Space-Time Coding And Transmission Optimization for Wireless Channels[1]

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Abstract

Space-time codes have recently been introduced to improve mobile system performance in a multipath fading environment. Here, we consider an L-multipath, m mobile-antenna system in which the base station has a phased n-antenna array (ie. fading at the base station antennas is completely correlated). We show that when the channel has no ISI, then adaptive antennas, in the form of beamforming, can be combined with delay-diversity coding to achieve a diversity gain of mL and a large coding gain, whenever $n \geq L$. When the channel has ISI, or for the rapid fading channel; beamforming can be used to achieve a coding gain over a SIMO system, although both have the same diversity gain. Novel concepts of beamforming are derived in the process.

1 Introduction

Space-time coding for the general case of Multiple-Input Multiple-Output (MIMO) channels has been studied in[1, 2, 3, 4, 5, 6] and space-time codes have been recently introduced in a multipath fading environment [7, 8, 9, 10] to improve mobile system performance. In the block time-invariant environment (where channel is time-invariant during transmission of one block of data) [11], it has been shown that using multiple antennas at the base station and mobile allows one to achieve a maximum diversity of mn, where m and n are the number of mobile and base station antennas respectively. Delay diversity codes were shown to be special cases of space-time codes, that were capable of achieving maximum diversity. However, all the work to date in this subject assumes a multipath channel model in which the fading from each base station antenna to any mobile antenna is independent, or at least non-degenerate. i.e. If we collect the mn fading gains into a vector, then the autocovariance matrix of the vector is full rank. This assumption requires that both the base station and the mobile must be surrounded by local scatterers, or that the base station antennas must be spaced far apart, so that their signals are uncorrelated. In several situations, the base station is placed high above the ground, and practicalities dictate that its antennas are spaced close together. In this situation, the base antenna array will be a phased array. i.e. having completely correlated signals. In that case, an L-multipath channel model such as that proposed in [2] would be valid. In this paper we will consider the above scenario (Figure 1). The mobile, being at ground level, is assumed to have a diversity array. Thus, the fades of each of the L-multipaths at the base are completely correlated, but those at the mobile are uncorrelated. The case that assumes complete correlation at the mobile also, would give essentially the same results as those derived in this paper. We will discuss schemes that would achieve diversity as well as coding gain in this scenario. We will only consider the downlink case (Base station to mobile link) in this paper. The uplink would require a different approach, which we believe would be more along the lines of classical beamforming. We have considered three distinct cases. The first is the case in which all the the multipaths have the same delay, which results in a channel that is free of Inter-Symbol Interference (ISI). In the second case, the multipaths have different delays, thus causing ISI in the receiver, and in the third case we have considered the channel with rapid fading. In this paper we will discuss the results obtained for the above cases and some of the proofs and further results are presented in [NMC:1999]. The organization of the paper is as follows. The problem is formulated for the ISI-free case in Section 2. In Section 3 we show that for the ISI-free case, an appropriate scheme for achieving the maximum diversity of mL, and obtaining a coding gain, is to combine beamforming with a space-time code such as a delay diversity code. The optimum beamformer is derived for this case and is seen to be very different from the classical beamformer. In section 4 we formulate the problem for the case with ISI, and show that whereas using more than one base antenna does not increase the diversity, it does provide a large coding gain. We will provide suboptimal results for the coding gain solution. In section 5 we will formulate the problem for the rapid fading case, and show that using more than one base station antenna can again provide a large coding gain. Section 6 illustrates these ideas with simulations. We finally conclude in Section 7.

