STANFORD

Lecture 15

Decision Feedback Equalizers

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Announcements & Agenda

- Announcements
  - PS7 due next Wed (March 6) – last required
  - PS8 is optional (but relatively easy) and due March 12 (solutions immediate)

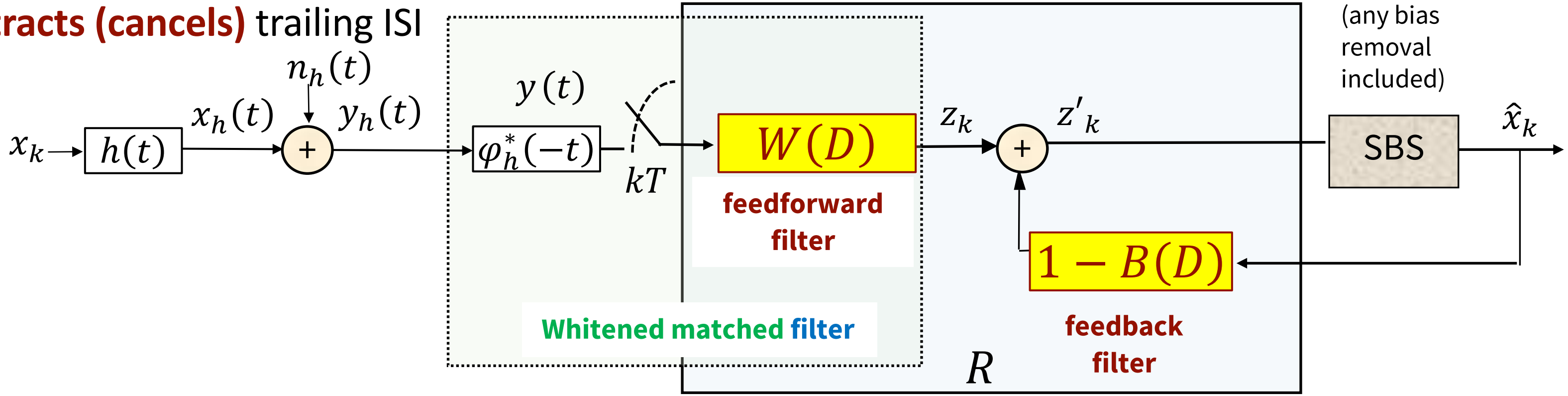
- Today
  - Continue Decision Feedback MMSE
  - Examples
  - The Whitened Matched Filter and Zero Forcing
  - FIR Implementation

Decision Feedback (successive decoding)

Section 3.6

Add previous-decision use

- **DFE Subtracts (cancels)** trailing ISI



- MMSE optimizes both feedforward, $W(D)$, and feedback, $B(D)$, filters.

Definition 3.6.1 [Mean Square Error (for DFE)] *The* **MMSE-DFE error** *signal is*

$$e_k = x_k - z'_k \quad . \tag{3.218}$$

The MMSE *for the* MMSE-DFE *is*

$$\sigma^2_{MMSE-DFE} \triangleq \min_{w_k, b_k} \mathbb{E}\left[|x_k - z'_k|^2\right] \quad . \tag{3.219}$$

- Subtraction of ISI eliminates noise enhancement (almost entirely, even though only “past” ISI),
  - but lowers received message energy w.r.t. MFB.

MMSE-DFE Solution

- MMSE estimates are linear (see Appendix D) , so for any given $B(D)$ acting on x_k:

$$W_{MMSE-DFE}(D) = B(D) \cdot W_{MMSE-LE}(D) = \frac{B(D)}{\|h\| \cdot \left[Q(D) + \frac{1}{SNR_{MFB}}\right]}$$

$$E_{MMSE-DFE}(D) = B(D) \cdot E_{MMSE-LE}(D)$$

Full math details in 3.6

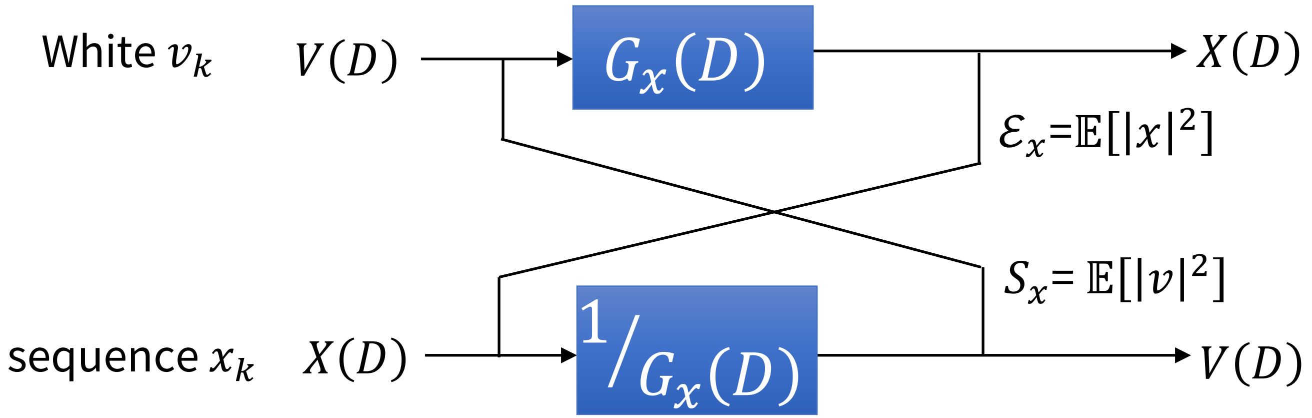
- Further $\underbrace{R_{ee,dfe}(D)}_{DFE\ error\ autocorellation} = B(D) \cdot \underbrace{R_{ee,le}(D)}_{LE\ error\ autocorellation} \cdot B^*(D^{-*})$

- Optimization considers all causal monic ($b_0 = 1$) $B(D)$.

- The LE had autocorrelation $R_{ee,le}(D) = \dfrac{\frac{\mathcal{N}_0}{2}}{\|h\|^2 \cdot (Q(D) + 1/SNR_{MFB})}$

USE: Canonical (Spectral) Factorization (D.3.4)

- Realizes a spectrum magnitude with white input into a causal, and 1-to-1 invertible, filter.



stationary

- The power spectral density of x_k is $R_{xx}(D)$ with $D = e^{-j\omega T}$.
- Find $G_x(D)$?

