average # of nearest neighbor $\frac{2}{M^2} + 2\left(1 - \frac{2}{M^2}\right)$

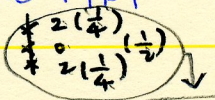
$P_e \le N_e Q\left(\frac{d_{min}}{2\sigma}\right)$

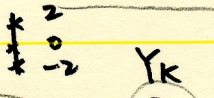
$d_{min} = 2$

The precoded partial response sys. → ① PR channel $\overline{P_e} = 2\left(1-\frac{1}{M^2}\right)Q\left(\frac{d}{2\sigma}\right)$ ≤ ZF-DFE (with EP): $\overline{P_e} = 2\left(1-\frac{1}{M}\right)MQ\left(\frac{d}{2\sigma}\right)$

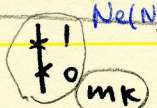
$\overline{N_e}$ of PR precoder — $\overline{N_e}$ of ZF-DFE — N_e (error prop) — N_e (No error prop)

② • No feedback
③ • No power loss (T_x)
④ • No EP
⑤ • slight neighbor increase





$\frac{2}{4}(1) + \frac{1}{2}(2) = \frac{3}{2}$ instead of 1



For a general PR channel $H(D)$, the differential encoder is

$$\overline{m_k} = m_k \overset{\vee}{\underset{i=1}{\oplus}} (-h_i \overline{m}_{k-1})$$

The noiseless outputs $\overline{y_k}$ can be mapped to the transmit message m_k as

$$m_k = \left(\frac{\overline{y_k}}{d} + \overset{\vee}{\underset{i=1}{\sum}} h_i \left(\frac{M-1}{2}\right)\right) \bmod M$$

It can also be shown that

$$N_e \le 2\left(1-\frac{1}{M^{\gamma+1}}\right)$$

$$\overline{P_e} = \overline{N_e} Q\left(\frac{d}{2\sigma}\right)$$

⑨ Diversity

$$y(t) = \sum_k x_k \underline{p}(t-kT) + \underline{n}(t) = \|p\| \sum_k x_k \underline{\varphi}_p(t-kT) + \underline{n}(t)$$

$$y(t) = \sum_{l=0}^{L-1} y_l(t) * \varphi_{p,l}^*(-t) = \|p\| \sum_k x_k q(t-kT) + n(t)$$

$\checkmark \Rightarrow$ same as |S| model where $q(t)$ contains the effect of diversity

$$Y(D) = \|p\| Q(D) X(D) + N(D)$$

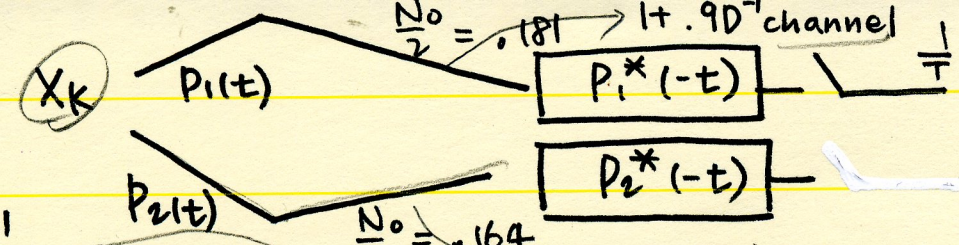
where $\|p\|^2 = \sum_{l=0}^{L-1} \|p_l\|^2$

$$q(t) = \left(\frac{\sum_{l=0}^{L-1} p_l(t) * p_l^*(-t)}{\|p\|^2} \right)$$

$$\overline{R}_{nn}(D) = \frac{N_0}{2} Q(D)$$

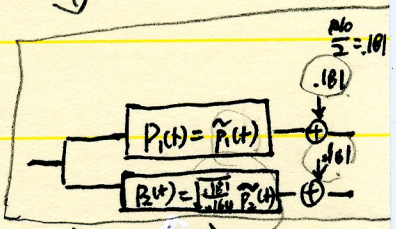
- $\checkmark$ Exactly the same form as the sampled output in single channel case.
- $\checkmark$ All previously developed equalizers apply directly to the diversity channels. (FIR ZFE, MMSE-LE, MMSE-DFE, ZF-DFE)

Diversity Example



$\bar{\mathcal{E}}_x = 1$
$M = 2$
$SNR_{mfb} = 10dB$

- ✓ whiten noise $\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{.181}{.164}} \end{bmatrix}$



Need to make the noise variance equal to proceed

Multiply channel 2 with $\sqrt{\frac{.181}{.164}}$

So, the channel set up for iid gaussian noise is $p_1(t) = p_1(t)$

and $p_2 = \sqrt{\frac{.181}{.164}}\,\tilde{p}_2(t)$ $\quad \|p\|^2 = \|p_1\|^2 + \|\tilde{p}_2\|^2 = 1.81 + \left(\frac{1.81}{.164}\right)1.64$

$= 2(1.81)$

$$SNR_{MFB} = \frac{\bar{\mathcal{E}}_x \|p\|^2}{\frac{N_0}{2}} = 13\,dB$$

$$Q(D) = \frac{1}{2(1.81)}\left[(1+.9D)(1+.9D^{-1}) + \frac{.181}{.164}(1+.8D)(1+.8D^{-1})\right]$$

$$= .492D^{-1} + 1 + .492D \quad \text{"less ISI"}$$

$$= n_0(1+.835D)(1+.835D^{-1})$$

$$\checkmark\ SNR_{ZF-DFE} = \frac{SNR_{MFB}}{1+.835^2} = 10.8\,dB \Leftarrow 10^{-4}$$

13dB − 10.8dB = 2.2dB loss