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Supplementary Lecture 4B

Waveguide Channels

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Announcements & Agenda

- Goals
  - Familiarize, or reacquaint, interested students/researchers to obtain an $H(f)$ for a waveguide
  - Permit construction of real or complex baseband-equivalent channels for analysis.

This is second of 3 short lectures online

S4A: Transmission Line

S4B: Waveguide

S4C: Wireless (with antennas)

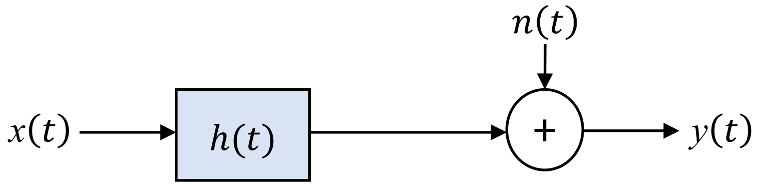
- Topics
  - Filtered AWGN
  - Waveguide cutoff frequency and model
  - Transfer function
  - Non-coherent special case
  - Coax as 2-port

Finding $H(f)$

Section 1.3.8

Computation of $H(f)$

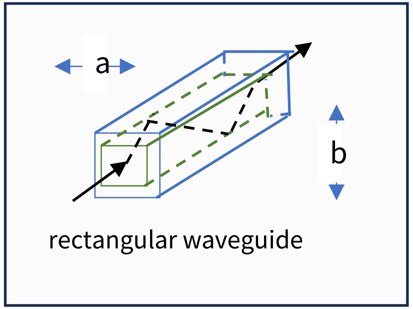
- **Channel filter** $h(t) \leftrightarrow H(f)$ from electromagnetic modelling.

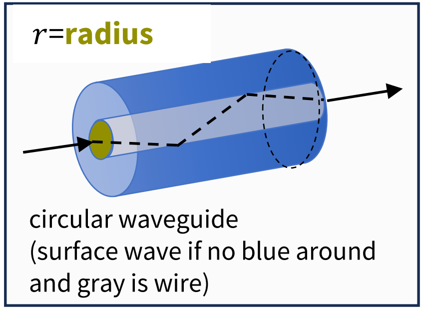


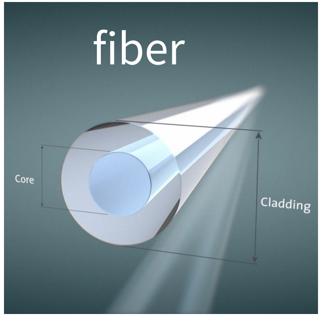
- **Noise:** AWGN often specifies a PSD, like -174 dBm/Hz
  - This is room temperature noise $k \cdot T$ (Boltzman constant x Kelvin temperature).
  - Often ADC quantization and/or background noises lifts this to numbers like -150 dBm/Hz , or "noise figure" in dB increases this level
  - Unrelated radio noises can "color" the noise (must measure $S_{noise}(f)$) and then can model with noise-whitening receiver that changes $H(f) \leftrightarrow S_{noise}^{-1/2}(f) \cdot H(f)$

Waveguides (metallic & fiber)

- Waveguides are intermediate to wireline and wireless.
  - There is a single ``wire'' (which is not necessarily metal, e.g. fiber).







- Cutoff-frequency (wavelength small enough to fit), so high/band-pass!

$$\begin{aligned}\omega_{c,rect} &= \pi \cdot c \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \text{ and} \\ \omega_{c,circ} &= \frac{c}{r} \cdot \chi'_{m.n} \text{ for TE}_{m,n} \\ &= \frac{c}{r} \cdot \chi_{m.n} \text{ for TM}_{m,n} \quad ,\end{aligned}$$

m	n	$\mathcal{J}_m(\chi_{m,n}) = 0$	$\mathcal{J}'_m(\chi'_{m,n}) = 0$
0	1	2.405	3.832
0	2	5.520	7.016
0	2	8.654	10.173
1	1	3.832	1.841
1	2	7.016	5.331
2	1	5.136	3.054
2	2	8.417	6.706
3	1	6.380	4.201

$\chi_{m,n}$ is n^{th} root/zero of $\mathcal{J}_m$, *the* m^{th} Bessel function; $\chi'_{m,n}$ is derivative's n^{th} root/zero.

Waveguide Transfer Function (Coherent passband)

$$H(\omega) = e^{-\overbrace{(\alpha+\beta(\omega))}_{\gamma}\cdot l}$$

$$\gamma = \alpha + \beta(\omega)$$

- Length is l , γ works same as transmission line formulas, so can sometimes reuse trans-line 2 ports.
- α is attenuation (tabulated for waveguides, like 0.05 dB/m, usually a constant over passband in use (min at 1.55 μm for fiber).
- β is wave number and is a function of frequency, group distortion (min at 1.3 μm for fiber).