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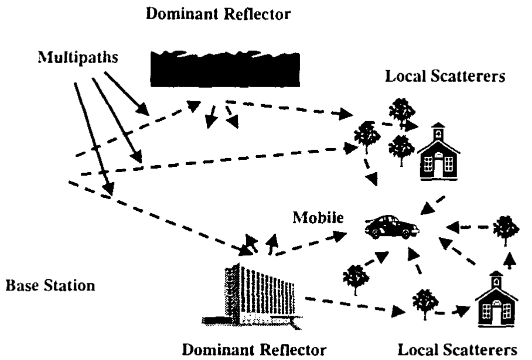

Figure 1: Block diagram of the MIMO system

2 Problem Formulation

The problem is to design 'space-time coding schemes' [12] that achieve low frame error rates. In various papers [7, 8, 12, 11, 10], space-time coding schemes have been discussed that essentially assume that the elements of the channel matrix **FA** are non-degenerate, even if they are dependent. In the scenario we consider however, this is not true. Further, for our case, we show explicitly how the concepts of beamforming and space-time coding can be combined. It will be shown that such a combination can give one diversity as well as coding gain.

2.1 MIMO Wireless System

The downlink system equation in the time domain can be written as

$$\mathbf{y}_k = \sum_{p=1}^{L} \boldsymbol{\alpha}_p \cdot \mathbf{a}(\theta_p) \cdot \mathbf{x}_{k-\tau_p} + \mathbf{v}_k \qquad (1)$$

where L is the number of multipaths, τ_p is the delay in symbol periods (assumed integer) associated with the p th multipath, $\mathbf{x}_k$ is the $n \times 1$ signal vector transmitted at symbol time k using the n base station antennas, $\mathbf{y}_k$ and $\mathbf{v}_k$ are the $m \times 1$ vector of the signal and noise received using the m mobile antennas. $\boldsymbol{\alpha}_p$ is the $m \times 1$ vector of the fading channel gains at the m mobile antennas, for the p th multipath, while $\mathbf{a}(\theta_p)$ is the $1 \times n$ vector of the base station antenna array response to the p th multipath, that is incident on the base array at an angle θ_p. For example, for a linear array: $\mathbf{a}(\theta) = (1 \;\; e^{j2\pi\delta sin(\theta)} \;\; \cdots \;\; e^{j2\pi\delta(n-1)sin(\theta)})$

2.2 Assumptions

The system model (1) is valid because it is assumed that the base station does not have any local scatterers and that the base antennas are spaced close together, and hence the base array is a phased array (ie. completely correlated response at all antennas for each multipath). Also, the mobile is assumed to be at ground level, so that the local scatterers cause uncorrelated fading at its antennas. Further, the L multipaths are assumed uncorrelated and having equal power, since they're presumably reflected by objects well separated in space. Thus, the fading channel gains $\boldsymbol{\alpha}_p$s have i.i.d. elements, which are assumed complex gaussian random variables of variance σ_f^2 each. The angles $\{\theta_p\}$ are assumed to be distinct. Note that if the power of the L multipaths is measured and unequal, this can be easily incorporated in the theory that follows, by absorbing the powers in the $\mathbf{a}(\theta_p)$s . This trivial case is not considered for simplicity of presentation. The noise vector $\mathbf{v}_k$ is assumed to consist of zero mean i.i.d. complex gaussian random variables (white noise) of variance σ_n^2 each. The transmitter is assumed to have knowledge of the multipath angles $\{\theta_p\}$, since these are expected to change slowly and so can be estimated. However, it does not know the fading channel gains $\boldsymbol{\alpha}_p$s due to the high doppler. The mobile is assumed to have knowledge of the entire channel state information (CSI).