$G_x(D) = 1 + g_1 \cdot D + g_2 \cdot D^2 + \cdots$
All poles/zeros outside unit circle

$G_x^*(D^{-*}) = 1 + g_1^* \cdot D^{-1} + g_2^* \cdot D^{-2} + \cdots$
All poles/zeros inside unit circle

$$R_{xx}(D) = \underbrace{G_x(D)}_{\substack{monic \\ min-phase}} \cdot \underbrace{S_x}_{\substack{>0 \\ real}} \cdot G_x^*(D^{-*})$$

Spectral factorization of $R_{xx}(D)$

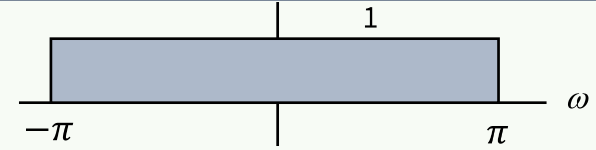
Paley Weiner Criterion

- Well, but we cannot always do this factorization (even if it is a power spectral density); it must satisfy:

$$\frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} |\ln R_{xx}(e^{-\jmath\omega})| d\omega < \infty$$

- No "dead zones"

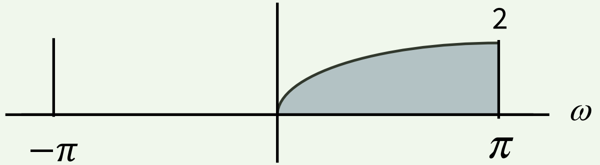
(a) - yes



$S_x\left(e^{-j\omega T}\right) = 1$

$G_x(D) = 1$; $S_x = \bar{S}_x = 1$

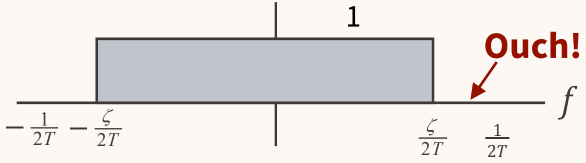
(b) - yes



$S_x\left(e^{-j\omega T}\right) = 2 - 2 \cdot \cos(\omega T)$

$G_x(D) = 1 - D$; $S_x = \bar{S}_x = 1$

(c) - no



Needs resampling @ $\frac{\zeta}{T}$

Then @ $\frac{\zeta}{T}$; $G_x(D) = 1$; $\bar{S}_x = 1$

Back to DFE MMSE (error autocorrelation)

$$Q(D) + \frac{1}{SNR_{MFB}} = \gamma_0 \cdot G(D) \cdot G^*(D^{-*}),$$

- Always satisfies PWC for finite $SNR_{MFB} > 0$.

$$\bar{R}_{ee}(D) = \frac{B(D) \cdot B^*(D^{-*})}{Q(D) + {}^1/_{SNR_{MFB}}} \cdot \frac{\frac{\mathcal{N}_0}{2}}{\|h\|^2}$$

$$= \frac{B(D)}{G(D)} \cdot \frac{B^*(D^{-*})}{G^*(D^{-*})} \cdot \frac{\frac{\mathcal{N}_0}{2}}{\gamma_0 \cdot \|h\|^2} = \frac{\frac{\mathcal{N}_0}{2} \cdot \|b/g\|^2}{\gamma_0 \cdot \|h\|^2}$$

The fractional polynomial inside the squared norm b/g is necessarily monic and causal, and therefore the squared norm has a minimum value of 1.

Minimum when $B(D) = G(D)$

$$\bar{r}_{ee,0} \geq \frac{\frac{\mathcal{N}_0}{2}}{\gamma_0 \cdot \|h\|^2}$$

Minimum value for MSE-DFE

MMSE-DFE Best Settings

- Detailed math to check previous slides sketch is in Section 3.6.

Lemma 3.6.1 [MMSE-DFE] *The MMSE-DFE has* **feedforward section**

$$W(D) = \frac{1}{\|h\| \cdot \gamma_0 \cdot G^*(D^{-*})} \quad (3.232)$$

(realized with delay, as it is strictly noncausal) and **feedback section**

$$B(D) = G(D) \quad (3.233)$$

where $G(D)$ is the unique **canonical factor** *of:*

$$Q(D) + \frac{1}{SNR_{MFB}} = \gamma_0 \cdot G(D) \cdot G^*(D^{-*}) \quad . \quad (3.234)$$

*This text also calls the joint matched-filter/sampler/$W(D)$ combination in the forward path of the DFE the "***Mean-Square Whitened Matched Filter (MS-WMF)***". These settings for the MMSE-DFE minimize the MSE as was shown above.*

MS-WMF =
WMF, if infinite SNR

The MMSE value itself

- Take log of factorization:

$$\frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \ln\left(Q(e^{-\jmath\omega T}) + \frac{1}{SNR_{MFB}}\right)\cdot d\omega \;=\; \ln(\gamma_0) + \frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \ln\left(G(e^{-\jmath\omega T})\cdot G^*(e^{-\jmath\omega T})\right)\cdot d\omega$$

Periodic over one period (0 ave)

$$= \ln(\gamma_0) \quad . \qquad (3.236)$$

- Salz Formula (1977):

$$\sigma^2_{MMSE-DFE} = \frac{\frac{\mathcal{N}_0}{2}}{\|h\|^2}\cdot e^{-\frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \ln\left(Q(e^{-\jmath\omega T}) + \frac{1}{SNR_{MFB}}\right)\cdot d\omega}$$

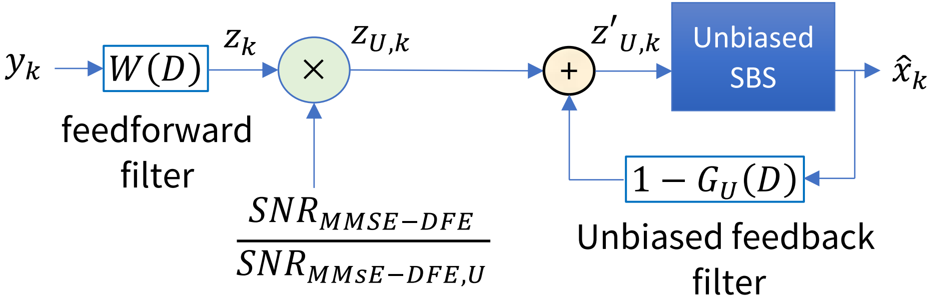
- Finally, back to our receiver SNR-based analysis:

$$SNR_{MMSE-DFE} \;=\; \frac{\bar{\mathcal{E}}_{\boldsymbol{x}}}{\sigma^2_{MMSE-DFE}} = \gamma_0 \cdot SNR_{MFB}$$

$$SNR_{MMSE-DFE,U} = SNR_{MMSE-DFE} - 1 \le SNR_{MFB}$$

Unbiased MMSE-DFE

- Final result



$$G_U(D) = 1 + \frac{SNR_{MMSE-DFE}}{SNR_{MMSE-DFE,U}} \cdot [G(D) - 1]$$

- Can absorb unbiasing multiply into feedforward filter
- Must always perform at least as well as LE, why?

