Bassam El Khoury Seguias
Email bassamk@stanford.edu
5335069

Description: The following problem tries to show the detrimental effects introduced by a Zero Forcing Equalizer in terms of Noise Enhancement.

Extra Credit Problem

Consider at the transmitter, a basis function $\varphi(t)$ and consider a channel with impulse response $h(t)$ such that the resulting pulse response is $p(t) = e^{\frac{-t}{2}} u(t)$

1) Compute the norm of $p(t) = \|p\|$

2) Compute $q(t)$ and the corresponding $Q(f)$

3) Derive an expression for $W_{ZFE}(f)$

4) Suppose that the Channel is subject to AWGN with PSD = N W/Hz (assume the noise is added at the entrance of the equalizer) and that we can transmit a 50 KHz rectangular base-band signal. Calculate the noise enhancement caused by the ZFE equalizer.

5) Design a 3-tap FIR filter for the above equalizer

Solution

1) We first form $P(f)$ which is found to be equal to $P(f) = \frac{1}{0.5 + 2\pi jf} = \frac{2}{1 + 4\pi jf}$

$$\|p\|^2 = \int_{-\infty}^{\infty} P(f) \times P^*(f) df = \int_{-\infty}^{\infty} \frac{4}{1 + (4\pi f)^2} df = 4\arctan(4\pi f)_{-\infty}^{\infty} / 4\pi = 1$$

And so $\|p\| = 1$

2) $q(t) = \frac{p(t) \otimes p^*(-t)}{\|p\|^2}$

But $p^*(-t) = e^{\frac{t}{2}} u(-t)$ and so we get:

$$P^*(f^{-*}) = \frac{1}{0.5 - 2\pi jf} = \frac{2}{1 - 4\pi jf}$$

And we can compute $Q(f)$ as follows:

$$Q(f) = \frac{\frac{4}{1+(4\pi f)^2}}{1} = \frac{4}{1+(4\pi f)^2} = \frac{1}{0.25+(2\pi f)^2} = \frac{2\times(\frac{1}{2})}{(\frac{1}{2})^2+(2\pi f)^2}$$

This gives: $q(t) = e^{-\left|\frac{t}{2}\right|}$

We note that $q(0)$ = 1 and that $q(t) = q^*(-t)$

3)

$$W_{ZFE}(f) = \frac{1}{Q(f)\times\|p\|} = \frac{1}{4} + 4(\pi f)^2$$

4) Without the presence of the equalizer, the noise power would be equal to N*2*50K watts
By adding the equalizer, the noise power becomes:

$$\int_{-50K}^{50K} N\left|W_{ZFE}(f)\right|^2 df = N\int_{-50K}^{50K}(\frac{1}{4}+4\pi f^2)^2 df = 2\times N\int_{0}^{50K}(\frac{1}{16}+(4\pi f^2)^2+2\pi f^2)df = 2N(\frac{1}{16}f+\frac{16\pi^2 f^5}{5}+\frac{2\pi f^3}{3})_0^{50}$$

So we get a noise power equal to

$$2N\left(\frac{50\times10^3}{16}+\frac{16\pi^2\times(50\times10^3)^5}{5}+\frac{2\pi(50\times10^3)^3}{3}\right)$$

Taking the ratio of the noise power with equalization to that without equalization we get: 1.97×10^{20}, which is seen to be very large. This proves that ZFE equalization can drastically enhance noise power.

5) From the expression we found for $W_{ZFE}(f)$, we can see that it is itself a 3-tap equalizer with Coefficients [4*pi 0 0.25]