Discrete and Continuous Time Sampling and Convolution Review and Relationships

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Background

Conversion factor of T is required when the continuous time signal $p(t)$ is sampled and represented in discrete format p_k, where T is the sample interval used to sample $p(t)$. However T factor is also required, when we try to model the continuous time convolution and power sum calculation with it discrete time samples. This part is a little odd and this note tries to answer this both in time domain and frequency domain.

First recall the continuous time Fourier transform (CTFT) and discrete time Fourier transform (DTFT) has following relationship. (This can be found in any text books)

Relation of CTFT and DTFT pairs

Now given a continuous time signal x(t) run through a channel p(t) and the output signal is

$$y(t) = x(t) * p(t) \tag{1}$$

This can be calculated from x_k and p_k as

$$y(kT) = T \cdot x_k * p_k \tag{2}$$

To see this first let's do it in Frequency domain, assuming sample theory is satisfied and $Y(\omega)$ is non zero only in ,-$\frac{-\pi}{T} < \omega < \frac{\pi}{T}$, then we have

$$Y(e^{-j\omega T}) = \frac{1}{T} \cdot Y(\omega) = \frac{1}{T} \cdot X(\omega) \cdot P(\omega) \tag{3}$$

Noted,

$$X(\omega) = T \cdot X(e^{-jwT}) \tag{4}$$

$$P(\omega) = T \cdot P(e^{-jwT}) \tag{5}$$

Stick this into (3), we got

$$Y(e^{-j\omega T}) = T \cdot X(e^{-j\omega T}) \cdot P(e^{-j\omega T}) \tag{6}$$

And,

$$y(kT) = T \cdot x_k * p_k \tag{7}$$

To see this in time domain,

$$y(t) = \int_{-\infty}^{\infty} x(\lambda) p(t-\lambda) d\lambda \tag{8}$$

Noted that integration can be approximated with summation of slice of rectangle, the same result as (7) follows. This is better illustrating in the picture next page. And perhaps easier to understand why there is T factor for convolution.

Same process can be applied to see why power sum need a T factor as well.

$$\|p\|^2 = T \cdot \sum_{-\infty}^{\infty} \cdot |P|^2 \tag{9}$$

$y(t) \cong \sum T \cdot x_k \cdot p_{t-k} = T \cdot x_k * p_k$
$area = T \cdot x_1 \cdot p_k$
Xk
X2
X1
X0
P(t)
Pk
P2
P1
P0
t
T
T
T