STANFORD

Lecture 5

MIMO and Statistical Channels

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Announcements & Agenda

- Announcements
  - PS2 due tomorrow
  - PS1 solutions at canvas (will try to link to web page)
  - HWH – use them as needed (HWH3 is now also at web site).

- Today
  - MIMO Channels (1.5)
  - Probabilistic Channel Models (1.6)
  - Models and Programs

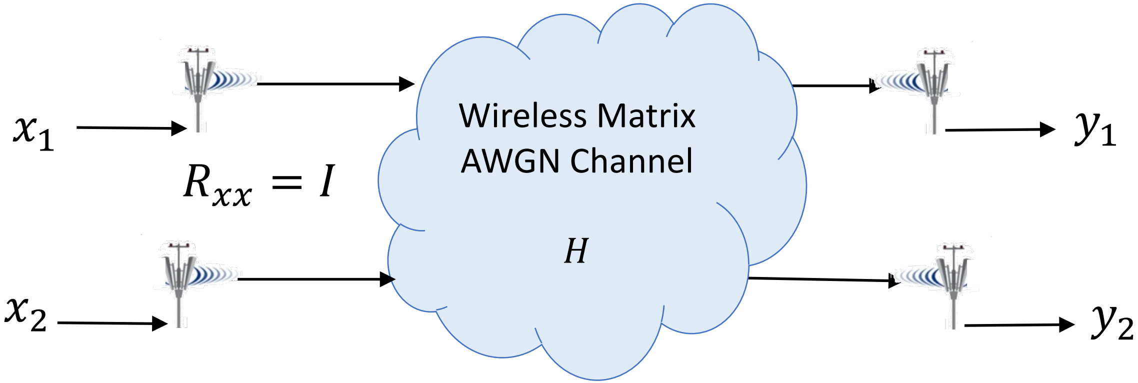
- Problem Set 3 = PS3 due Tuesday January 30 at 17:00, no late
  1. 1.63 Coherence Time and Bandwidth
  2. 1.65 Coherence Bandwidth
  3. 2.3 Log Likelihood ratios and codes
  4. 2.5 Design – Mapping 16 QAM into DMCs
  5. 2.8 Bandwidth Expansion

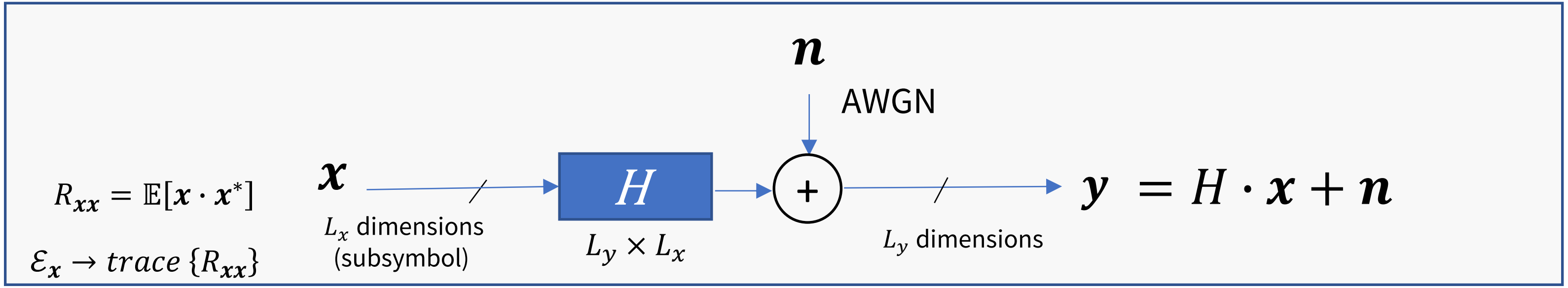
Now that we know the basic AWGN, the rest of this course (and B) focus on various collections of "little" indexed AWGN channels. How do we generate and use them well?

MIMO Channel

Section 1.5

MIMO Channel – dimensions in space (antennas)





- The dimensions are in space, so "L" (see in L6 that they best be ½ wavelength apart in most MIMO).
  - Instead of N, overall $N \cdot L$.

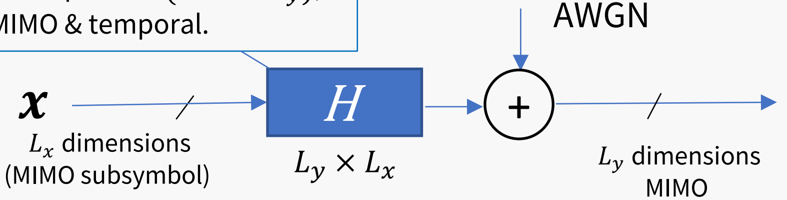
Matrix AWGN channel, time, freq, and/or space

Could also be "temporal" codewords ($N \sim L_x$): with intervening guard periods $(N + \nu \sim L_y)$, or both MIMO & temporal.

ML Detect $\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{y} - H \cdot \boldsymbol{x}\|^2$

$R_{\boldsymbol{xx}} = \mathbb{E}[\boldsymbol{x} \cdot \boldsymbol{x}^*]$

$\mathcal{E}_{\boldsymbol{x}} \rightarrow trace\,\{R_{\boldsymbol{xx}}\}$



$\boldsymbol{y} = H \cdot \boldsymbol{x} + \boldsymbol{n}$

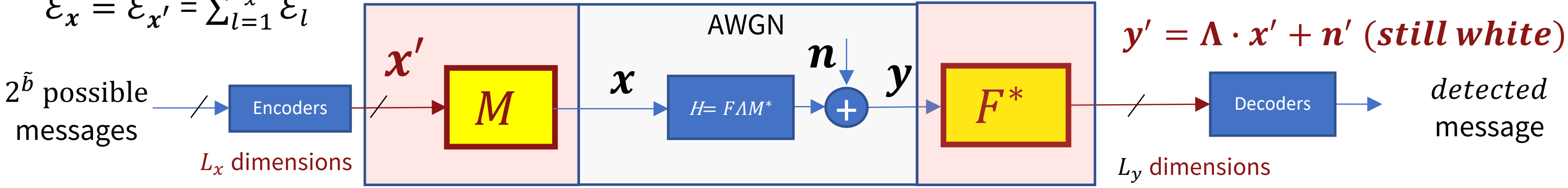
$H = F \cdot \Lambda \cdot M^*$

singular value decomposition (svd in matlab)