3 ISI-free block time-invariant MIMO system

Consider a Multiple-Input Multiple-Output (MIMO) system with L multipaths, all of which have the same delay $\tau = 0$. The downlink system equation in the time domain can be written as

$$\mathbf{y}_k = \sum_{p=1}^{L} \boldsymbol{\alpha}_p \mathbf{a}(\theta_p) \mathbf{x}_k + \mathbf{v}_k \qquad (2)$$

All notations are as defined in Section (2.1). Consider a block transmission scheme, wherein a frame of data consisting of l vector symbols $\{\mathbf{x}_k, k = 0, 1, \ldots, l-1\}$ is transmitted we can write the block transmission in matrix form as below

$$\mathbf{Y} = \mathbf{FAX} + \mathbf{V} \qquad (3)$$

where $\mathbf{Y} \doteq (\mathbf{y}_0 \;\cdots\; \mathbf{y}_{l-1})$, $\mathbf{X} \doteq (\mathbf{x}_0 \;\cdots\; \mathbf{x}_{l-1})$, $\mathbf{F} \doteq (\boldsymbol{\alpha}_1 \;\cdots\; \boldsymbol{\alpha}_L)$, and $\mathbf{A} \doteq \left(\mathbf{a}(\theta_1)^T \;\cdots\; \mathbf{a}(\theta_L)^T\right)^T$. Since the fading is assumed constant over a frame, system equation (3) is valid. The performance criterion is obtained in a manner similar to [12]. Since the mobile knows the ideal CSI, and the noise is assumed white gaussian, hence the probability of the decoder deciding in favor of code matrix $\mathbf{X}_e$ when the code matrix $\mathbf{X}_0$ was transmitted (ie. matrix $\mathbf{X}$ in (3)) is approximated by

$$P(\mathbf{X}_0 \to \mathbf{X}_e|\mathbf{FA}) \leq \exp(-||\mathbf{FA}(\mathbf{X}_0 - \mathbf{X}_e)||_F^2/4\sigma_n^2) \quad (4)$$

Now, if $n = 1$ (SIMO case), then **FA** collapses into a column vector of m independent random variables, and its clear that a diversity of m is achievable. Whenever $n \geq L$, a maximum diversity of mL is achievable. This is because we can write

$$||\mathbf{FA}(\mathbf{X}_0 - \mathbf{X}_e)||_F^2 = \mathrm{tr}[\mathbf{FV\Lambda V^* F^*}]$$

where $\mathbf{A}(\mathbf{X}_0 - \mathbf{X}_e)(\mathbf{X}_0 - \mathbf{X}_e)^*\mathbf{A}^* = \mathbf{V\Lambda V}^*$ is the singular value decompostion. $\mathbf{FV}$ is an $m \times L$ matrix with i.i.d. complex Gaussian elements. If one designs the codebook such that for every pair of codewords, $\mathbf{V\Lambda V}^*$ has full rank L, then the exponent in (4) is a χ^2 random variable with $2mL$ degrees of freedom ([13]) and hence the diversity gain of mL. This is possible if and only if $n \geq L$.
We will assume subsequently that a diversity gain of mL is always the target. In that case, it is easy to show that

$n = L$ and a simple delay-diversity code achieves the diversity of mL. The reason for which is provided in [14].

Now the question arises as to the benefit of using more than L antennas at the base station. We can show that by using an appropriate concept of beamforming at the base, we can get a coding gain over the system that uses only L antennas. Both systems however, have the same diversity gain of mL, since that's the maximum achievable. However when using more than L transmit antennas the coding advantage is not insignificant, especially when the target frame error rate is high. Hence, the advantage of using $n > L$. In the following, we derive the beamforming concept that is applicable in this case.

When $n > L$, to achieve a diversity of mL, we begin with an L-delay diversity code as the core code. Call the $L \times l$ toeplitz code matrix of this code as $\mathbf{C}$. Now we map the $L \times 1$ vector symbol at each transmission (i.e. a column of $\mathbf{C}$) into an $n \times 1$ transmit vector using a linear transform represented by the $n \times L$ matrix $\mathbf{W}$. Thus, the final code matrix transmitted from the n antennas is $\mathbf{X} = \mathbf{WC}$ If (and only if) $\mathbf{W}$ is chosen such that the product $\mathbf{AW}$ is full rank, then this coding scheme will achieve a diversity of mL. Now we optimize $\mathbf{W}$ so as to get the largest coding gain possible. To this end, note that the coding gain is maximized by maximizing the determinant [12]

