STANFORD

Supplementary Lecture 4C

Wireless Channels

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Announcements & Agenda

- Goals
  - Familiarize, or reacquaint, interested students/researchers to obtain an $H(f)$ for a wireless channel (with antennas)
  - Permit construction of real or complex baseband-equivalent channels for analysis.

This is second of 3 short lectures online

S4A: Transmission Line

S4B: Waveguide

S4C: Wireless (with antennas)

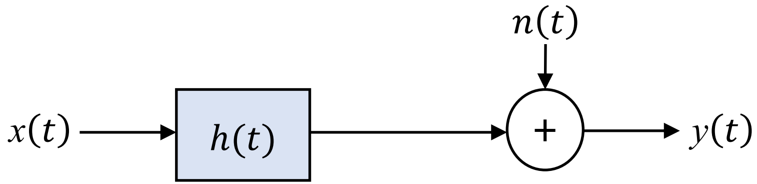
- Topics
  - Filtered AWGN
  - Isotropic and Dipole Antennas
  - Antenna arrays
  - MIMO Beam forming/steering
  - 2D array

Finding $H(f)$

Section 1.3.8

Computation of $H(f)$

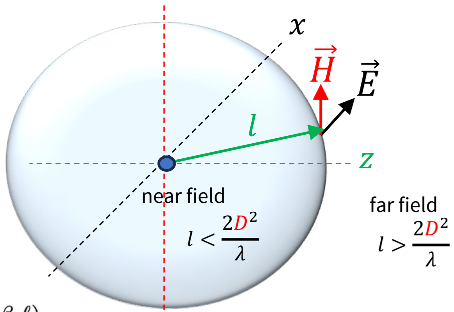
- **Channel filter** $h(t) \leftrightarrow H(f)$ from electromagnetic modelling.



- **Noise:** AWGN often specifies a PSD, like -174 dBm/Hz
  - This is room temperature noise $k \cdot T$ (Boltzman constant x Kelvin temperature).
  - Often ADC quantization and/or background noises lifts this to numbers like -150 dBm/Hz , or "noise figure" in dB increases this level
  - Unrelated radio noises can "color" the noise (must measure $S_{noise}(f)$) and then the channel model with noise-whitening receiver is $H(f) \leftrightarrow S_{noise}^{-1/2}(f) \cdot H(f)$.

Ideal antennas

Ideal istotropic (monopole)

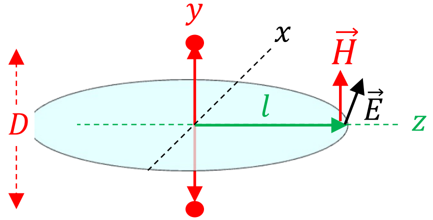


$$\vec{E} = E_0 \cdot \frac{e^{-\jmath(\omega \cdot t - \beta \cdot \ell)}}{\sqrt{4\pi \cdot \ell^2}} \quad .$$

- Isotropic baseband equivalent

$$\varphi(\omega) = E_0 \cdot \frac{e^{+\jmath\omega \cdot \ell/(\ell_o \cdot c)}}{2 \cdot (\ell/\ell_0) \cdot \sqrt{\pi}}$$

- Z_0 free space = $\sqrt{\mu_0/\epsilon_0} = 377\ \Omega$



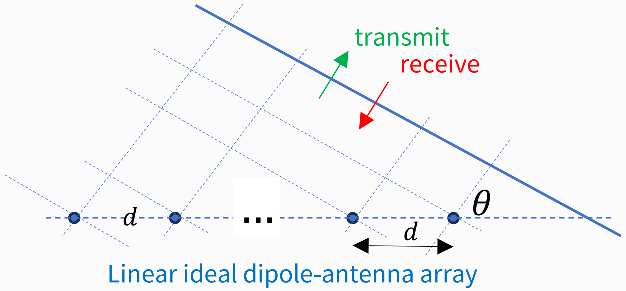
Dipole planar slice
wire vertical to plane

- Dipole planar baseband equivalent
  - Has an unnormalized basis function, and
  - Is typically fed with voltage in middle.
  - A **cross-dipole** supports 2 polarizations, 2nd in vertical disk, so directional

$$\varphi(\omega) = E_{0,dipole} \cdot \frac{e^{+\jmath\omega \cdot \ell/(\ell_o \cdot c)}}{2 \cdot (\ell/\ell_0) \cdot \sqrt{\pi}}$$

- Z_0 dipole $= {}^{377}/_{6\pi} = 73\ \Omega$
  - Whence, 75 ohm often used in cable connects.