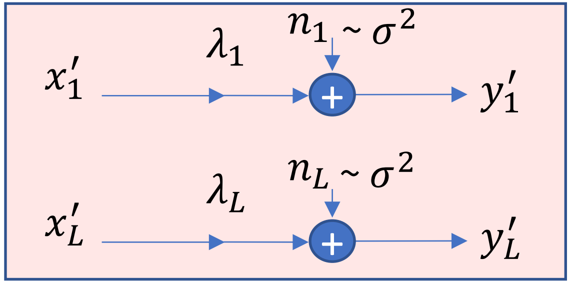
$F \cdot F^* = F^* \cdot F = I_{L_y}$; $M \cdot M^* = M^* \cdot M = I_{L_x}$

$\Lambda.\ (L_y \times L_x)$ is "diagonal" (real)

$\mathcal{E}_{\boldsymbol{x}} = \mathcal{E}_{\boldsymbol{x}'} = \sum_{l=1}^{L_x} \mathcal{E}_l$



$\boldsymbol{y}' = \boldsymbol{\Lambda} \cdot \boldsymbol{x}' + \boldsymbol{n}'$ (*still white*)



$L \leq min(L_x, L_y)$ independent dimensions

Each subchannel has its own ML detector

Overall is ML

Vector Coding (MIMO)

Kasturia 1989

Probabilistic Channels

Section 1.6

The statistically parametrized channel model

- The channel has a parameter a , so $p_{[\boldsymbol{y}\ a]/\boldsymbol{x}}$ where a is random.
  - Deterministic-parameter examples are σ^2 for the AWGN or p for the BSC -- but now a can vary randomly.
  - If a were just another channel output, then $[\boldsymbol{y}\ a] \rightarrow \boldsymbol{y}$ and all previous analysis applies.

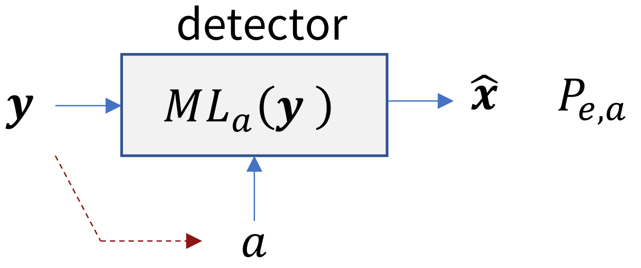
$$p_{[\boldsymbol{y}\ a]/\boldsymbol{x}} = p_{\boldsymbol{y}/[\boldsymbol{x}\ a]} \cdot \underbrace{p_{a/\boldsymbol{x}}}_{p_a}$$

Instead, $\boldsymbol{x}$ and a are independent.

- The parameter a , is somewhat like an additional message to estimate, but not exactly.
  - a can be a (continuous/discrete) random process $a(t)$, whose probability density is stationary (or "quasi-stationary").
  - The channel "varies" with the random-variable selection $a = \alpha \in \mathcal{A}$ with distribution p_a.

- The ML/MAP receiver is a function of a.
  - It has random error-probability $P_{e,a}$:

$$\langle P_e \rangle \triangleq \mathbb{E}\left[P_{e,a}\right]$$



Ergodic Analysis

- Mean value

$$\mathbb{E}[a] = \sum_{\alpha \in \mathcal{A}} \alpha \cdot p_a(\alpha)$$

- **Sample Mean**

$$\langle a \rangle_J = \frac{1}{J} \cdot \sum_{j=1}^{J} \alpha_j$$

Applies also to $f(a)$

- **Ergodic** if

$$\langle a \rangle = \lim_{J \to \infty} \langle a \rangle_J = \mathbb{E}[a]$$

- Traditional deterministic analysis has $a = constant$, so the channel represents an average over parameters.

- Ergodic analysis averages $P_{e,a}$ over a so the performance,. i.e, Compute ML's $P_{e,a}$ for each sample value.
  - Then average it.
- **Monte Carlo Analysis** – pick a values from $p_a(\alpha)$, determine $P_{e,a}$ for each, and average results.
  - Some $p_a(\alpha)$ admit a closed form expression for $\langle P_e \rangle$.

AWGN Statistical model

- The channel-transfer amplitude h now becomes random (in addition to the noise).
  - Each dimension (real or almost always complex) has a random amplitude (& phase).

$$y = h \cdot x + n$$

h ↑ random n ↑ AWGN

- This equation omits the dimensional indices: time (k), frequency (n), or space (l).
  - This will be true in every dimension (the amplitude variable may have different $p_h(\alpha)$ in different dimensions).

- **Channel gain:** Remains the important (now random) quantity:

$$g = \frac{|h|^2}{\sigma^2}$$

- **Channel gain distribution:** $p_g(v)$ derives from $p_h(\alpha)$ presuming fixed noise power spectral density.

$$p_g(v)$$

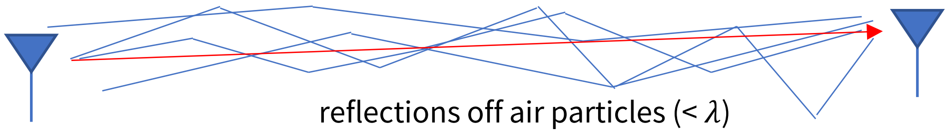
- Many statistical models find use in wireless.
- Code/modulator design can compensate for the uncertainty of h , as well as for n.

Ergodic and Confidence Analysis

- Ergodic average error probability is: $$\langle P_e \rangle = \int_{v=0}^{\infty} P_e(v) \cdot p_g(v) \cdot dv$$
- Ergodic average bits/dimension is: $$\langle \bar{b} \rangle = \int_{u=0}^{\infty} \frac{1}{2} \cdot \log_2\left(1 + \frac{\mathcal{E}(v) \cdot v}{\Gamma}\right) \cdot p_g(v) \cdot dv$$
- Outage probability P_{out} (confidence-interval that SNR is too low) is: $$P_{out} = \int_{v=0}^{g_{out}} p_g(v) \cdot dv$$
- Above threshold $\int_{v=g_{out}}^{\infty} p_g(v) \cdot dv$ is 1- P_{out} and corresponds to the usual P_e when not in outage.
  - And the outage is bad, design presumes "coin-flip" bit-error probability basically.