Examples

Section 3.6.4

Return to $H(D) = 1 + .9 \cdot D^{-1}$

- Refresh $\|h\|^2 = 1.81$, $SNR_{MFB} = 10$

$$H(\omega) = \begin{cases} \sqrt{T} \cdot \left(1 + .9e^{\jmath\omega T}\right) & |\omega| \leq \frac{\pi}{T} \\ 0 & |\omega| > \frac{\pi}{T} \end{cases}$$

$$Q(D) + \underbrace{1/10}_{1/SNR_{MFB}} = \left(\frac{1}{\underbrace{1.81}_{\|h\|^2}}\right) \cdot [9 \cdot D^{-1} + 1.991 + .9 \cdot D]$$

$$= \underbrace{.785}_{\gamma_0} \cdot \underbrace{(1 + .6334 \cdot D)}_{G(D)=B(D)} \cdot \underbrace{(1 + .6334 \cdot D^{-1})}_{G(D^{-1})}$$

$$W(D) = \frac{1}{\sqrt{1.81}(.785)(1 + .633 \cdot D^{-1})} = \frac{.9469}{1 + .633 \cdot D^{-1}}$$

% Q is positive real – roots are in conjugate-reciprocal pairs.

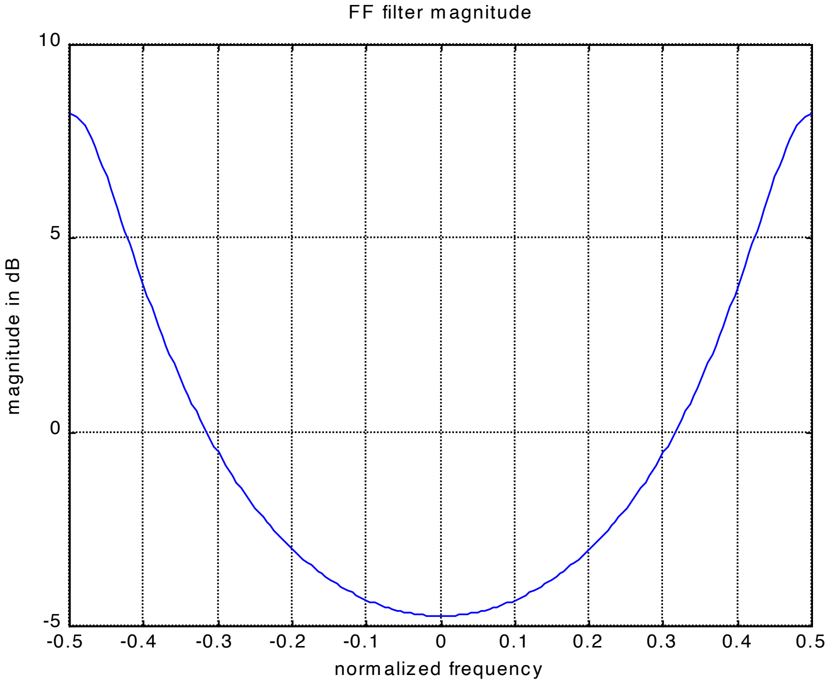
```
>> roots([.9 1.991 .9]) =

  -1.5788
  -0.6334

>> .9/(1.81*.6334) =   0.7850
```

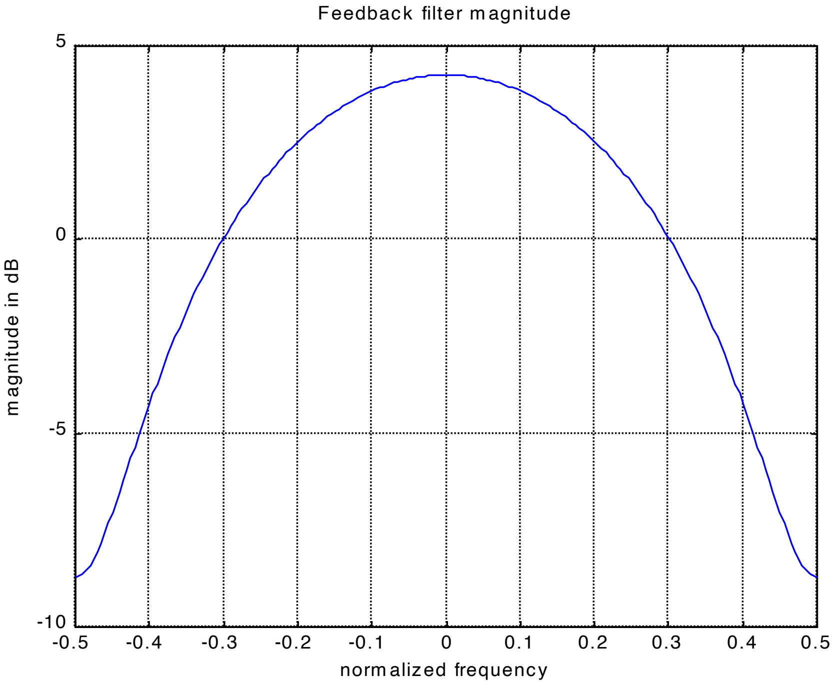
% follows from last (or first) coefficient

- Performance $SNR_{MMSE-DFE,U} = .785 \cdot 10 - 1 = 6.85 \cdot (8.4\ dB)$
  - **1.6 dB less than MFB –**
  - $G_U(D) = (1 + .7259 \cdot D)$

```
>> 7.85/6.85*.6334 = 0.7259
>> .9469*(7.85/6.85)= 1.0851
```

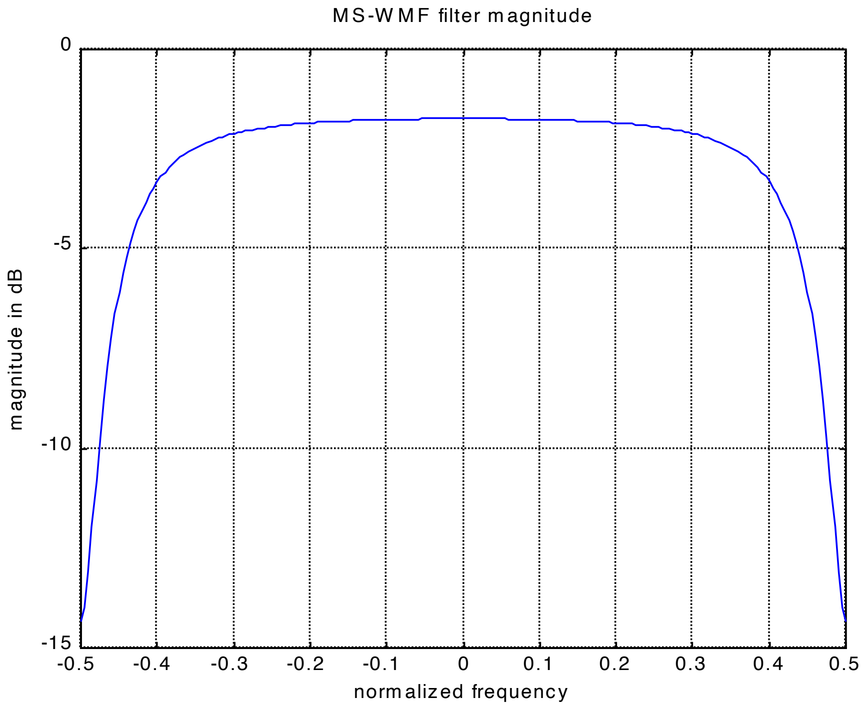
Example continued





- FF is single pole (anticausal) IIR filter, while FB is two-tap causal FIR filter (really 1 tap implemented).
- The anticausal FF is implemented by FIR approximation with delay.