$$\det[\mathbf{AW}(\mathbf{C}_0 - \mathbf{C}_e)(\mathbf{C}_0 - \mathbf{C}_e)^*\mathbf{W}^*\mathbf{A}^*] \quad (5)$$

for any pair of code matrices $\{\mathbf{C}_0, \mathbf{C}_e\}$. Since the code has already been chosen to be delay diversity code, we can take $\det[(\mathbf{C}_0 - \mathbf{C}_e)(\mathbf{C}_0 - \mathbf{C}_e)^*]$ as a constant. Therefore, the optimization problem reduces to

$$\begin{aligned} &\underset{\mathbf{W}}{\text{maximize}} \quad \det[\mathbf{AWW}^*\mathbf{A}^*] \quad (6) \\ &\text{subject to} \quad \text{tr}[\mathbf{WW}^*] = L \end{aligned}$$

The constraint (6) arises due to total transmitted power constraint $\text{tr}[\mathbf{W}\ E[\mathbf{c}_k\mathbf{c}_k^*]\ \mathbf{W}^*] = P$, which occurs because the delay diversity code used by the L base antenna system allots power P/L to each antenna.

The solution to the maximization problem in 6 is found using standard linear algebra to be $\mathbf{W} = \mathbf{Q}_+$ where $\mathbf{Q}_+$ is found from the SVD of $\mathbf{A}$ as:

$$\mathbf{A} \;=\; \mathbf{T\Sigma}\boldsymbol{Q}^* \;=\; \mathbf{T}\ (\mathbf{\Sigma}_+ \mid \mathbf{0})\ (\underbrace{\mathbf{Q}_+}_{n \times L} \mid \Box\,)^*$$

and the value of the maximum is $g(n, L) = (\det[\mathbf{\Sigma}_+])^2$. The coding gain when using an n antenna system, over an L antenna one, is given by $\{g(n, L)/g(L, L)\}^{1/L}$. Its clear that this is more than 1. A proof of this is shown in [14]. Note that a similar inequality would hold in a variety of antenna array configurations, where the array response matrix has the key structure noted in [14].

4 MIMO Wireless System with ISI

We now develop the case where the MIMO channel has ISI. We shall see that the issues that come up in this case are qualitatively different from the case without ISI. The multipath channel with ISI can be modeled as below:

$$\mathbf{y}_k = \sum_{p=1}^{L} \boldsymbol{\alpha}_p \cdot \mathbf{a}(\theta_p) \cdot \mathbf{x}_{k-\tau_p} + \mathbf{v}_k$$

All notations are as defined in Section 2.1. Except for the introduction of ISI, all other assumptions in Section 2.2 are assumed to be valid. As in Section 3, if we consider a block of l vector symbols $\{\mathbf{x}_k, k = 0, 1, \dots, l-1\}$ transmitted, then the channel equation can be written in matrix form as below:

$$\mathbf{Y} = \mathbf{FBU} + \mathbf{V} \quad (7)$$

where $\mathbf{B} \doteq \mathbf{diag}\,(\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_L))$ and

$$\mathbf{U} \doteq \begin{pmatrix} \mathbf{x}_{0-\tau_1} & \mathbf{x}_{1-\tau_1} & \cdots & \mathbf{x}_{l-1-\tau_1} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{x}_{0-\tau_L} & \mathbf{x}_{1-\tau_L} & \cdots & \mathbf{x}_{l-1-\tau_L} \end{pmatrix}$$