Scattering leads to h variation

- micro

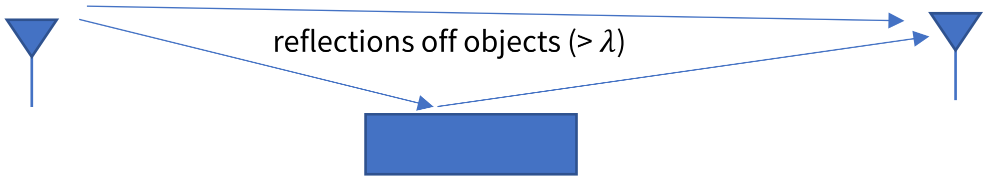


Central Limit adding on
inphase and independently
on quadrature at receiver
(each Gaussian, uniform phase)

Amplitude is Rayleigh Dist'n

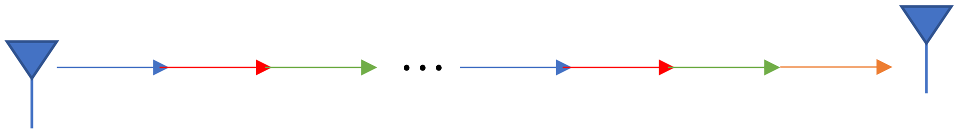
With line-of-sight (LoS) mean, Ricean

- multi-path



Sum of Rayleighs/Riceans
each with own delay ~ T
Power-Delay Profile

- macro (shadow)



Product of several attenuations
Log-product has Central Limit Thm
applying to overall gain (dB)
Lognormal, so applies to LoS mean
"link budget"

Micro Scattering: Rayleigh Fading

- **Rayleigh Fading** (micro-scattering) model uses:

$$h = \sqrt{h_I^2 + h_Q^2} \qquad \mathcal{E}_h = \mathbb{E}[|h|^2] \qquad p_h(u) = \frac{u}{\bar{\mathcal{E}}_h} \cdot e^{-\left(\frac{u^2}{2 \cdot \bar{\mathcal{E}}_h}\right)}$$

Channel gain g then has χ-squared Distribution (2 degrees) as:

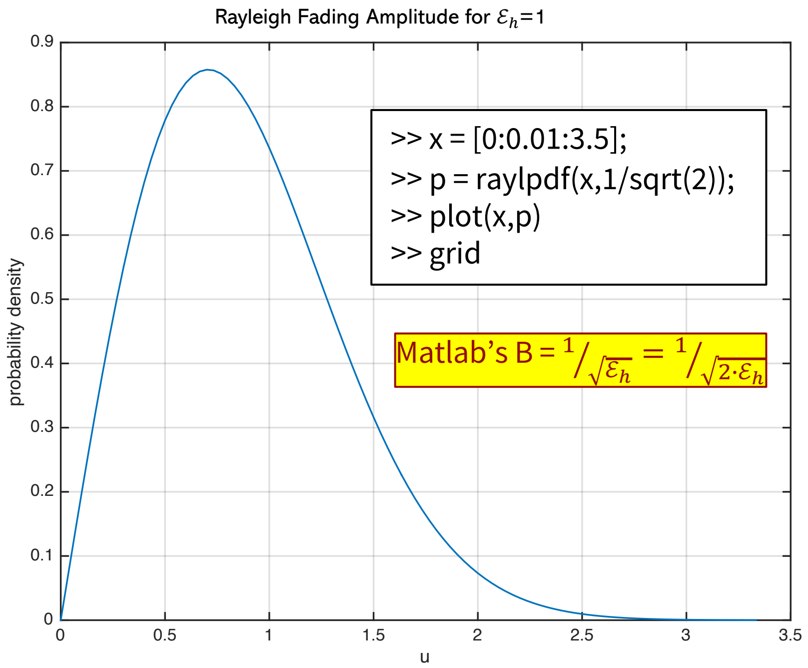
$$p_g(v) = \frac{1}{\mathcal{E}_g} \cdot e^{-\frac{v}{\mathcal{E}_g}}$$

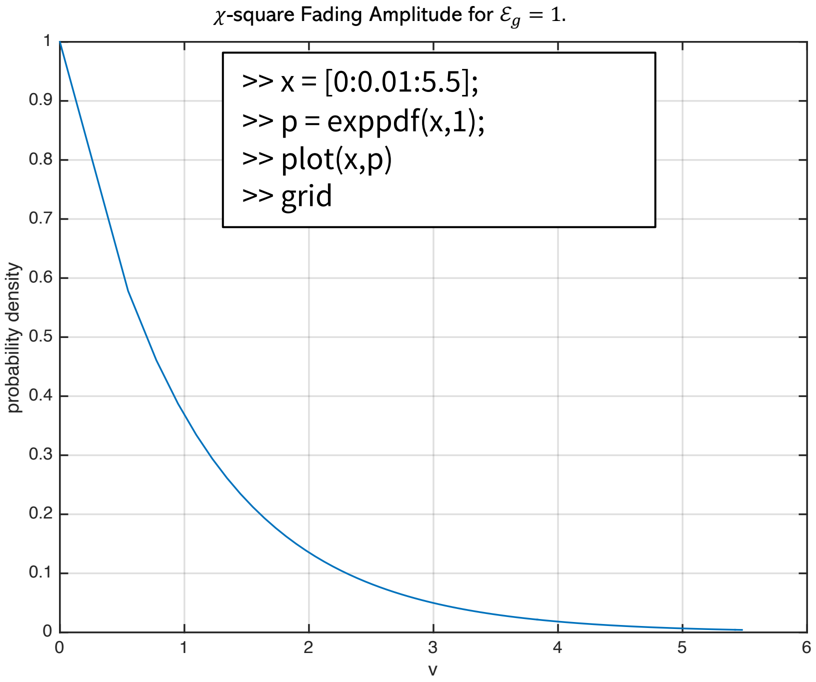
It's also called "exponential."

$$\mathbb{E}[g] = \mathcal{E}_g = \frac{\mathbb{E}[|h|^2]}{\sigma^2} |_{\bar{\mathcal{E}}_x = 1}$$

- P_e becomes random.
  - Time average = statistical average (ergodic).

Distribution Plots





- Squaring small h value makes it smaller yet (when <1), forcing more probability to the left above.

Ave Error Prob $<P_e>$

- For QPSK, $\kappa = 1$, the **average error probability** is

$$\langle \bar{P}_e \rangle = \int_0^{\infty} Q(\sqrt{\kappa \cdot g}) \cdot p_g(g) \cdot dg \; .$$

- For Rayleigh ⇨

$$\frac{1}{2} \cdot \left(1 - \sqrt{\frac{\kappa \cdot SNR}{\kappa \cdot SNR + 1}} \right) \cong \frac{1}{4\kappa \cdot SNR}$$

For large SNR

$\kappa = \frac{3}{M-1}$ for square QAM

- $\langle P_e \rangle$ only decays linearly with SNR. For 10^{-6} with QPSK, $SNR = 54$ dB >> 13.5 dB (fixed a).
  - This is misleading in that there is small probability that channel is really bad – this dominates $\langle P_e \rangle$.
  - A little help is needed when the channel is in "outage."