Eliminates Noise Enhancement

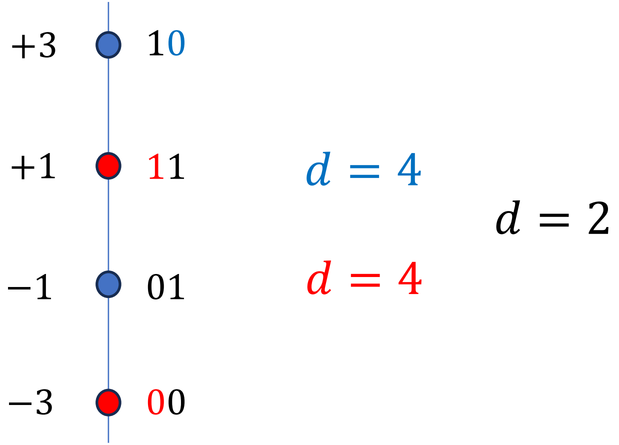


Includes MF also

- The MS-WMF is nearly flat (so noise/error stays pretty flat, and error itself is white.
- It is nearly an "all-pass" filter (MS-WMF just adjusts phase to minimum phase channel):
  - Minimum phase channel has maximum energy at left for all phase equivalents (helpful for best "feedback" to have no advance ISI).

Add a Code?

- BICM, 64-state code with 4PAM
  - Maintains $\bar{b} = 1$, $r = \frac{1}{2}$
- $\mathcal{E}_{4PAM} = 5 = 5 \cdot \mathcal{E}_{2PAM}$, so 7dB loss
- $d_{free} = 10$, and each code gets r =½ resources
  - $r \cdot d_{free} \cdot SNR = 6.4 + 7 = 13.4$ dB



Thanks to DFE we've maintained the data rate
And reduced error prob to $P_e \cong 10^{-6}$

- See a problem perhaps?
  - Decision delay of code means FB input could be incorrect

Complex Example?

$$h_k = \frac{1}{\sqrt{T}} \cdot \left[-\frac{1}{2} \quad \left(1 + \frac{\jmath}{4}\right) \quad -\frac{\jmath}{2} \right] \quad . \qquad (3.$$

The $SNR_{MFB} = 10$ dB. Then,

$$\tilde{Q} = \frac{-.25\jmath \cdot D^{-2} + .625(-1+\jmath) \cdot D^{-1} + 1.5625(1+.1) - .625(1+\jmath) \cdot D + .25\jmath \cdot D^2}{1.5625} \qquad (3.$$

```
>>p= transpose(roots([-i/4 (5/8)*(-1+i) 1.5625*(1+.1) (5/8)*(-1-i) i/4])) =

  2.2130 - 0.1356i  -0.1356 + 2.2130i   0.4502 - 0.0276i  -0.0276 + 0.4502i

>> gamma0=(i/4)/(p(3)*p(4)*1.5625) =  0.7865 - 0.0000i

>> SNRdfeu=10*log10(10*gamma0-1) =  8.3665  dB
```

```
>> conv([1 -p(3)],[1 -p(4)]) =
 1.0000 + 0.0000i  -0.4226 - 0.4226i  -0.0000 + 0.2034i
>> nh=sqrt(1.5625);
>> 1/(nh*gamma0) =  1.0171
```

$$W(D) = \frac{1.0171}{1 - .4226(1+j) \cdot D^{-1} + .2034j \cdot D^{-2}}$$

$$G(D) = (1 - (.4502 - j.0276) \cdot D) \cdot (1 - (-.0276 + .4502j) \cdot D)$$

- $SNR_{MMSE-DFE,U} = .7865 \cdot 10 - 1 = 7.88 \cdot$ (~8.4 dB also – coincidence).

```
>> GU=1+(SNRdfeu+1)/(SNRdfeu)*(1-conv([1 -p(3)],[1 -p(4)])) =  1.0000   2.5926 + 0.4731i   2.1195 - 0.2277i
```

Whitened Matched Filter

and the ZF-DFE

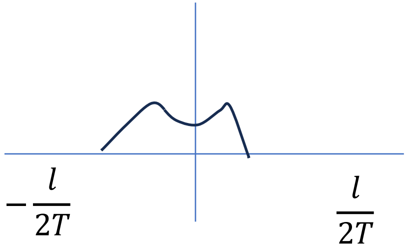
Section 3.6.3

ZF-DFE

- Set $SNR_{MFB} = \infty$, so then the spectral factorization is directly of $Q(D) = \eta_0 \cdot P_c(D) \cdot P_c^*(D^{-*})$.
  - $B(D) = P_c(D) \quad ; \eta_0 = {}^1/_{\|P_c\|^2}$
  - $W(D) = [\eta_0 \cdot \|h\| \cdot P_c^*(D^{-*})]^{-1}$
  - $SNR_{ZF-DFE} = \eta_0 \cdot SNR_{MFB}$

- However, the spectral factorization is not guaranteed to exist (might not satisfy PWC).
- Subtle mistake is “ZF-DFE has no noise enhancement so same as MMSE-DFE” ?
  - Zero noise enhancement is true ONLY if the entire **transmit** band is energized.
  - And still not as good as MMSE-DFE even then.

The issue is that best transmit spectrum almost never satisfies PWC



- We’ll see as 379’s progress that this optimized-input modulator needs care with DFEs.
  - Mistakes dwarf coding-gain improvements.
  - Indeed, using nonero-gap code magnifies the loss.

