Intuitive understanding of the Hilbert Transform

For parts a-c, use sketches of the Fourier Transform of the following signals to determine the Hilbert Transform. Sketch the unscaled analytic Fourier Transform.

a) h(t) = 2T sinc(2Tt)

b) h(t) = 2T sinc(2Tt) sin(t)

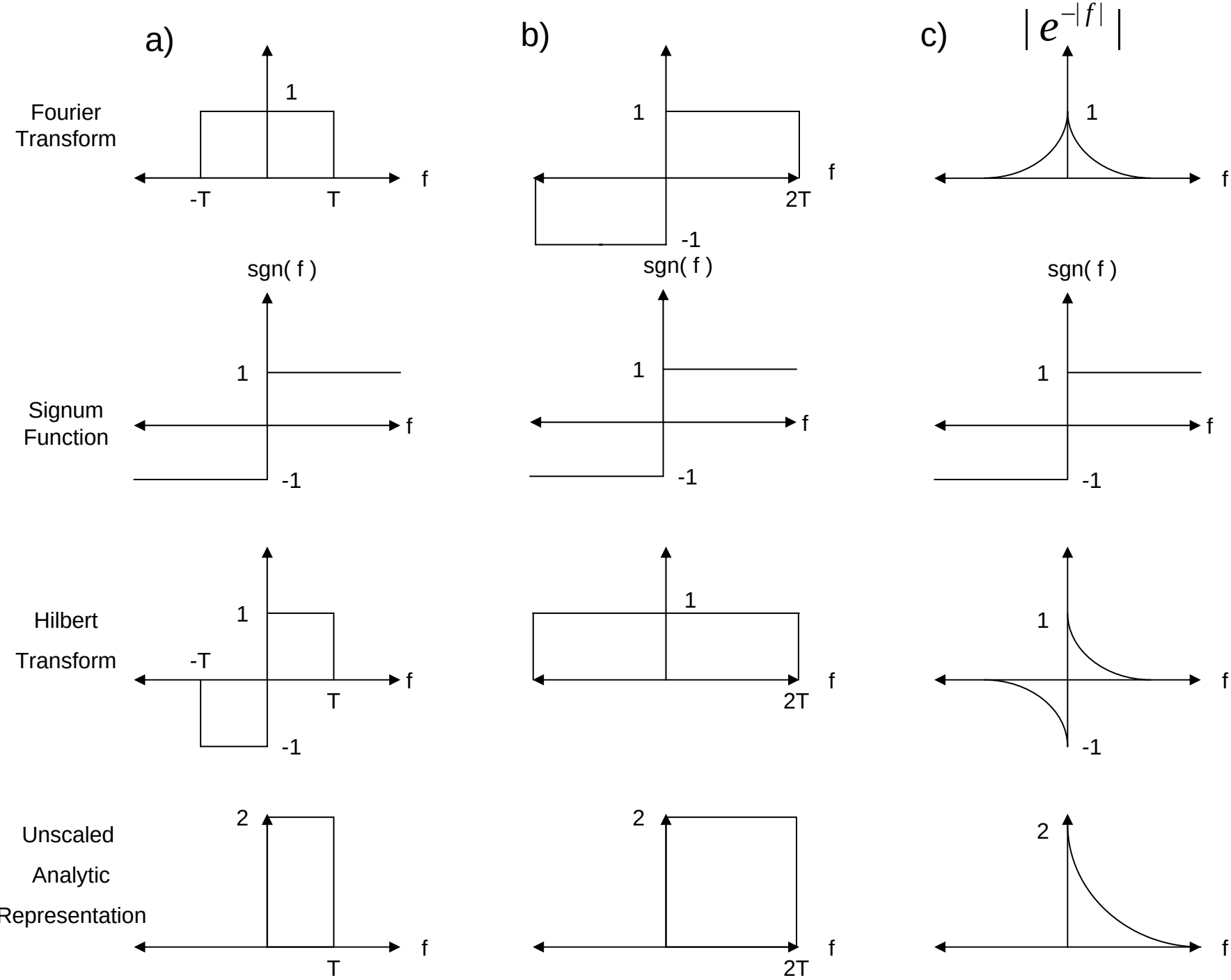
c) h(t) = $\dfrac{2}{1+(2\pi t)}$

d) Using inspection and the intuition gained from parts a-c what is the Hilbert transform of

$$h(t) = \sum_{i=-\infty}^{\infty} \cos(\omega_i t + \theta_i)\cos(t)\,?$$

Comment on the effect of the Hilbert Transform on hermitian symmetric and symmetric functions.

Hint: Remember that the Hilbert transform is defined as $\hat{H}$(f) = –j SGN(f) H(f).


a)
b)
c)
$|e^{-|f|}|$
Fourier Transform
Signum Function
Hilbert Transform
Unscaled Analytic Representation
sgn(f)
sgn(f)
sgn(f)
1
-1
-T
T
2T
2
f

a) $\hat{h}$ (t) = T sinc(Tt) sin(t)

b) $\hat{h}$ (t) = 2T sinc(2Tt) cos(t)

c) $\hat{h}$ (t) =sin(t) $\dfrac{2}{1+(2\pi t)}$

d) $\hat{h}$ (t) = $\sum_{i=-\infty}^{\infty} \sin(\omega_i t + \theta_i)\sin(t)$

The signum function will modify the Fourier Transform of the cosine function, turning it into the Fourier Transform of the sine function.

Functions with hermitian symmetry have Hilbert Transforms that are symmetric and symmetric functions have Hilbert Transforms that are hermitian symmetric.