- *That is, the design will need good codes (which spread redundancy over all dimensions).*
  - That is, with **diversity** d (think number of message repeats for now).

$$\left[\frac{1}{4 \cdot \kappa \cdot g} \right]^{\lceil (d+1)/2 \rceil}$$

The rcvr must make ~$\lceil (d+1)/2 \rceil$sample errors to cause a symbol error, but rate decreases as $1/d$.

A little redundancy can go a long way to correct Rayleigh outages if not "too often."

Outage Probability (stability)

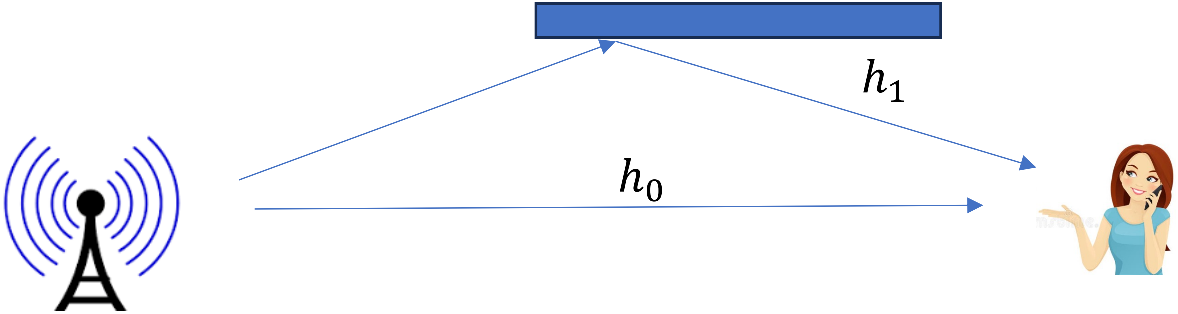
- The average $\langle P_e \rangle$ *alone* is less helpful in that the instantaneous values are also more important.
- There is a minimum SNR, and corresponding g, for which the system has too-high instantaneous P_e .
  - This **outage probability** is

$$P_{out}(\delta) = \Pr\{\, P_{e,g} > \delta \,\}.$$

- Outage is more important than single errors when its likelihood (P_{out}) is high.
  - Outages are often not like single symbol errors or bit errors that are caused by isolated large noises.
    - Outage is essentially a guarantee that (*100 x* P_{out}) % of symbols in an s-b-s decoder will be unreliable, so "flip coins."
  - Something is lost - don't want it too often. It measures user experience.
  - Most of the dimensions within the "**coherence** dimensions" will also be lost (see next slide).
- Typical outage probabilities
  - 5% is good enough for most internet traffic (translates to about a hour a day, depends on when).
  - 1% for video – only a few minutes a day when video does not work.
  - FIVE 9's (industrial or "carrier" grade – almost no faults) – .00001 -- less than 1 second per day.

Power-Delay Profile

- This generalizes the basic bandlimited channel impulse response $h(t)$ to itself a random process.



$$h(t) = h_0 \cdot \delta(t) + h_1 \cdot \delta(t - T_1) + \ \ldots$$ where h_0 and h_1 are random (~Rayleigh).

- The magnitude (power) is random, but the delays T_i are deterministic.

- So basically, the channel response sums fading AWGN's with delays → random filtered channel.

Coherence – how many "looks" have the same channel?

- **Coherence Types:**
  - **Time:** how correlated in time are the amplitude variables → the "**coherence time.**"
  - **Frequency:** how correlated in frequency are the amplitude variables → the "**coherence bandwidth.**"
  - **Space:** how correlated in space are the amplitude variables → "**spatial coherence**" (coherence length in optics).

Coherence is often measured by the correlation coefficient $\rho = \frac{\mathbb{E}\,[x^* y]}{\sigma_x \cdot \sigma_y}$.

- **Coherence Time:** T_Δ Problem 3.1 (1.63)
  - 3dB point in $\rho \propto \cos 2\pi f_d t$, so phase shift for doppler frequency f_d of moving vehicle.
    - The local receiver clock appears to shift w.r.t. best phase of the moving signal's clock.
    - Where $f_d = (v/c) \cdot f_c$ = ratio speed/light-speed times carrier-freq (m/s = .44704 x mph).
  - Symbols should occur much faster so $T \ll T_\Delta$, or at least $T < T_\Delta$ for trivial designs.

$$T_\Delta < \frac{.125}{f_d \cdot (2^{2 \cdot \bar{b}} - 1)}$$

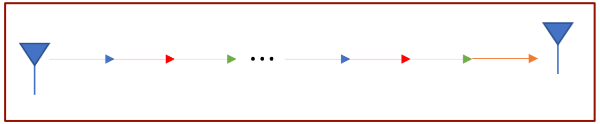
- **Coherence Bandwidth:** $W_\Delta = {}^1/_{\tau_{rms}}$ Problem 3.1 (1.63)
  - Where the rms delay spread τ_{rms} uses the power-delay profile as a probability distribution (normalizes it) to compute variance of delay around a nominal (mean) delay.
  - τ_0 is the distribution's mean value.

$$\tau_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} (t - \tau_0)^2 \cdot |h(t)|^2 \cdot dt}{\int_{-\infty}^{\infty} |h(t)|^2 \cdot dt}}$$

- **Spatial Coherence:** antenna spacing needs to be more than ½ wavelength for independence of noise
  - So signal (which is correlated) can increase amplitude coherently versus random noise
  - This concept applies to the "far field" of antenna (receiver is more than a few wavelengths removed from xmit).

$c = 3 \times 10^8$ m/s

Macro ("Shadow") Fading Model



- Lognormal is most common model for the macro fade – distribution of gain-scale factor $e^{\mu_h + \sigma_h \cdot Z}$.
  - Essentially determines a multiplier for the Rayleigh/Ricean average value, so micro is about this average value
  - Cascade of transfer functions multiply, so their logs' add. Sum many random variables and get "normal" (Gaussian) by Central Limit Theorem. So "log" is "normal" → lognormal.