Return to $H(D) = 1 + .9 \cdot D^{-1}$

- Refresh $\|h\|^2 = 1.81$, $SNR_{MFB} = 10$

$$H(\omega) = \begin{cases} \sqrt{T} \cdot \left(1 + .9 e^{\jmath \omega T}\right) & |\omega| \leq \frac{\pi}{T} \\ 0 & |\omega| > \frac{\pi}{T} \end{cases}$$

$$Q(D) + {}^{1}\!/_{10} = \left(\frac{1}{1.81}\right) . [9 \cdot D^{-1} + 1.81 + .9 \cdot D]$$

$$= \underbrace{.5525}_{\eta_0} \cdot \underbrace{(1 + .9 \cdot D)}_{P_c(D)=B(D)} \cdot \underbrace{(1 + .9 \cdot D^{-1})}_{P_c(D^{-1`})}$$

$$W(D) = \frac{\sqrt{1.81}}{1 + .9 \cdot D^{-1}} \qquad MWF = \frac{1 + .9 \cdot D}{1 + .9 \cdot D^{-1}}$$ (all pass filter, exactly)

- Performance $SNR_{ZF-DFE} = .5525 \cdot 10 = 5.25 \cdot (7.4\ dB)$

L phases, FSE – same, but for FF only

- There are l phases of input samples for each symbol-sample-time output ("polyphase system")

$$y_i(kT) = y(kT - iT/l) \ , i = 0, \ldots, l-1$$

- Each phase has a D-Transform $Y_i(D) = H_i(D) * X(D) + N_i(D)$
  - This is a form of what is called "diversity" where several channels carry the same input to different outputs

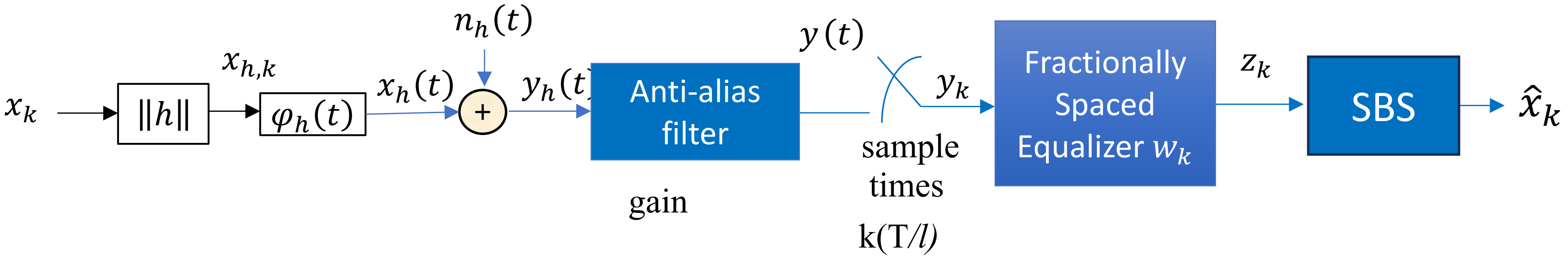
$$\underbrace{\boldsymbol{Y}(D)}_{l\times 1} = \underbrace{\boldsymbol{H}(D)}_{l\times 1} \cdot X(D) + \underbrace{\boldsymbol{N}(D)}_{l\times 1}$$

- Retains all timing-offset benefits, matched-filter absorption, etc.

FIR Implementation

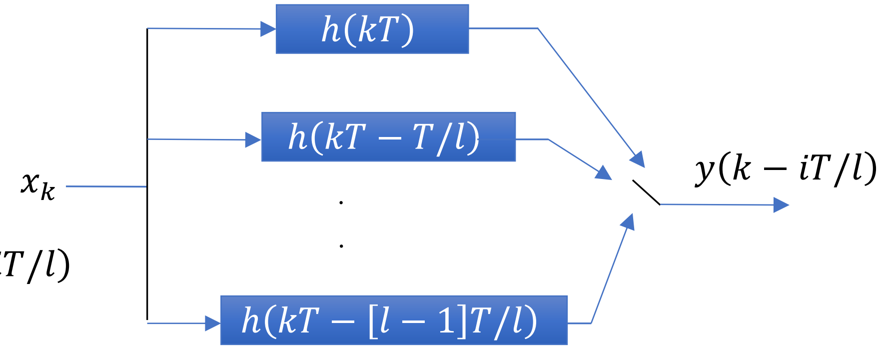
Section 3.7

Sample Fast enough – absorb MF



- Continue polyphase channel model $^{l}/_{T}$ where $l > 1$:
  - Initial example is integer (more generally rational fraction).
  - Channel model is FIR (like with DMT earlier).

$$y(k - iT/l) = \sum_{m=k-\nu}^{k} x_m \cdot h\left(kT - \frac{[l-1]T}{l} - mT\right) + n(k - iT/l)$$



- Each mini channel corresponds to one of the l sampling phases per symbol period.

Creating a matrix FIR channel

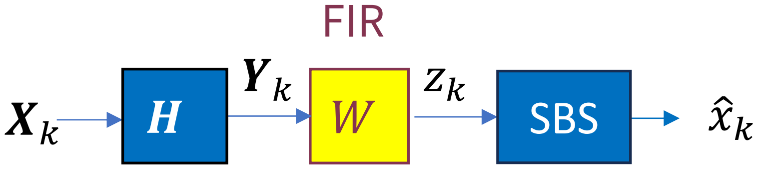
- Creates a vector channel $\boldsymbol{y}_k = \sum_{m=k-\nu}^{k} x_m \cdot \boldsymbol{h}_{k-m} + \boldsymbol{n}_k = \sum_{m=0}^{\nu} x_{k-m} \cdot \boldsymbol{h}_m + \boldsymbol{n}_k$
  - where

$$\boldsymbol{y}_k = \begin{bmatrix} y(kT) \\ y(k-T/l) \\ \vdots \\ y(k-(l+1)T/l) \end{bmatrix} \quad \boldsymbol{h}_k = \begin{bmatrix} h(kT) \\ h(k-T/l) \\ \vdots \\ h(k-(l+1)T/l) \end{bmatrix} \quad \boldsymbol{n}_k = \begin{bmatrix} n(kT) \\ n(k-T/l) \\ \vdots \\ n(k-(l+1)T/l) \end{bmatrix}$$

- channel model $\boldsymbol{y}_k = \underbrace{[\boldsymbol{h}_0 \boldsymbol{h}_1 \ \cdots \ \boldsymbol{h}_\nu]}_{\boldsymbol{h}} \cdot \underbrace{\begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-\nu} \end{bmatrix}}_{\boldsymbol{X}_k} + \boldsymbol{n}_k$

