STANFORD

Supplementary Lecture 10

Other MU Channels

May 4, 2026

JOHN M. CIOFFI

Hitachi Professor Emeritus of Engineering

Instructor EE379B – Spring 2026

Agenda

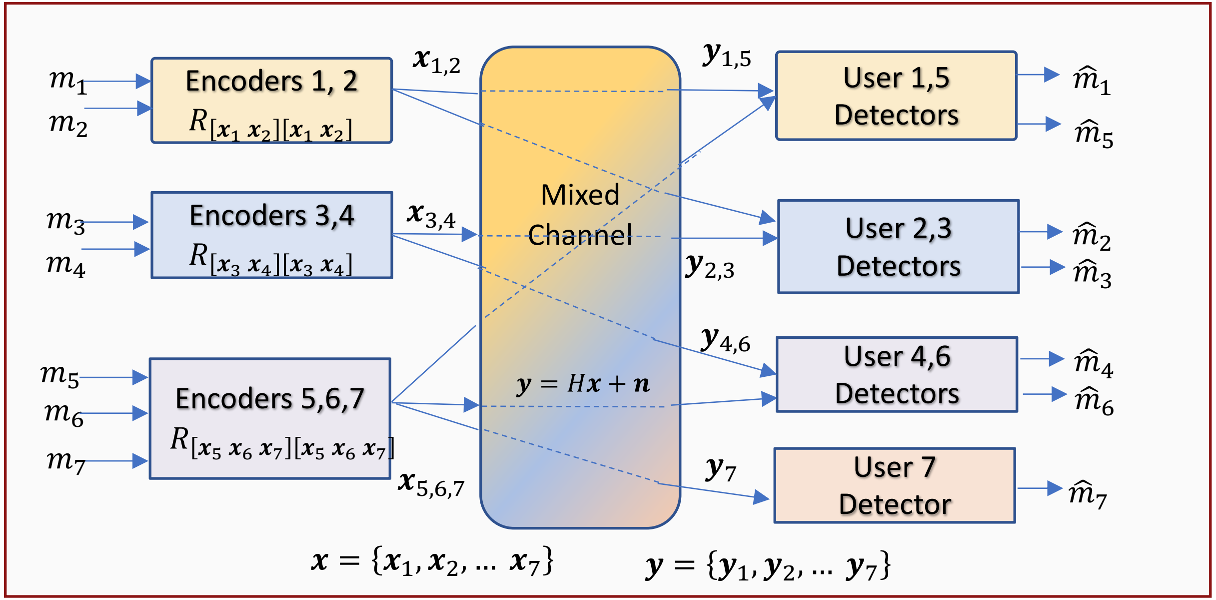
- General MU memoryless
- Relay
- Reflective Intelligent Surfaces

Much in this S10 and also Sections 2.10 and 2.11 is not yet fully developed.

There is abundant opportunity for multiple good dissertations arising from these suggestions.

Those interested (anywhere), feel free to contact me, cioffi@Stanford.edu .

General: May need all 2^{U-1} subuser atoms

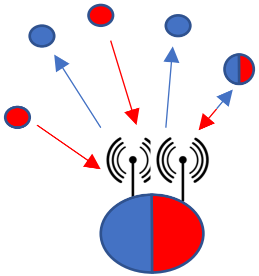


"Memoryless" Channel

$$p_{y/x} \cdot \prod_{u=1}^{U} p_x(u)$$

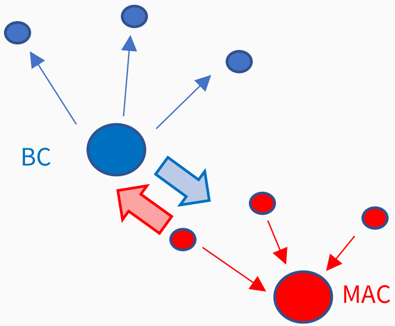
- Max is 127 atoms/user, so 869 total, but here there are user groups that reduce this.
- Likely atomization is $3 \cdot (2^4 - 1) = 45$ total atoms (3 macro transmitters and 4 macro receivers).
  - "Multiuser Nesting"