$\mathbf{Y}, \mathbf{F}, \mathbf{V}$ are as defined in Section 2. If $\{\tau_1, \tau_2, \dots, \tau_L\}$are all distinct, both SIMO and MIMO will have the same diversity mL, but MIMO potentially will have higher coding gain, which depends upon $\{\theta_1, \theta_2, \dots, \theta_L\}$. e.g. If $\{\theta_1 = \theta_2 = \cdots = \theta_L\}$, then the coding gain would be n. This is the case we consider here. In general, as in (6), assuming a diversity target of mL, we need to maximize:

$$\text{maximize} \quad \det[\mathbf{BUU}^*\mathbf{B}^*] \quad (8)$$

Now suppose we use a powerful code for the SIMO system that achieves diversity mL (this implies that $\mathbf{U}_{SISO}$ defined below has full rank) that has the $L \times l$ code matrix

$$\mathbf{U}_{SISO} = \begin{pmatrix} x_{0-\tau_1} & x_{1-\tau_1} & \cdots & x_{l-1-\tau_l} \\ \vdots & \vdots & \cdots & \vdots \\ x_{0-\tau_L} & x_{1-\tau_L} & \cdots & x_{l-1-\tau_L} \end{pmatrix}$$

Then we can use the same code for the MIMO case, by mapping the scalar input x_k to the $n \times 1$ vector $\mathbf{x}_k = \mathbf{b}\, x_k$ using the beamforming vector $\mathbf{b}$, the gain of the MIMO system can be written as:

$$\det\left[\hat{\mathbf{B}}\mathbf{U}_{SISO} \cdot \mathbf{U}_{SISO}^*\hat{\mathbf{B}}^*\right] \quad (9)$$

where $\hat{\mathbf{B}} \doteq \mathbf{diag}\,(\mathbf{a}(\theta_1)\mathbf{b}, \cdots, \mathbf{a}(\theta_L)\mathbf{b})$. Therefore the coding gain over the SIMO case is $\left(\prod_{p=1}^{L} \mid \mathbf{a}(\theta_p)\mathbf{b} \mid^2\right)^{1/L}$. Thus, for optimum performance, we need to

$$\begin{aligned} &\underset{\mathbf{b}}{\text{maximize}} \quad P(\mathbf{b}) \;\doteq\; \left(\prod_{p=1}^{L} \mid \mathbf{a}(\theta_p)\mathbf{b} \mid^2\right)^{1/L} \quad (10) \\ &\text{subject to} \quad \parallel \mathbf{b} \parallel^2 = 1 \end{aligned}$$

The normalization $\parallel \mathbf{b} \parallel^2 = 1$ ensures that the transmitted power is the same as the SIMO case. The maximization problem in 10 has a non-convex cost function, which seems difficult to solve exactly. However, a sub-optimal solution for $\mathbf{b}$ is given by:

$$\mathbf{b} = \frac{\sum_{p=1}^{L} \pm\mathbf{a}^*(\theta_p)}{\parallel \sum_{p=1}^{L} \pm\mathbf{a}^*(\theta_p) \parallel} \quad (11)$$

where the $\pm$ sign indicates that we choose that sign for each $\mathbf{a}^*(\theta_p)$, such that $P(\mathbf{b})$ is maximized. Another solution using relaxation of the none-convex constraint is presented in [14]. In simulations, we search over all 2^L sign combinations of $\mathbf{a}^*(\theta_p)$s, and choose the one that leads to the maximum gain $P(\mathbf{b})$.

5 Rapid Fading MIMO Wireless System

Here we will consider the case where the MIMO channel experiences rapid fading, but has no ISI. The multipath channel with rapid fading can be modeled as below:

$$\mathbf{y}_k = \sum_{p=1}^{L} \boldsymbol{\alpha}_{p,k} \cdot \mathbf{a}(\theta_p) \cdot \mathbf{x}_k + \mathbf{v}_k$$