LOG NORMAL with its mean and variance μ_h, σ_h^2 related to original Gaussian's h mean/variance $m_{h_0}, \sigma_{h_0}^2$ by:

$$\mu_h = \ln\left(\frac{m_{h_0}}{\sqrt{1+\frac{\sigma_{h_0}^2}{m_{h_0}^2}}}\right) \text{ and}$$

$$\sigma_h^2 = \ln\left(1+\frac{\sigma_{h_0}^2}{m_{h_0}^2}\right) .$$

Lognormal's mean and ***standard deviation*** usually specified in dB (so 10·log10 of σ_h , not σ_h^2 in this rare case of 10 log10 for s.d.).

This is "gross attenuation" that applies to all – longer channels have more attenuation.

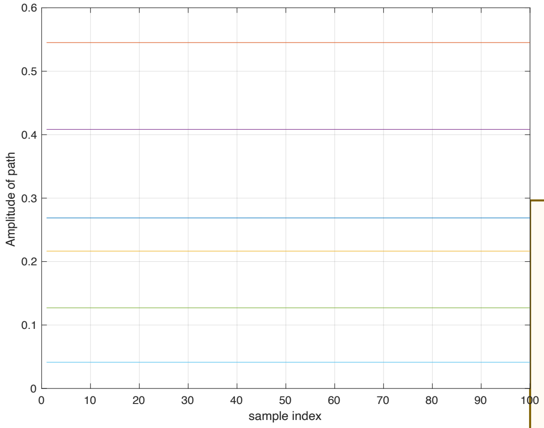
Models and Programs

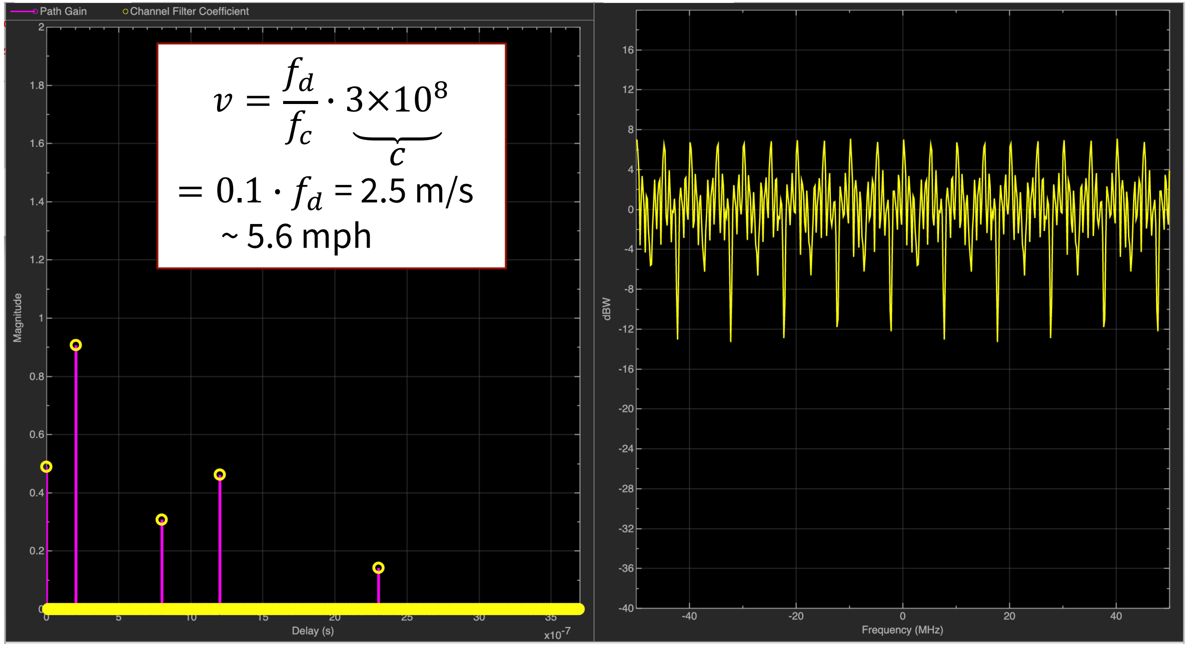
Matlab's RayleighChannel.m

- Typical mobile carrier ~ 3 GHz:
  - Need this to compute doppler from vehicle speed
  - But program Input is doppler itself, and sampling freq.

```
reset(rayleighchan)
fs = 1e8;                    % Sample rate in Hz
pathDelays = [0 200 800 1200 2300 3700]*1e-9; % in seconds
avgPathGains = [0 -0.9 -4.9 -8 -7.8 -23.9];   % dB
fD = 25;                     % Max Doppler shift in Hz

rayleighchan = comm.RayleighChannel('SampleRate',fs, ...
   'PathDelays',pathDelays, ...
   'AveragePathGains',avgPathGains, ...
   'MaximumDopplerShift',fD, ...
   'ChannelFiltering',false, ...
   'Visualization','Impulse and frequency responses');
```

```
>> pathgains=rayleighchan();
plot(abs(pathgains(1:100,:)))
```





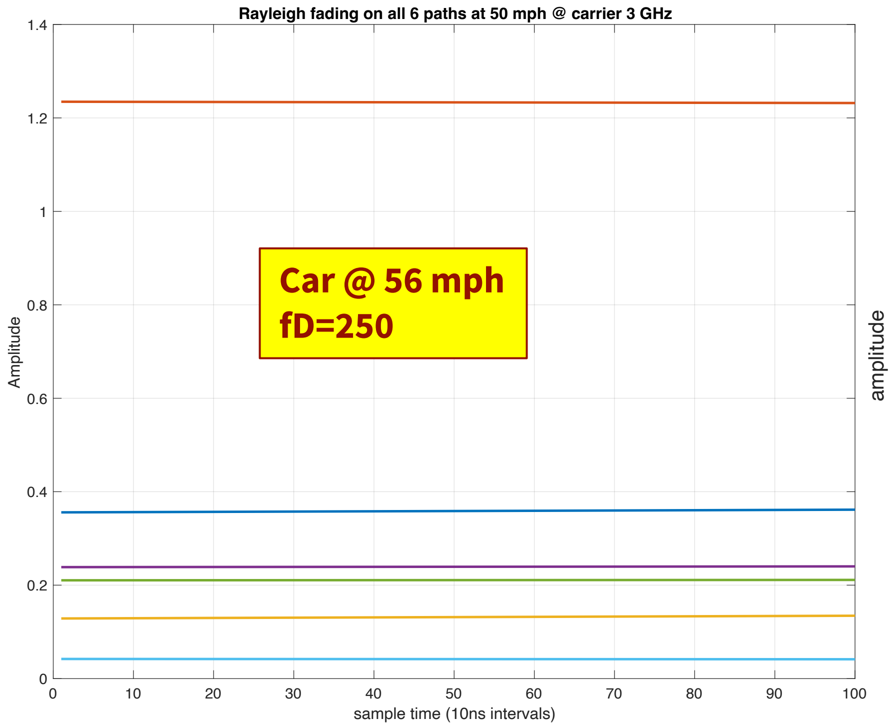
$$v = \frac{f_d}{f_c} \cdot \underbrace{3\times10^8}_{c}$$
$$= 0.1 \cdot f_d = 2.5 \text{ m/s}$$
~ 5.6 mph