Antenna Arrays

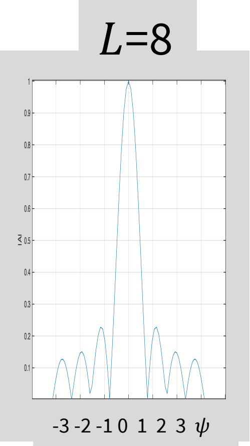


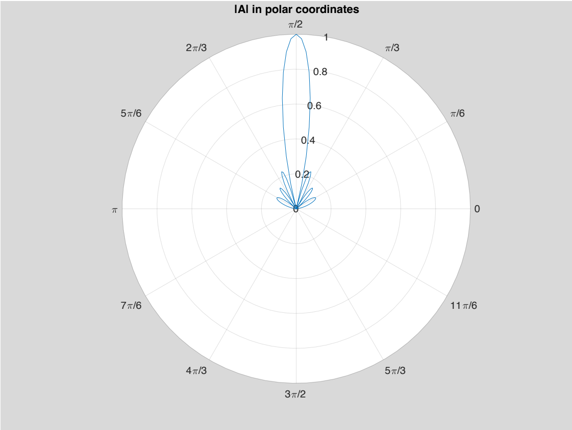
$$A(\theta) = \sum_{l=0}^{L-1} \phi_l \cdot e^{j\beta \cdot l \cdot d \cdot cos(\theta)}$$

Simplest when ϕ_l=1, so add at receiver, or transmit with equal Unit gain on all antennas.

- For receiver, the inter-antenna delays allow multiple looks.
  - Noise is uncorrelated if antenna spacing exceeds $^{\lambda}/_{2}$
  - Largest amplitude is L when $\theta = {}^{\pi}/_{2}$ with unit gains
- For transmitter, the amplitudes can be set to direct energy in any θ.
- Arrays allow transmitter and receiver to point at one another.

Equal-gain array patterns with L



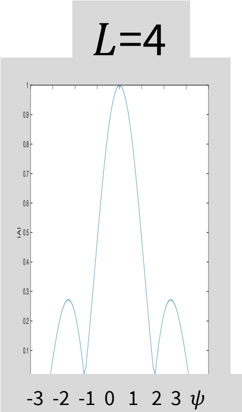


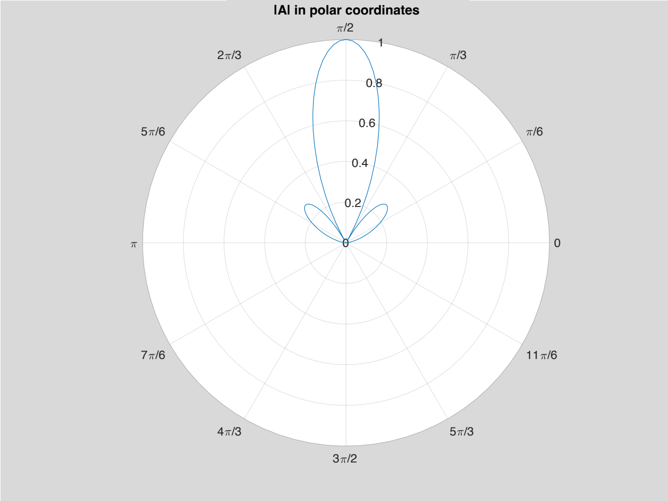
- Equal gains $\alpha_l = 1$

$$|A(\theta)| = \left| \frac{\sin(L \cdot \psi/2)}{L \cdot \sin(\psi/2)} \right|$$

$$\psi \triangleq \beta \cdot d \cdot \cos(\theta).$$

beamform (xmit)
or
beamsteer (rcvr)





MIMO Beam Forming and Steering

- Each path has $H_{c,k} \cdot \delta(t - \tau_k)$.
- **MIMO** beamform has a linear combination of different symbol-dimensions/antenna.

$$\underbrace{A_{xmit}}_{L_x \times L_x} = \begin{bmatrix} \boldsymbol{a}_0 \\ \boldsymbol{a}_1 \\ \vdots \\ \boldsymbol{a}_{L-1} \end{bmatrix}$$

Each row can steer in its own direction

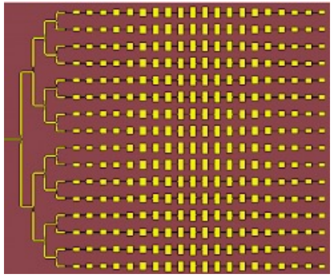
- Similarly, the receiver can linearly combine (beam steer) $L_x \times L_y$ A_{rcvr}.
- Then

$$H_k = A_{rcvr,k} \cdot H_{c,k} \cdot A_{xmit}, k$$

- Reflections are good – “rich scattering.”

2D Antenna Array

- Usually a rectangular array:



$$A(\theta,\phi) = \sum_{\ell} \mathfrak{a}_\ell \cdot e^{\jmath \cdot \beta \cdot \boldsymbol{d}_\ell \bullet \boldsymbol{r}_0}$$

$$\boldsymbol{r}_0 = \begin{bmatrix} \cos(\phi) \\ \sin(\phi)\cdot\sin(\theta) \\ \sin(\phi)\cdot\cos(\theta) \end{bmatrix}$$

spherical coordinates

$$|A(\theta,\phi)| = \left|\frac{\sin(L_x \cdot \psi_x/2)}{L_x \cdot \sin(\psi_x/2)}\right| \cdot \left|\frac{\sin(L_y \cdot \psi_y/2)}{L_y \cdot \sin(\psi_y/2)}\right|$$

- Designs adapt array coefficients to create many beam-form or beam-steer possibilities, see Chapters 4 and 5, and EE379B

End Supplementary Lecture 4C