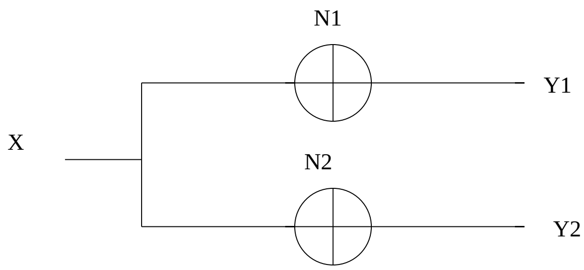
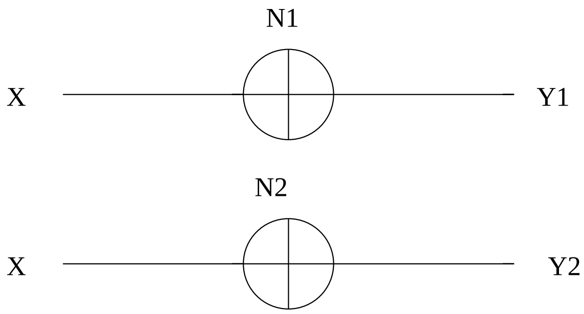
So, in this document we would like to talk about decision regions. You had a homework problem where a signal x was put through a parallel channel, so that the reciever recieved a vector output x+n_1 and x+n_2. Where n_1 and n_2 are independent. Now, the question was what were the decision regions? Well, there was an alternate way of looking at the problem, which ties into the concepts of dimension and energy.

It is common knowledge that on an AWGN channel, two independent looks at an output gives you 3dB performance gain. This can be seen by going through the mathematics, or by a more abstract way of looking at the system.

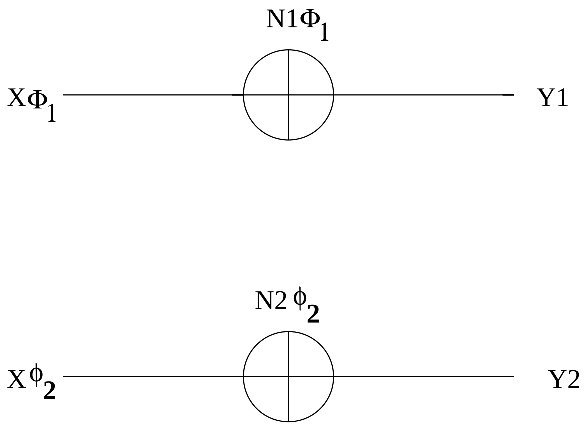
Consider the system:



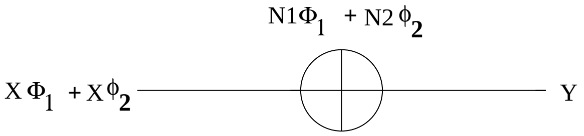
Now, this can be drawn as:



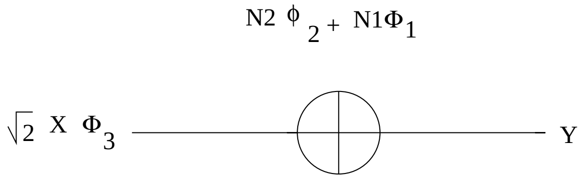
Clearly, these two channels do not depend on one another (for independent noise sources). So, we can also think of this as the analog equivalent to the above figure



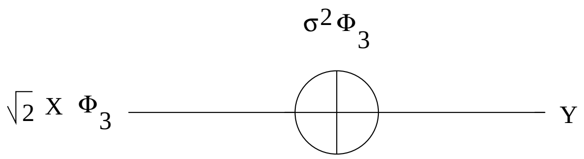
Now, since ϕ_1 and ϕ_2 are orthogonal to one another. We can rewrite the system as (Figure 4).



Now, let's consider a new basis function $\phi_3 = \frac{1}{\sqrt{2}}(\phi_1 + \phi_2)$. Then we can express the above system as



What does this say?? This shows that we are essentially sending a QAM signal whose 'x' and 'y' coordinates have the same energy as before, meaning, the total energy of the signal is twice that amount. Now, for an AWGN channel, this will give you a 3dB gain, since the projection of AWGN on any 1 dimensional space is still just $\frac{N_o}{2}$, but the energy has doubled, so the SNR has 3dB gain.



Now, this should tell you something about dimension. As long as N1 and N2 are independent, we can treat any system with 'independent looks' as a multidimensional system.

So, dimension has many important conceptual aspect to it. In the AWGN noise case, it turns out that orthogonality of the basis function implies independence of the noise, hence it makes sense to treat each orthogonal basis as another dimension, and that is the convention we use. But, it may turn out that the noise is correlated between two 'dimensions' (orthogonal basis), in which case, we must be more careful.

Finally, we discuss the implications that this has to the homework problem. We have written a program that allows the user to input a set of constellation points (in 2-D) and the noise statistics(make sure that everything is properly normalized). The noise statistics are in a .m file that are called by the program decreg.m. Look at the source code to see how it is done. To use this program, with the given problem, we need to convert the problem into the equivalent 2-dimensional model. That is, we transmit 1+i and -1-i. The noise statistics are input by the user into .m files. Once we do this, we see that the decision regions are indeed what is shown in the homework solutions. We recommend playing around with the programs to get a feel for decision regions using different noise density functions.