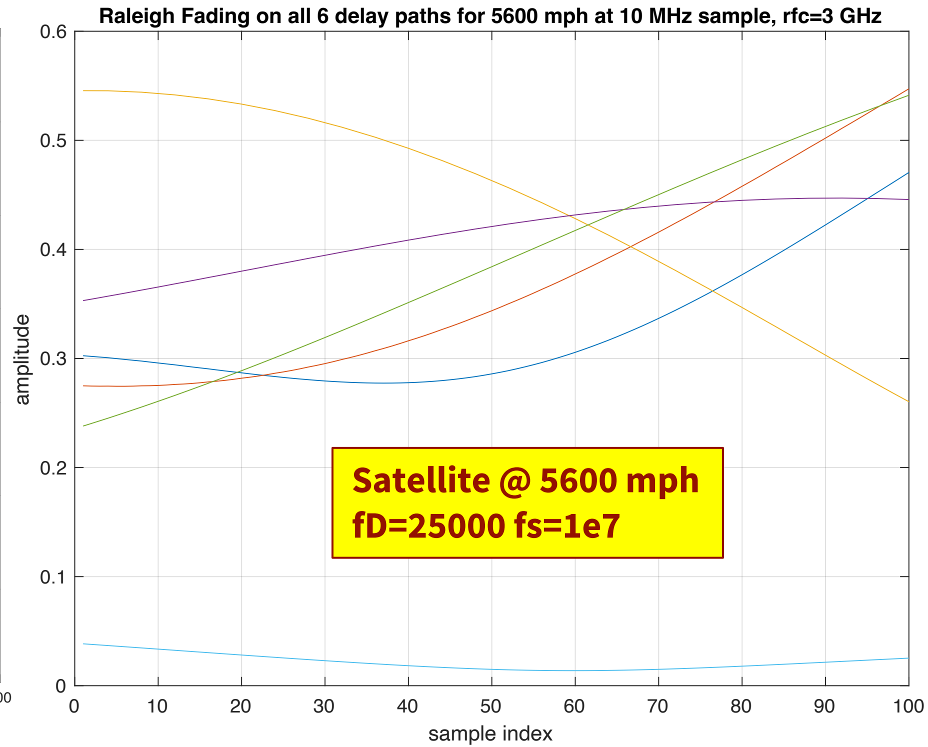
```
rayleighchan = comm.RayleighChannel('SampleRate',fs, ...
  'PathDelays',pathDelays, ...
  'AveragePathGains',avgPathGains, ...
  'MaximumDopplerShift',fD, ...
  'ChannelFiltering',false);
  pathgains=rayleighchan();
  plot(abs(pathgains(1:100,:)))

% Must reset(rayleight) if you want consistent plot here
```

"walking" movement of 5.6 mph
Fairly slow relative to 100 Msamples/sec (really no fading over reasonable-length packets), but some frequencies are notched.

Fading at different speeds

- Same channel, carrier, but faster dopper/speeds





- So wider bandwidth (100 MHz sampling) for car helps make its channel appear stationary.
- For satellite doppler has faster Rayleigh fading – note a high-redundancy code would help.

To generate Rayleigh faded outputs

```
reset(rayleighchan)

fs = 1e8;                    % Sample rate in Hz
pathDelays = [0 ]; % in seconds
avgPathGains = [0];  % dB – everything else relative to 0 dB on multipath
fD = 250;                    % Max Doppler shift in Hz
SNR=10 ;  %(dB)

rayleighchan = comm.RayleighChannel('SampleRate',fs, ...
   'PathDelays',pathDelays, ...
   'AveragePathGains',avgPathGains, ...
   'MaximumDopplerShift',fD, ...
   'NormalizePathGains',0, ...
   'PathGainsOutputPort',1);
```

Configure to generate data

```
Ex=2;  % this is kind of important to set at 2 and scale noise to it via SNR
N0=10^(-SNR/10)*Ex;  % handle channel gain through SNR w.r.t. 0 dB
   % don't need the above two for following command, but helps to remind
x=qammod(randi(4,1000,1)-1,4);
[xfade , pathgain] = rayleighchan(x);  % then this produces what you want
```

PS3.2 – need to know this

faded x values

Complex fade amplitudes

Input to fading channel;
Matlab appears to scale any input so that $\mathcal{E}_x$=2
(1 unit of energy per real dimension)

Example – "simple" Wi-Fi model

- Multipath's with Ricean/Rayleigh scattering has model:

$$h = \sum_k h_k$$

$$h_k = 10^{-L(d)/20} \cdot \sqrt{P_{h,k}} \cdot \left[\underbrace{\sqrt{\frac{K}{K+1}}}_{LOS} \cdot \delta_k + \underbrace{\sqrt{\frac{1}{K \cdot \delta_k + 1}} \cdot u_k}_{Rayleigh} \right]$$

u_k is unit-variance Gaussian

K	delay (ns)	cluster 1 $P_{1,h,k}$ (dB)	cluster 2 (dB) $P_{2,h,k}$ (dB)
0	0	0	–
1	10	-5.4	-
2	20	-10.8	-3.2
3	30	-16.2	-6.3
4	40	-21.7	-9.4
5	50	–	-12.5
6	60	–	-15.6
7	70	–	-18.7
8	80	–	-21.8

- $K = 1$ is small home; $K = 4$ is big home/office.
- $L(d)$ is the amplitude on overall (macro/shadow) fade/scattering = Path Loss (d in meters)

$$\frac{1}{\sqrt{2\pi\sigma_Z^2}} \cdot e^{-[L_{shadow}(d)]^2/(2\sigma_Z^2)}$$

$$\begin{aligned} L(d) &= L_{path}(d) + L_{shadow}(d) \quad \text{dB} \quad d \le d_{bp} \\ &= L_{path}(d_{bp}) + L_{shadow}(d_{bp}) + 35\log_{10}\left(\frac{d}{d_{bp}}\right) \text{dB} \quad d > d_{bp} \end{aligned}$$

$\sigma^2 = 3\ dB$ for L_{shadow}

(log normal)

$\sigma^2 = 4\ dB$ for L_{shadow}

Above the Break-point-distance value

where the break-point distance is $d_{bp} = 5$m for smaller homes and

$$L_{path}(d) = 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) - 147.5 \text{ dB}$$

- Run Monte Carlo simulations on this.