Multiuser Nesting



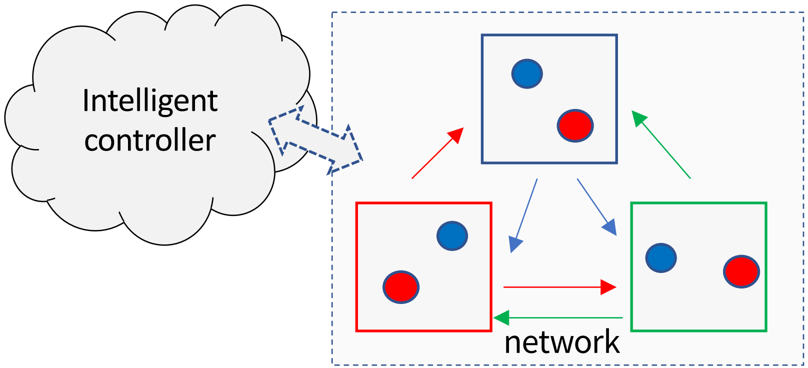
a) radio node edge
(base station or
Access point)

~



b) two subnetworks
Down (BC) and up (MAC)
IC network between them

Macro User

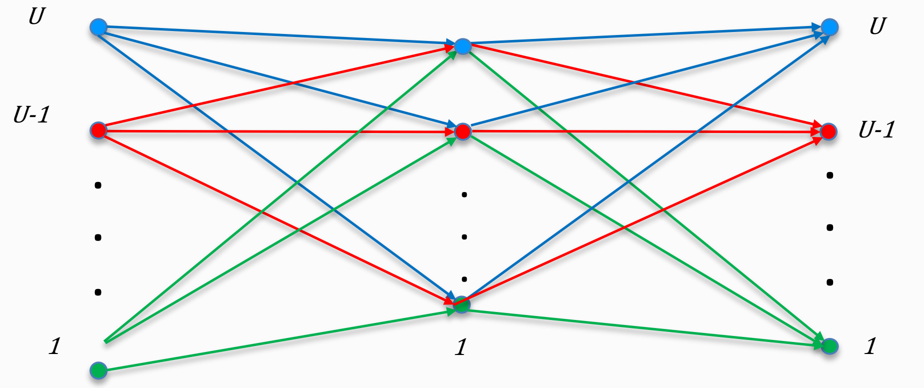


c) three nested IC: (BC, MAC)
Like those in b), nested into
3x3 IC network

Section 5.6.2.3 later
Also has
suboptimal for this.

Single-Stage (IC's) Relay Channel

- Conceptually uses what we know already and introduces sub-users at k_r relay points



$\boldsymbol{B}^{(1)}$ is $(2^{k_r}-1)\cdot U$ atoms $\boldsymbol{B}^{(2)}$ is $(2^{U}-1)\cdot k_r$ atoms

$$\mathcal{A}^{(1)}\left(\boldsymbol{B}^{(1)}, R_{\boldsymbol{xx}}^{(1)}\right)=\cup_{\boldsymbol{\Pi}^{(1)}}^{conv}\left\{\boldsymbol{B}_{min}^{(1)}\left(\boldsymbol{\Pi}^{(1)}, R_{\boldsymbol{xx}}^{(1)}\right)\right\} \triangleq \beta_k^{(1)} \qquad \mathcal{A}^{(2)}\left(\boldsymbol{B}^{(2)}, R_{\boldsymbol{xx}}^{(2)}\right)=\cup_{\boldsymbol{\Pi}^{(2)}}^{conv}\left\{\boldsymbol{B}_{min}^{(2)}\left(\boldsymbol{\Pi}^{(2)}, R_{\boldsymbol{xx}}^{(2)}\right)\right\} \triangleq \beta_u^{(2)}$$

$$b_u\left(R_{\boldsymbol{xx}}^{(1)}, R_{\boldsymbol{xx}}^{(2)}\right)=\left\{b_u \mid b_u \in \sum_{i=1}^{k_r} \min_{\left\{\beta_k^{(1)}\in\boldsymbol{\mathcal{B}}_k^{(1)} \wedge \beta_k^{(2)}\in\boldsymbol{\mathcal{B}}_k^{(2)}\right\}}\left[\beta_k^{(1)}\left(u, R_{\boldsymbol{xx}}^{(1)}\right);\beta_u^{(2)}\left(k, R_{\boldsymbol{xx}}^{(2)}\right)\right]\right\}$$