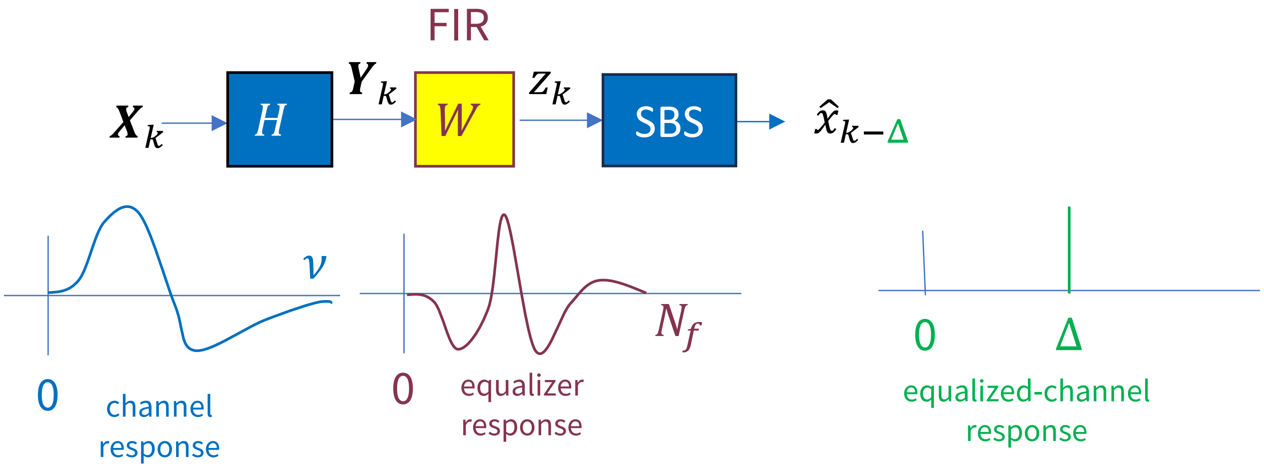
- The values in the FIR equalizer's span are

$$\boldsymbol{Y}_k = \begin{bmatrix} \boldsymbol{y}_k \\ \boldsymbol{y}_{k-1} \\ \vdots \\ \boldsymbol{y}_{k-N_f+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \boldsymbol{h}_0 & \boldsymbol{h}_1 & \cdots & \boldsymbol{h}_\nu & 0 & \cdots & 0 \\ 0 & \boldsymbol{h}_0 & \boldsymbol{h}_1 & \cdots & \boldsymbol{h}_\nu & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \cdots & 0 \\ 0 & \cdots & 0 & \boldsymbol{h}_0 & \boldsymbol{h}_1 & \cdots & \boldsymbol{h}_\nu \end{bmatrix} \end{bmatrix}}_{\boldsymbol{H}} \cdot \boldsymbol{X}_k + \boldsymbol{N}_k$$



Delay Δ – FIR is causal, estimates $\hat{x}_{k-\Delta}$

- Implemented receivers must have **delay** Δ (making FIR filters causal)



- Try to estimate $x_{k-\Delta}$

$$\Delta \approx \frac{N_f + \nu}{2}$$

Not (usually) possible to have Δ=0

- Precise delay may need to be optimized by trying a few (or via algorithm later).

MMSE-LE Design

- The error signal is $e_k = x_{k-\Delta} - z_k$; $\sigma^2_{MMSE-LE} = \mathbb{E}[|e_k|^2]$
- Orthogonality principle: $\mathbb{E}[e_k \cdot \boldsymbol{Y}_k] = 0$ → MMSE
  - $\boldsymbol{w} = \boldsymbol{R}_{xY} \cdot \boldsymbol{R}_{YY}^{-1}$ **and**
    - $\boldsymbol{R}_{xY} = \mathbb{E}[\boldsymbol{x}_{k-\Delta} \cdot \boldsymbol{Y}_k^*]$
    - $\boldsymbol{R}_{YY} = \mathbb{E}[\boldsymbol{Y}_k \cdot \boldsymbol{Y}_k^*]$

Detailed algebra in 3.7.1

- $\boldsymbol{R}_{xY} = \bar{\mathcal{E}}_x \cdot H^*_{\Delta+1}$ basically energy x $(\Delta+1)^{th}$ row of $\boldsymbol{H}^*$
- $\boldsymbol{R}_{YY} = \bar{\mathcal{E}}_x \cdot \boldsymbol{H} \cdot \boldsymbol{H}^* + l \cdot \frac{\mathcal{N}_0}{2} \cdot \boldsymbol{R}_{NN}$

$$\boldsymbol{w} = H^*_{\Delta+1} \cdot \left(\boldsymbol{H} \cdot \boldsymbol{H}^* + \frac{l}{SNR} \cdot \boldsymbol{R}_{NN}\right)^{-1}$$

$$\sigma^2_{MMSE-LE} = \bar{\mathcal{E}}_x - \boldsymbol{w} \cdot \boldsymbol{R}_{Yx}$$

$$SNR_{MMSE-LE} = \frac{\bar{\mathcal{E}}_x}{\sigma^2_{MMSE-LE}}$$

$$SNR_{MMSE-LE,U} = SNR_{MMSE-LE} - 1$$

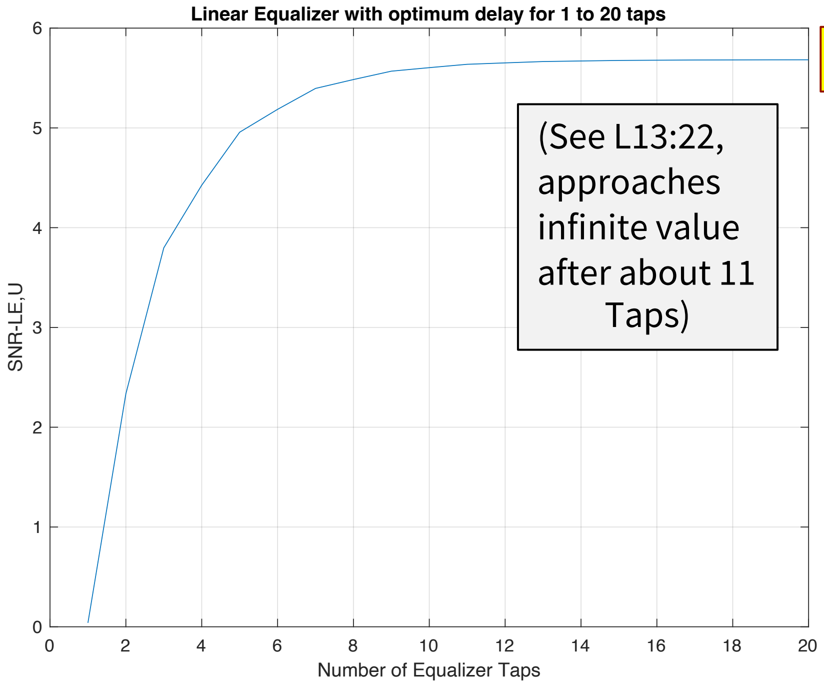
$$\gamma_{MMSE-LE} = \frac{SNR_{MMSE-LE,U}}{SNR_{MFB}}$$

Return to 1+.9D^{-1} example

- Oversampling $l = 1$, 3 taps $N_f = 3$, $\bar{\mathcal{E}}_x = 1$, and $\nu = 1$