Thanks Samsung, modified for 379A

```
function [h, tap_delay] = get_channel(model_letter,…
N_tx, N_rx, index)

  Inputs:
  ------------------------------------------------------
  model_letter: Channel model letter, 'A' to 'F'
  N_tx        : Number of transmit antennas
  N_rx        : Number of receive antennas
  index       : index of the channel, 1 to 5000

  Outputs:
  ------------------------------------------------------
  h           : The channel matrix, N_rx x N_tx x N_taps
  tap_delay   : the tap delays in s
  tap_delay_10ns   : the tap delays in 10ns
```

- Channel Model Letters (next page) – `B' is the model below and on next page.
- The lognormal macro fading is not included and needs to be applied outside, table below

Table 3.4 Path loss model parameters

		Path loss slope		Shadow fading std. dev. (dB)		Channel conditions	
Channel model	Breakpoint distance d_{BP} (m)	Before d_{BP}	After d_{BP}	Before d_{BP}	After d_{BP}	Before d_{BP}	After d_{BP}
A	5	2	3.5	3	4	LOS	NLOS
B	5	2	3.5	3	4	LOS	NLOS
C	5	2	3.5	3	5	LOS	NLOS
D	10	2	3.5	3	5	LOS	NLOS
E	20	2	3.5	3	6	LOS	NLOS
F	30	2	3.5	3	6	LOS	NLOS

	RMS delay		
Model	Spread (ns)	Environment	Example
A	0	N/A	N/A
B	15	Residential	Intra-room, room-to-room
C	30	Residential/small office	Conference room, classroom
D	50	Typical office	Sea of cubes, large conference room
E	100	Large office	Multi-story office, campus small hotspot
F	150	Large space (indoors/outdoors)	Large hotspot, industrial, city square

This program provides samples @ 100 MHz, so needs resampling to 20, 40, 80, 160, 320 … MHz Wi-Fi channel bandwidths ∝ symbol rate of 250 kHz.

Eldad Perahia & Robert Stacey ,
2013, Cambridge Press

Different Letters

Table 3.5 Channel model A (Erceg *et al.*, 2004)

Tap index	Excess delay [ns]	Power [dB]	Cluster 1 AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]
1	0	0	45	40	45	40

Table 3.6 Channel model B (Erceg *et al.*, 2004)

Tap index	Excess delay [ns]	Cluster 1 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]	Cluster 2 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]
1	0	0	4.3	14.4	225.1	14.4					
2	10	−5.4	4.3	14.4	225.1	14.4					
3	20	−10.8	4.3	14.4	225.1	14.4	−3.2	118.4	25.2	106.5	25.4
4	30	−16.2	4.3	14.4	225.1	14.4	−6.3	118.4	25.2	106.5	25.4
5	40	−21.7	4.3	14.4	225.1	14.4	−9.4	118.4	25.2	106.5	25.4
6	50						−12.5	118.4	25.2	106.5	25.4
7	60						−15.6	118.4	25.2	106.5	25.4
8	70						−18.7	118.4	25.2	106.5	25.4
9	80						−21.8	118.4	25.2	106.5	25.4

Table 3.7 Channel model C (Erceg *et al.*, 2004)

Tap index	Excess delay [ns]	Cluster 1 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]	Cluster 2 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]
1	0	0	290.3	24.6	13.5	24.7					
2	10	−2.1	290.3	24.6	13.5	24.7					
3	20	−4.3	290.3	24.6	13.5	24.7					
4	30	−6.5	290.3	24.6	13.5	24.7					
5	40	−8.6	290.3	24.6	13.5	24.7					
6	50	−10.8	290.3	24.6	13.5	24.7					
7	60	−13.0	290.3	24.6	13.5	24.7	−5.0	332.3	22.4	56.4	22.5
8	70	−15.2	290.3	24.6	13.5	24.7	−7.2	332.3	22.4	56.4	22.5
9	80	−17.3	290.3	24.6	13.5	24.7	−9.3	332.3	22.4	56.4	22.5
10	90	−19.5	290.3	24.6	13.5	24.7	−11.5	332.3	22.4	56.4	22.5
11	110						−13.7	332.3	22.4	56.4	22.5
12	140						−15.8	332.3	22.4	56.4	22.5
13	170						−18.0	332.3	22.4	56.4	22.5
14	200						−20.2	332.3	22.4	56.4	22.5

Table 3.8 Channel model D (Erceg *et al.*, 2004)

Tap index	Excess delay [ns]	Cluster 1 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]	Cluster 2 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]	Cluster 3 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]
1	0	0	158.9	27.7	332.1	27.4										
2	10	−0.9	158.9	27.7	332.1	27.4										
3	20	−1.7	158.9	27.7	332.1	27.4										
4	30	−2.6	158.9	27.7	332.1	27.4										
5	40	−3.5	158.9	27.7	332.1	27.4										
6	50	−4.3	158.9	27.7	332.1	27.4										
7	60	−5.2	158.9	27.7	332.1	27.4										
8	70	−6.1	158.9	27.7	332.1	27.4										
9	80	−6.9	158.9	27.7	332.1	27.4										
10	90	−7.8	158.9	27.7	332.1	27.4										
11	110	−9.0	158.9	27.7	332.1	27.4	−6.6	320.2	31.4	49.3	32.1					
12	140	−11.1	158.9	27.7	332.1	27.4	−9.5	320.2	31.4	49.3	32.1					
13	170	−13.7	158.9	27.7	332.1	27.4	−12.1	320.2	31.4	49.3	32.1					
14	200	−16.3	158.9	27.7	332.1	27.4	−14.7	320.2	31.4	49.3	32.1					
15	240	−19.3	158.9	27.7	332.1	27.4	−17.4	320.2	31.4	49.3	32.1	−18.8	276.1	37.4	275.9	36.8
16	290	−23.2	158.9	27.7	332.1	27.4	−21.9	320.2	31.4	49.3	32.1	−23.2	276.1	37.4	275.9	36.8
17	340						−25.5	320.2	31.4	49.3	32.1	−25.2	276.1	37.4	275.9	36.8
18	390											−26.7	276.1	37.4	275.9	36.8

Table 3.11 Channel model F, clusters 1, 2, and 3 (Erceg *et al.*, 2004)