All notations are as defined in Section 2.1. All other assumptions in Section 2.2 are assumed to be valid, except for the $\alpha_{p,k}$s which are now time dependent. Therefore the channel is no more block time-invariant. As in Section 3, if we consider a block of l vector symbols $\{\mathbf{x}_k, k = 0, 1, \ldots, l-1\}$ transmitted, then the channel equation can be written in matrix form as below:

$$\mathbf{Y} = \mathbf{FBU} + \mathbf{V} \tag{12}$$

where $\mathbf{F}_k \doteq (\boldsymbol{\alpha}_{1,k}\ \boldsymbol{\alpha}_{2,k}\ \cdots\ \boldsymbol{\alpha}_{L,k})$, $\mathbf{F} \doteq (\mathbf{F}_1\ \mathbf{F}_2\ \cdots\ \mathbf{F}_l)$, $\mathbf{B} \doteq \mathbf{diag}(\mathbf{A}, \cdots, \mathbf{A})$, and $\mathbf{U} \doteq \mathbf{diag}(\mathbf{X}_1, \cdots, \mathbf{X}_l)$. $\mathbf{A}, \mathbf{X}$ and $\mathbf{V}$ are as defined in Section 2.1. In general, as in [12] , assuming a diversity target of mL, we need to maximize:

$$\left(\prod_{t=1}^{l} \mid \mathbf{A}(\mathbf{x}_t^o - \mathbf{x}_t^e) \mid^2\right)^{1/l} \tag{13}$$

Now suppose we use a code for the SIMO system that achieves diversity mL, then as in the ISI case, we can use the same code for the MIMO case, by mapping the scalar input x_k to the $n \times 1$ vector $\mathbf{x}_k \doteq \mathbf{b}x_k$ using the beamforming vector $\mathbf{b}$, the MIMO system gain can be written as:

$$\left(\prod_{t=1}^{l} \mid \mathbf{Ab}(x_t^o - x_t^e) \mid^2\right)^{1/l} \tag{14}$$

Therefore the coding gain over the SIMO case is $\left(\prod_{t=1}^{l} \mid \mathbf{Ab} \mid^2\right)^{1/l}$. Thus, for optimum performance, we need to

$$\begin{array}{lcl} \underset{\mathbf{b}}{\text{maximize}} \quad P(\mathbf{b}) & \doteq & \left(\prod_{p=1}^{l} \mid \mathbf{Ab} \mid^2\right)^{1/l} \\ \text{subject to} & & \parallel \mathbf{b} \parallel^2 = 1 \end{array} \tag{15}$$

The normalization $\parallel \mathbf{b} \parallel^2 = 1$ ensures the same transmitted power as the SIMO case. The solution to this problem is easily found to be the normalized eigenvectors of matrix A.

6 Simulation Results

Since the results obtained for the ISI and ISI-free cases are so different qualitatively, we report the simulation results for each case separately.

6.1 The ISI-free Case

The ideas described in Section 3 were tested by simulating a MIMO system with $m = 2, L = 2$ and different values of n. The base station was assumed to use a linear array with antenna spacing $d = 0.5$ wavelengths.
The entire MIMO transmission system was simulated by transmitting and maximum-likelihood (Viterbi) decoding, using the delay diversity code with a QPSK signal constellation. The frame length was chosen as 100, and 10,000 frames were transmitted. Two different multipath angle pairs, $\{50^o, 60^o\}$ and $\{20^o, 60^o\}$, were chosen to illustrate the performance. Figures 2,3 show the frame error rate as a function of the SNR for the cases $n = 2, 4, 8$. SNR is defined as $SNR = L\mathcal{E}_x\sigma_f^2/\sigma_n^2$, where $\mathcal{E}_x$ is the transmitted signal energy per transmission. Thus SNR is the SNR per mobile antenna per symbol, assuming omnidirectional transmission.
It can be seen that the coding gain allows better performance at a given SNR when n increases. The coding gain is significant, especially in the case of the angle pair $\{50^o, 60^o\}$, where the multipaths are not well separated in space. This again illustrates the idea that using a higher n helps to resolve multipath better.