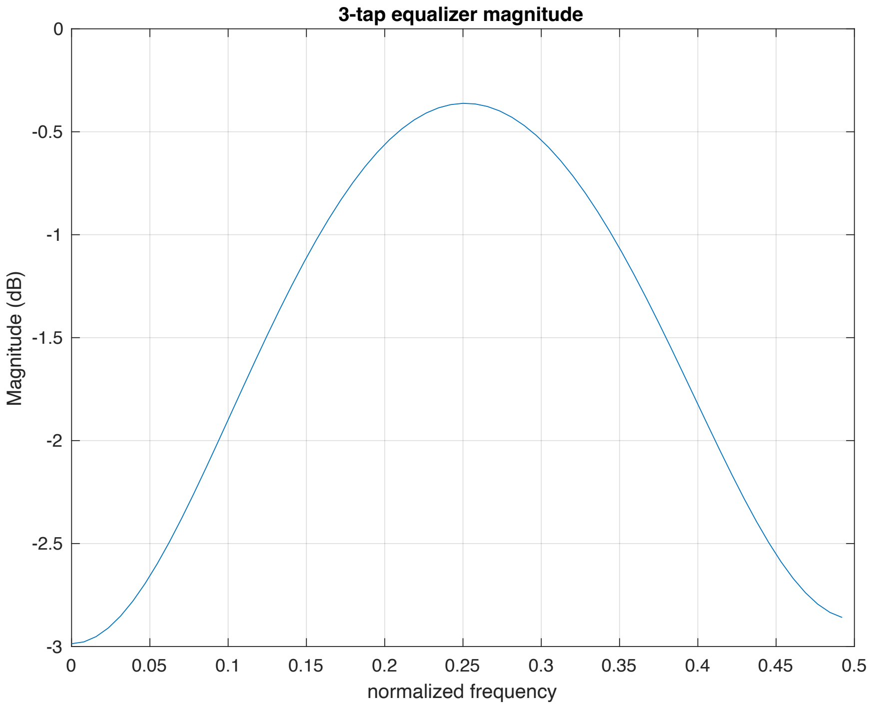
```
>> H=[.9 1 0 0
0 .9 1 0
0 0 .9 1];

>> Hdelstar=[0 0 1 0]*H' =  % 3rd row since Delta = 2
      0    1.0000    0.9000
>> RxY=Hdelstar;
>> RYY=H*H'+.181*eye(3) =
   1.9910    0.9000         0
   0.9000    1.9910    0.9000
        0    0.9000    1.9910

>> w=RxY*inv(RYY) =
  -0.2277    0.5038    0.2243

>> MMSE=1-w*RxY' =   0.2943

>> SNRU=10*log10(1/MMSE-1) =    3.7979  dB
```

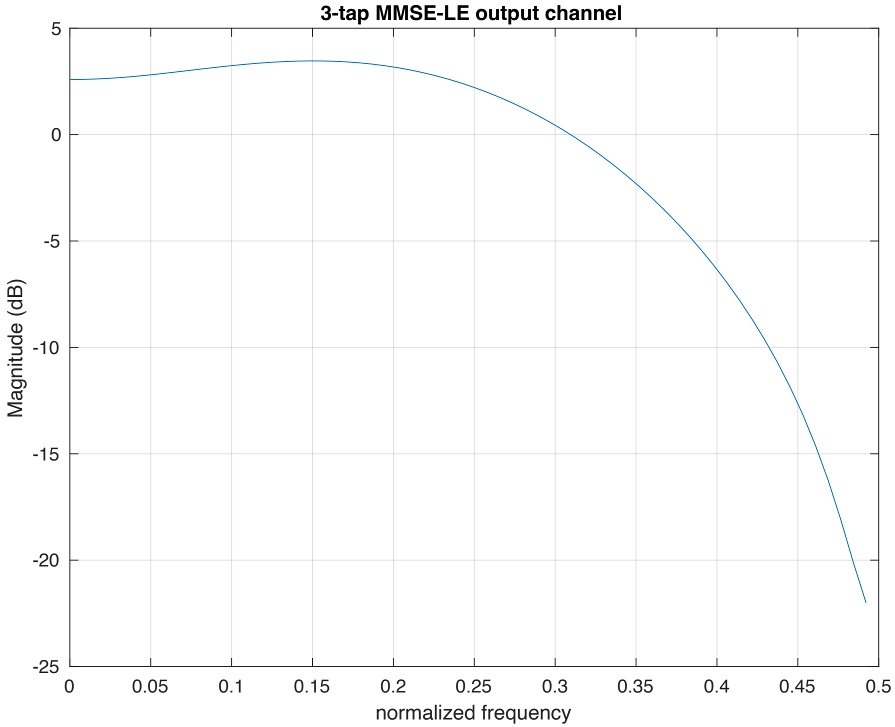
Δ = best value (by trial & error)