Tap index	Excess delay [ns]	Cluster 1 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]	Cluster 2 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]	Cluster 3 Power [dB]	AoA [°]	AS Rx [°]	AoD [°]	AS Tx [°]
1	0	−3.3	315.1	48.0	56.2	41.6										
2	10	−3.6	315.1	48.0	56.2	41.6										
3	20	−3.9	315.1	48.0	56.2	41.6										
4	30	−4.2	315.1	48.0	56.2	41.6										
5	50	−4.6	315.1	48.0	56.2	41.6	−1.8	180.4	55.0	183.7	55.2					
6	80	−5.3	315.1	48.0	56.2	41.6	−2.8	180.4	55.0	183.7	55.2					
7	110	−6.2	315.1	48.0	56.2	41.6	−3.5	180.4	55.0	183.7	55.2					
8	140	−7.1	315.1	48.0	56.2	41.6	−4.4	180.4	55.0	183.7	55.2					
9	180	−8.2	315.1	48.0	56.2	41.6	−5.3	180.4	55.0	183.7	55.2	−5.7	74.7	42.0	153.0	47.4
10	230	−9.5	315.1	48.0	56.2	41.6	−7.4	180.4	55.0	183.7	55.2	−6.7	74.7	42.0	153.0	47.4
11	280	−11.0	315.1	48.0	56.2	41.6	−7.0	180.4	55.0	183.7	55.2	−10.4	74.7	42.0	153.0	47.4
12	330	−12.5	315.1	48.0	56.2	41.6	−10.3	180.4	55.0	183.7	55.2	−9.6	74.7	42.0	153.0	47.4
13	400	−14.3	315.1	48.0	56.2	41.6	−10.4	180.4	55.0	183.7	55.2	−14.1	74.7	42.0	153.0	47.4
14	490	−16.7	315.1	48.0	56.2	41.6	−13.8	180.4	55.0	183.7	55.2	−12.7	74.7	42.0	153.0	47.4
15	600	−19.9	315.1	48.0	56.2	41.6	−15.7	180.4	55.0	183.7	55.2	−18.5	74.7	42.0	153.0	47.4
16	730						−19.9	180.4	55.0	183.7	55.2					

- Channel E has 4 clusters

Combine two programs – Wi-Fi

function [fad_mean_db, fad_std_db] = **get_fading**(model_letter, dist, fc)

get_fading provides log-normal distribution of path loss + shadow fading loss
(in dB).

TO GENERATE FADING SAMPLES: use fad_mean_db + fad_std_db*randn().

Inputs:
- model_letter: channel model letter, 'B' or 'D'
- dist: distance between Tx and Rx in meter
- fc: carrier frequency

Outputs:
- fad_mean_db: mean of the fading (in dB)
- fad_std_db: standard deviation of the log-normal fading (in dB)

Stanford student program:

Extra Credit Project:
expand get_fading to include other letters

$$h_k = \underbrace{10^{-L(d)/20}}_{get\_fading} \cdot \underbrace{\sqrt{P_{h,k}} \cdot \left[\underbrace{\sqrt{\frac{K}{K+1}}}_{LOS} \cdot \delta_k + \underbrace{\sqrt{\frac{1}{K \cdot \delta_k + 1}} \cdot u_k}_{Rayleigh} \right]}_{get\_channel}$$

- Can use these two programs to generate wireless “indoor” channels

IEEE Model - For wider bands and MIMO

Table 3.2 Channel sampling rate expansion (tap spacing reduction) factors (Breit *et al.*, 2010)

System bandwidth W (MHz)	Channel sampling rate expansion factor (k)	Tap spacing (ns)
$W \leq 40$ (802.11n)	1	10
$40 < W \leq 80$	2	5
$80 < W \leq 160$	4	2.5
$160 < W \leq 320$	8	1.25
$320 < W \leq 640$	16	0.625
$640 < W \leq 1280$	32	0.3125

- Multiple antennas

The shape of the PAS distribution commonly used for 802.11n is a truncated Laplacian. The PAS distribution over the angle for each tap is given by

$$\mathrm{PAS}(\phi) = \frac{1}{A}\sum_{k=1}^{N_C}\frac{p_k}{\sigma_k}\exp\left[\frac{-\sqrt{2}|\phi-\psi_k|}{\sigma_k}\right] \quad (3.18)$$

where N_C is the number of clusters, and for each cluster k, p_k is the tap power, σ_k is the tap AS, and ψ_k is the tap angle of incidence. Since the PAS is a probability density function, it must fulfill the requirement that $\int_{-\pi}^{\pi}\mathrm{PAS}(\varphi)d\varphi = 1$. Therefore A is equal to $\int_{-\pi}^{\pi}\sum_{k=1}^{N_C}(p_k/\sigma_k)exp[\sqrt{2}|\phi-\psi_k|/\sigma k]\,d\phi$. Figure 3.9 illustrates the distribution function for the third tap of channel model B for each cluster for Rx (using parameters from Table 3.6). The sum over the clusters at each angle results in PAS(φ).

For a uniform linear antenna array, the correlation of the fading between two antennas spaced D apart is described by Lee (1973). The correlation functions are given in Erceg *et al.* (2004), as follows:

$$R_{XX}(D) = \int_{-\pi}^{\pi}\cos\left(\frac{2\pi D}{\lambda}\sin\phi\right)\mathrm{PAS}(\phi)\,d\phi \quad (3.19)$$

and

$$R_{XY}(D) = \int_{-\pi}^{\pi}\sin\left(\frac{2\pi D}{\lambda}\sin\phi\right)\mathrm{PAS}(\phi)\,d\phi \quad (3.20)$$

$$\rho = R_{XX}(D) + \mathrm{j}\,R_{XY}(D)$$

Eldad Perahia & Robert Stacey, 2013, Cambridge Press

Outdoor (cellular) models

- Matlab has a lteFadingChannel.m program that is similar and includes doppler frequencies.
- I've not tested nor used it.
- There is a model 5Gmodel that is more complex, but follows same basics for those interested.
  - Does not appear to be in matlab yet.
- You now have the basic idea. EE359 appears to cover much more these models.

Each wireless channel – whatever sample, from whatever distribution, needs a design:

1. adaptive modulator choices
2. adaptive demodulator choices
3. code and data-rate choice

That design is 379A/B's focus. Certain types of channels need more or less complexity for such design.   379A/B builds your design insight. The models' details are less important than the ability to implement 1-3, after specific H (and noise) live identification.

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End Lecture 5