$$\beta_{m,n}(\omega) = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

rectangular waveguide

$$\beta_{m,n}(\omega) = \frac{\omega}{c} \cdot \sqrt{1 - \left(\frac{\omega_{cut}(m,n)}{\omega}\right)^2}$$

circular waveguide

$$v_g = c \cdot \sqrt{1 - (\omega_c/\omega)^2}$$

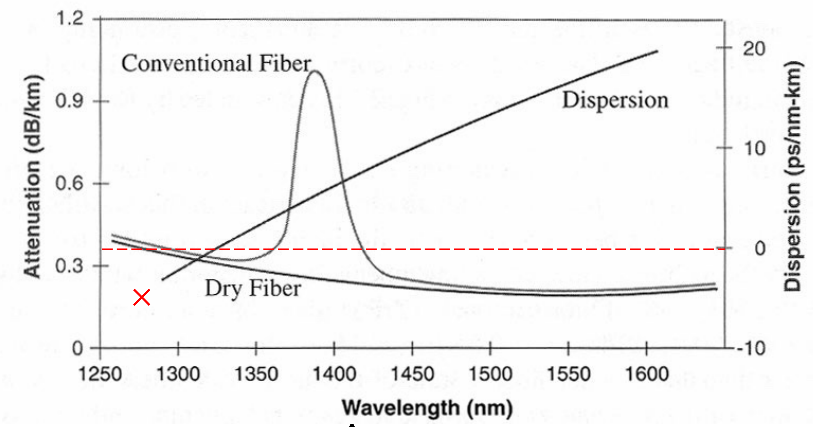
Multimode Fiber Channel – noncoherent/baseband

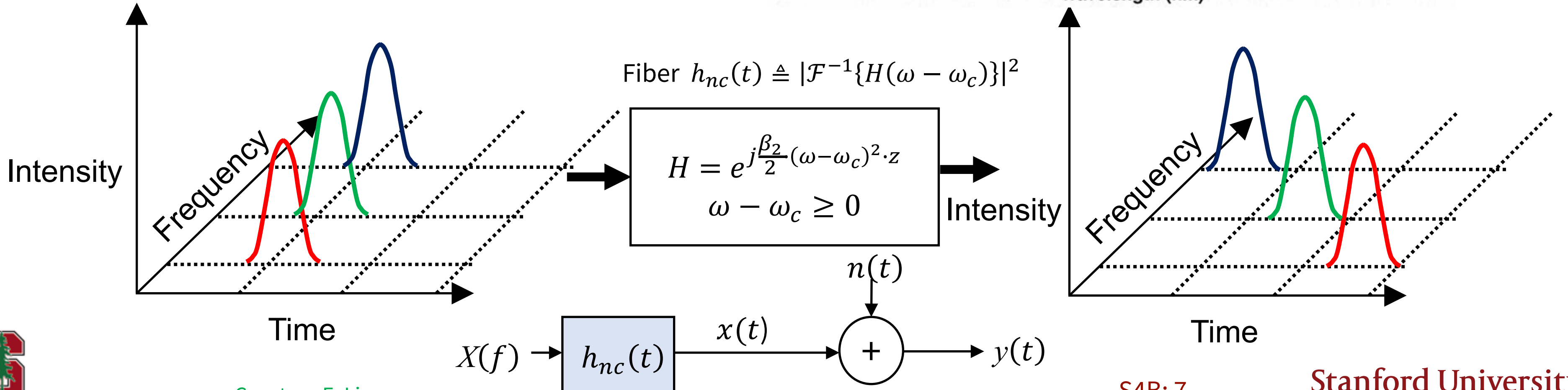
Fiber Chromatic Dispersion

- Group Delay:

$$T_g(\omega) \approx L\beta_1 + L\beta_2(\omega - \omega_c) + \cdots$$

  - β_1, β_2: power series coefficients of $\beta(\omega)$
- Focus is primarily on $\lambda = 1270$ nm:
  - $D(1270\ \mathrm{nm}) \approx -4\ \mathrm{ps/(nm \cdot km)}$
  - $D = -\frac{2\pi c}{\lambda^2} \cdot \beta_2$
  - Negative Dispersion → Positive β_2
- With no amplifier, AWGN
  - With optical amplifer, the AWGN gets squared too.



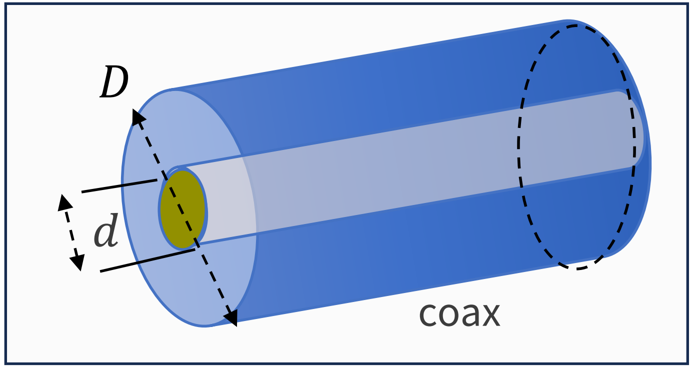


Courtesy E. Liang

Stanford University

Coax as 2-port transmission line

- Use $\gamma = \alpha + j \cdot \beta(\omega)$
- $Z_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{\mu}{\epsilon}} \cdot ln\left(\frac{D}{d}\right) = \frac{60\Omega}{\sqrt{\epsilon_r}} \cdot ln\left(\frac{D}{d}\right)$



$$\Phi_i = \begin{bmatrix} \cosh(\gamma_i \cdot l_i) & Z_{0i} \cdot \sinh(\gamma_i \cdot l_i) \\ Z_{0i}^{-1} \cdot \sinh(\gamma_i \cdot l_i) & \cosh(\gamma_i \cdot l_i) \end{bmatrix}$$

- For fiber, not this easy – model includes optical-electrical conversion.

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End Supplementary Lecture 4B