5.68dB

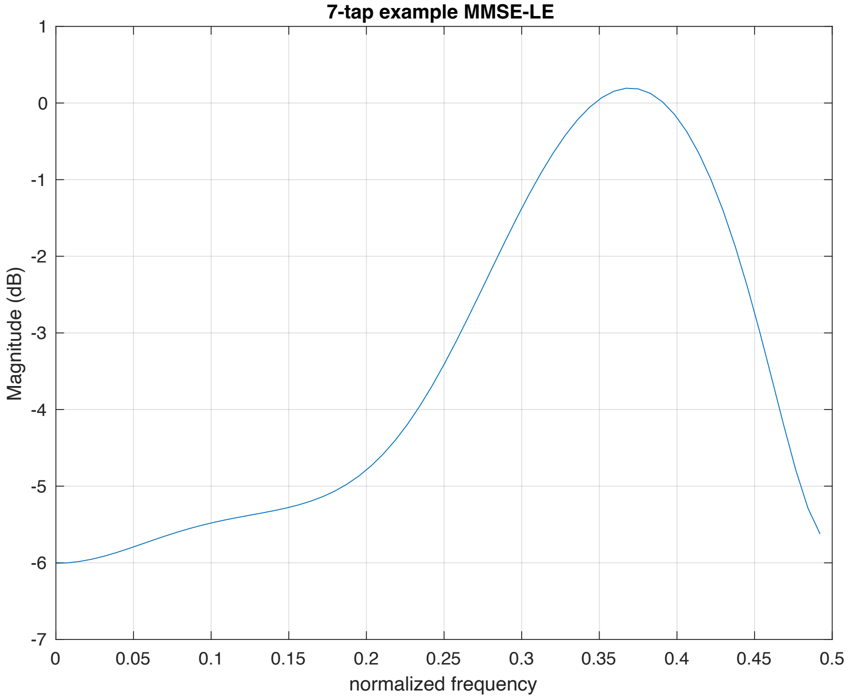
3-tap FIR MMSE-LE and output

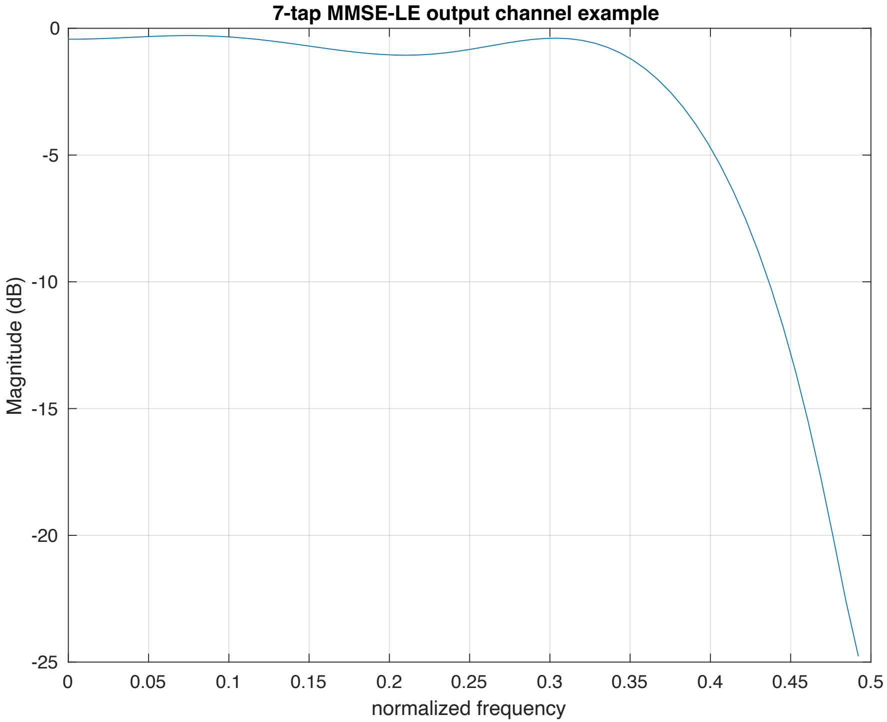




- This FIR MMSE LE tries to flatten channel, but not enough taps.

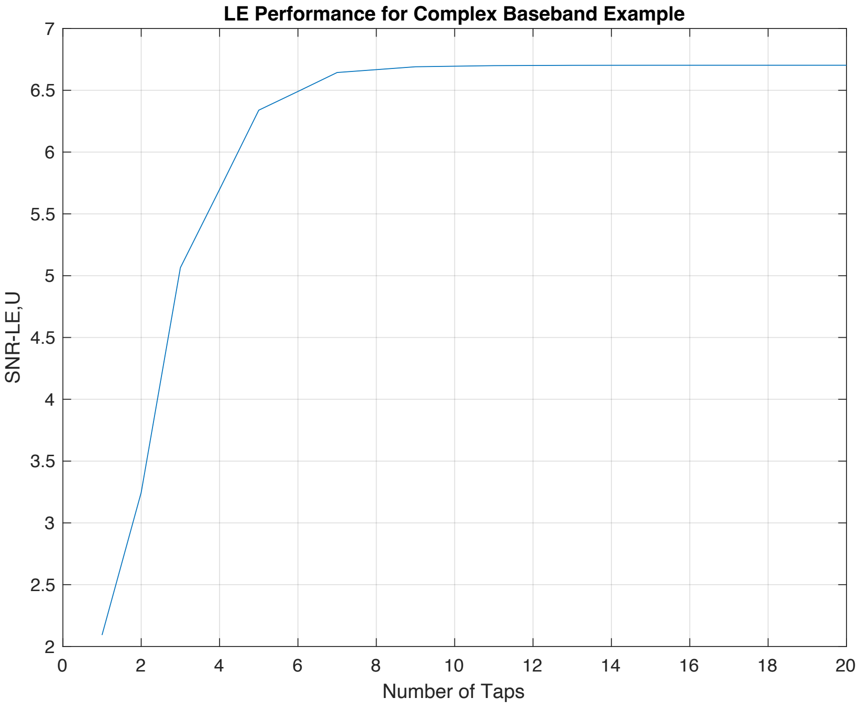
7-taps looks better





- There will be diminishing performance improvement as length increases.

For the complex example



6.7dB

```
>> for n=1:20
[dfeSNR(n),~ ]=dfecolor(1,[-1/2 1+i/4 -i/2],n,0,-1,1,.15625*[1 zeros(1,n-1)]);
end
```

- More on **dfecolor** appears shortly.
- Here 8 taps is enough.

FIR ZFE

- Design applies MMSE-LE with infinite SNR.
  - Finite taps cannot guarantee zero ISI, so there is:

$$\sigma^2_{ZFE-ISI} = \bar{\mathcal{E}}_x - \boldsymbol{w} \cdot R_{\boldsymbol{Y}x}$$

```
>> wzfe=H(:,4)'*inv(H*H') =   0.2702   -0.5434   0.8227

>> MMSEzf=1-wzfe*H(:,4) =  0.1773
```

- The FIR ZFE can still be biased – look at position of $x_{k-\Delta}$ so $\boldsymbol{w} \cdot \boldsymbol{H}$ in position $\Delta + 1$

chan=wzfe*H =   0.2432   -0.2189   0.1970   0.8227

  - Bias removal thus inverts, so $\omega_{FIR-ZFE} = 1 \,/\, \boldsymbol{w} \cdot \boldsymbol{H} \cdot \boldsymbol{1}_{\Delta}$ this means residual ISI and noise increase by this factor also

- Analysis must also add the scale the enhanced noise $\sigma^2_{ZFE} = \boldsymbol{w} \cdot R_{\boldsymbol{nn}} \cdot \boldsymbol{w}^*$
- Total is $\sigma^2_{FIR-ZFE} = \omega_{FIR-ZFE} \cdot (\bar{\mathcal{E}}_x - \boldsymbol{w} \cdot R_{\boldsymbol{Y}x} + \boldsymbol{w} \cdot R_{\boldsymbol{nn}} \cdot \boldsymbol{w}^*)$

```
>> chan = wzfe*H % =   0.2432   -0.2189   0.1970   0.8227
>> wnobias=1/chan(4)  %=   1.2155
>> MMSEzf=1-wzfe*H(:,4) % =  0.1773
>> SNRzfu=1/MMSEzf -1  % =   4.6402
>> SNRzfu=1/sigzfe2  % =   0.6730
>> (SNRzfu+1)/SNRzfu =   1.2155 % checks bias removal
>> enoise=.181*norm(wzfe)^2  =   0.1892
>> SNRall=10*log10(1/(wnobias*(MMSEzf+enoise)))  = 3.0813  dB (< 3.78 dB for MMSE-LE,U)
```

End Lecture 15