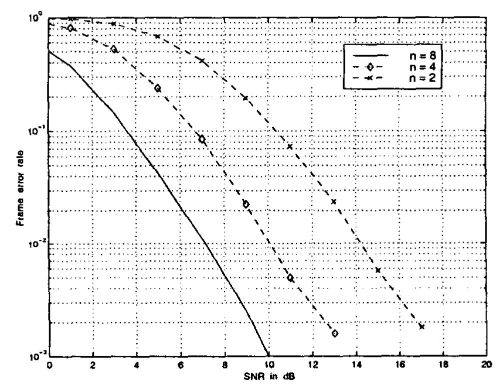

Figure 2: FER v.s. SNR; ISI-free case; angle pairs $\{20^o, 60^o\}$

6.2 The Case with ISI

As in Section 6.1, a MIMO system is simulated with $m = 2, L = 2$ and different values of n. The base station is assumed to use a linear array with antenna spacing $d = 0.5$ wavelengths. However it is assumed that one of the multipaths arrives with a one symbol delay with respect to the other, thus causing ISI.

In Section 4, we proposed a sub-optimal solution to the beamforming vector $\mathbf{b}$ that would maximize the coding gain over the SIMO case. The entire SIMO and MIMO transmission systems were simulated by transmitting and maximum-likelihood (Viterbi) decoding, using the delay diversity code with a QPSK signal constellation. The frame length was chosen as 100, and 10,000 frames were transmitted. Here we considered a multipath angle pair, different from the ones used in previous section, $\{30^o, 35^o\}$ to illustrate the performance of the system.

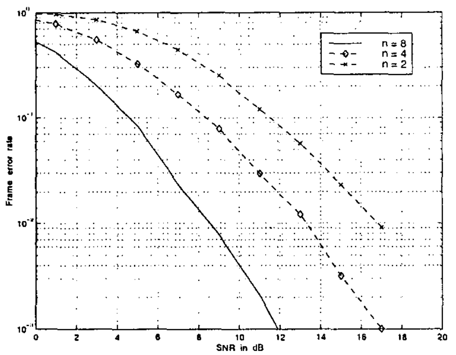

Figure 3: FER v.s. SNR; ISI-free case; angle pairs $\{50^o, 60^o\}$

Figure 4 shows the frame error rate as a function of the SNR for the above case, for $n = 1, 4, 8$. SNR is defined as in Section 6.1. It can be seen that when n increases, the coding gain allows better performance at a given SNR, even though the diversity is the same in all cases. And as discussed in Section 4, the coding gain is significant, especially in the case of the angle pair $\{30^o, 35^o\}$, where the multipaths are spaced close together.

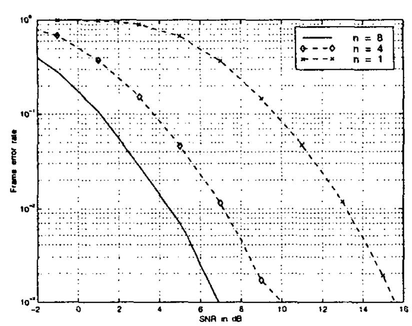

Figure 4: FER v.s. SNR; the ISI case; angle pairs $\{30^o, 35^o\}$

7 Conclusion

In this paper, we showed that the cases of an L-multipath downlink channel with or without ISI are qualitatively different, and require different approaches for optimum antenna array processing.
For the ISI-free channel, we can achieve an mL diversity gain, provided $n \geq L$ antennas are used at the base station. Further, we can get a coding gain by using an appropriate variation of beamforming. The optimal beamformer was derived, and was seen to be quite different from the classical concept. It was also shown that using a larger number of base station antennas improved the coding gain, by allowing the use of a more directive beam on the one hand, and by simultaneously allowing better resolvabilty of the multipaths on the other.
In cases of channel with ISI and rapid fading channel, even the SIMO system achieves an mL diversity gain. However, a MIMO system has additional coding gain due to beamforming. We derived a sub-optimal beamformer for these cases and presented simulation results using that for the case with ISI. As in the ISI-free case, it was shown that using a larger number of base station antennas improves the coding gain.

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