$$\mathcal{C}(\boldsymbol{b})=\sum_{u'=1}^{U}\left[\cup_{R_{\boldsymbol{xx}}^{(1)}\wedge R_{\boldsymbol{xx}}^{(2)}}^{\boldsymbol{hull}} \ \cup_{\boldsymbol{\Pi}^{(1)}\otimes\boldsymbol{\Pi}^{(2)}}^{\boldsymbol{hull}} \boldsymbol{b}\left(R_{\boldsymbol{xx}}^{(1)}, R_{\boldsymbol{xx}}^{(2)}\right)\right)\Big]_{u,u'}$$

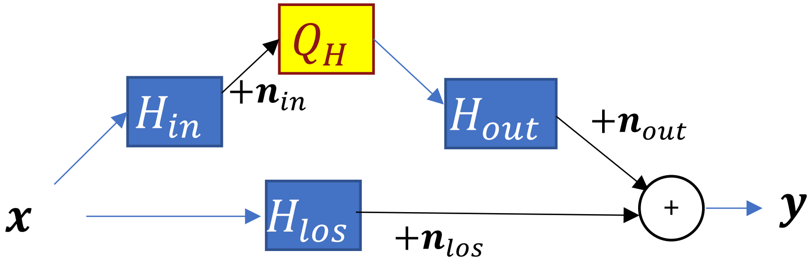
PS 5.5 (2.32) – simple relay channel

- Multi-stage tedious, but same principles apply recursively

Reflective Intelligent Surfaces (RIS)

Posed Project/Research
"maxRIS" or "minRIS"

$$\boldsymbol{y} = \underbrace{\begin{bmatrix} H_{los} \\ H_{out} \cdot Q_H \cdot H_{in} \end{bmatrix}}_{H_{RIS}} \cdot \boldsymbol{x} + \underbrace{\begin{bmatrix} \boldsymbol{n}_{los} \\ \boldsymbol{n}_{out} + Q_H \cdot \boldsymbol{n}_{in} \end{bmatrix}}_{\boldsymbol{n}_{RIS}}$$



- The RIS matrix Q_H satisfies $\|Q_H\|_F^2 \leq G_H$, the RIS gain – it may also satisfy
  - Q_H is unitary matrix (preserves energy)
  - Q_H is diagonal, and usually also unitary, to be phase/gain-only adjustment on each antenna port (in-to-out)
  - Q_H has individual elements restricted
- For a given $R_{\boldsymbol{xx}}$, maximize over Q_H

$$\mathcal{I}(\boldsymbol{y};\boldsymbol{x}) = \log_2 \left| R_{n,RIS} + H_{RIS} \cdot R_{\boldsymbol{xx}} \cdot H_{RIS}^* \right|$$

- For a given Q_H , maximize the same over $R_{\boldsymbol{xx}}$

$$R_{\boldsymbol{nn},RIS} = \begin{bmatrix} R_{\boldsymbol{nn}} & 0 \\ 0 & R_{\boldsymbol{nn},out} + Q_H \cdot R_{\boldsymbol{nn}in} \cdot Q_H^* \end{bmatrix}$$

- Cioffi postulates that $Q_H^{opt} = V_{out}^* \cdot V_{in}^*$ where
  - if $\boldsymbol{n}_{in}$ is white (noise whitening not allowed at RIS).
  - Gains are all 1 inside RIS (what Q_H implies / passive)

$$H_{in} \cdot R_{\boldsymbol{xx}} \cdot H_{in}^* + R_{\boldsymbol{n}_{in}\boldsymbol{n}_{in}} = V_{in} \cdot \Lambda_{in} \cdot V_{in}^*$$

$$R_{out}^{-1/2} \cdot H_{out} \cdot \Lambda_{in} \cdot H_{out}^* \cdot R_{out}^{-*/2} + I = V_{out} \cdot \Lambda_{out} \cdot V_{out}^*$$

End Supplementary Lecture 10